

Research Paper

IDENTIFYING CODE NUMBER OF NON-COMMUTING GRAPH OF A GROUP

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ABSTRACT. Let G be a finite non-abelian group with center $Z(G)$. The non-commuting graph of G , denoted by Γ_G , is the graph whose vertex set is $G \setminus Z(G)$, where two distinct vertices x and y are adjacent whenever $xy \neq yx$. For a graph Γ , a subset $C \subseteq V(\Gamma)$ is called an identifying code if, for every pair of distinct vertices x and y , the sets $N_\Gamma[x] \cap C$ and $N_\Gamma[y] \cap C$ are nonempty and distinct. The minimum cardinality of an identifying code in Γ is called the identifying code number and is denoted by $\gamma^{ID}(\Gamma)$. In this paper, we investigate the identifiability of non-commuting graphs associated with groups. Furthermore, for several classes of finite non-abelian groups, we determine the identifying code number of their non-commuting graphs.

1. INTRODUCTION

Throughout this paper, all graphs are assumed to be finite, simple, and undirected. For a graph Γ , the vertex set and edge set are denoted by $V(\Gamma)$ and $E(\Gamma)$, respectively. The order

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of Γ is the number of its vertices and is denoted by $|\Gamma|$. If two vertices u and v are adjacent, we write $u \sim v$.

For a vertex $x \in V(\Gamma)$, the open neighborhood of x is defined by

$$N_\Gamma(x) = \{y \in V(\Gamma) \mid x \sim y\},$$

while the closed neighborhood of x is

$$N_\Gamma[x] = N_\Gamma(x) \cup \{x\}.$$

The degree of a vertex x is denoted by $\deg_\Gamma(x)$ and equals $|N_\Gamma(x)|$. A graph is called regular if all of its vertices have the same degree.

The complement of a graph Γ , denoted by $\bar{\Gamma}$, is the graph with vertex set $V(\Gamma)$ in which two vertices are adjacent whenever they are non-adjacent in Γ .

A complete bipartite graph with parts of sizes r and s is denoted by $K_{r,s}$. More generally, a complete t -partite graph is denoted by $K_{n_1, \dots, n_t}$. For a subset $X \subseteq V(\Gamma)$, the induced subgraph on X is denoted by $\Gamma[X]$.

A subset $D \subseteq V(\Gamma)$ is called a dominating set of Γ if every vertex of Γ either belongs to D or is adjacent to a vertex in D . The domination number of Γ , denoted by $\gamma(\Gamma)$, is defined as

$$\gamma(\Gamma) = \min\{|D| : D \text{ is a dominating set of } \Gamma\}.$$

A subset $C \subseteq V(\Gamma)$ is called an identifying code of Γ if C is a dominating set and, for every pair of distinct vertices $u, v \in V(\Gamma)$,

$$N_\Gamma[u] \cap C \neq N_\Gamma[v] \cap C.$$

A graph admitting an identifying code is said to be identifiable. The identifying code number of Γ is defined by

$$\gamma^{ID}(\Gamma) = \min\{|C| : C \text{ is an identifying code of } \Gamma\}.$$

Two distinct vertices u and v are called twins whenever

$$N_\Gamma[u] = N_\Gamma[v].$$

It is well known that a graph is identifiable if and only if it is twin-free.

In recent years, domination theory and its variations have attracted considerable attention in graph theory. Several domination-related parameters, including rainbow domination, signed domination, and identifying codes, have been extensively studied. For further details, see [2, 13, 16, 17, 19].

The concept of identifying codes was introduced by Karpovsky et al. [12] in 1998. Since then, identifying codes have been investigated for many classes of graphs such as paths, cycles, and other graph families [6, 10].

Let G be a group and let $x \in G$. We denote by $Z(G)$ the center of G and by $C_G(x)$ the centralizer of x in G .

The symmetric and alternating groups of degree n are denoted by S_n and A_n , respectively. Moreover,

$$Q_8 = \langle a, b \mid a^4 = 1, a^2 = b^2, b^{-1}ab = a^{-1} \rangle,$$

and

$$D_{2n} = \langle a, b \mid a^n = b^2 = (ab)^2 = 1 \rangle,$$

denote the quaternion group of order 8 and the dihedral group of order $2n$ ($n > 2$), respectively. For a non-abelian group G , the non-commuting graph of G , denoted by Γ_G , is defined by

$$V(\Gamma_G) = G \setminus Z(G),$$

where two distinct vertices x and y are adjacent if and only if $xy \neq yx$.

The non-commuting graph was first introduced by ErdHos in 1975 [14]. Over the past several decades, graph-theoretic approaches to algebraic structures have become an active area of research. In particular, several graphs associated with groups and rings have been studied in order to better understand their algebraic properties; see, for example [1, 4, 5, 8, 9, 15, 18].

The paper is organized as follows. In Section 2, we establish several preliminary results required in the sequel. In Section 3, we derive a lower bound for the identifying code number of the non-commuting graph of a group and determine $\gamma^{ID}(\Gamma_{Q_8})$. Finally, in Section 4, we compute the identifying code number for some special classes of finite groups.

2. PRELIMINARIES

In this section, we will point out some facts that will be needed in the following sections.

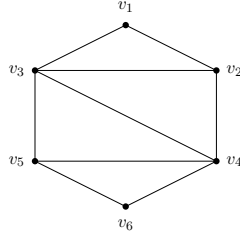
Definition 2.1. For two graphs $\Gamma_1 = (V_1, E_1)$ and $\Gamma_2 = (V_2, E_2)$, $\Gamma_1 \bowtie \Gamma_2$ is the join graph of Γ_1 and Γ_2 . Its vertex set is $V_1 \cup V_2$ and its edge set is $E_1 \cup E_2 \cup \{v_1v_2 \mid v_1 \in V_1, v_2 \in V_2\}$.

Definition 2.2. For an integer $k \geq 1$, let $A_k = (V_k, E_k)$ be the graph with vertex set $V_k = \{v_1, \dots, v_{2k}\}$ and edge set $E_k = \{v_iv_j \mid |i - j| \leq k - 1\}$.

Example 2.3. An illustration of the graph A_3 is given in Figure 1.

Theorem 2.4. [7] *Let $\mathcal{A}$ be the closure of $\{A_i \mid i = 1, 2, \dots\}$ with respect to operation $\bowtie$. Given a connected graph Γ , we have $\gamma^{ID}(\Gamma) = |V(\Gamma)| - 1$ if and only if $\Gamma \in \{K_{1,t} \mid t \geq 2\} \cup \mathcal{A} \cup (\mathcal{A} \bowtie K_1)$ and $\Gamma \not\cong A_1$.*

Theorem 2.5. [12] *Let Γ be an identifiable graph. Then $\gamma^{ID}(\Gamma) \geq \lceil \log_2(|\Gamma| + 1) \rceil$.*

FIGURE 1. The graph A_3 .

Theorem 2.6. [3] Let $n_i \geq 3$ and $\Gamma \cong K_{n_1, n_2, \dots, n_k}$ be a complete k -partite graph. Then $\gamma^{ID}(\Gamma) = n - k$, where $n = \sum_{i=1}^k n_i$.

Lemma 2.7. Let Γ be a graph and $X \subseteq V(\Gamma)$ with $|X| \geq 2$. Also let the induced subgraph on X be empty and $N_G(x) = N_G(x')$, for every $x, x' \in X$. If C is an identifying code of G with minimum cardinality, then $|X \cap C| \geq |X| - 1$.

Proof. Let $x, x' \in X$ and $\{x, x'\} \cap C = \emptyset$. Then $N_G[x] \cap C = N_G[x'] \cap C$. Which is a contradiction.

□

Lemma 2.8. Let Γ be a graph and C be an identifying code of Γ . If $N_\Gamma[x] \Delta N_\Gamma[y] = B$, then $|B \cap C| \geq 1$.

Proof. If $|B \cap C| = 0$, then $N_\Gamma[x] \cap C = N_\Gamma[y] \cap C$, which is not true. □

Lemma 2.9. Let G be a non-abelian group. Then $|Z(G)| \mid |\Gamma_G|$ and $\frac{|\Gamma_G|}{|Z(G)|} + 1$ is not a prime number.

Proof. Let $|Z(G)| = 1$ and $\frac{|\Gamma_G|}{|Z(G)|} + 1$ be a prime number. Then $|G|$ is a prime number and so G is an abelian group. Which is a contradiction.

Let $|Z(G)| \neq 1$. By the definition of the non-commuting graph a group, $|Z(G)| + |\Gamma_G| = |G|$. Since $|Z(G)| \mid |G|$, so $|Z(G)| \mid |\Gamma_G|$.

If $\frac{|\Gamma_G|}{|Z(G)|} + 1$ is a prime number, then $\frac{|\Gamma_G| + |Z(G)|}{|Z(G)|}$ is a prime number. Thus $\frac{G}{Z(G)}$ is a cyclic group and so G is an abelian group. Which is a contradiction. □

Lemma 2.10. Let $\Gamma \cong K_{n_1, \dots, n_{t-1}, n_t}$ be a complete t -partite graph such that $n_1 = \dots = n_{t-1} = 2$ and $n_t \geq 3$. Then $\gamma^{ID}(\Gamma) = n - 2$, where $n = \sum_{i=1}^t n_i$.

Proof. Let for every i , $1 \leq i \leq t-1$, $X_i = \{v_{i1}, v_{i2}\}$ and $X_t = \{v_{t1}, \dots, v_{tn_t}\}$ be t partitions of Γ .

Also let C be an identifying code of Γ with $\gamma^{ID}(\Gamma) = |C|$. By Lemma 2.7, $|X_i \cap C| \geq 1$, for every i , $1 \leq i \leq t-1$ and $|X_t \cap C| \geq r-1$.

Without loss of generality, we can assume that for every i , $1 \leq i \leq t-1$, $v_{i2} \in C$ and $\{v_{t2}, \dots, v_{tr}\} \subseteq C$.

Let $\{v_{i1}, v_{j1}\} \cap C = \emptyset$, for some $1 \leq i, j \leq t-1$. Then $N_\Gamma[v_{i2}] \cap C = N_\Gamma[v_{j2}] \cap C$, which is not true. Thus $|C| \geq n-2$ and so $\gamma^{ID}(\Gamma) \geq n-2$.

Let $D = V(\Gamma) \setminus \{v_{11}, v_{t1}\}$. Then

$$N_\Gamma[v_{11}] \cap D = D \setminus \{v_{12}\},$$

$$N_\Gamma[v_{12}] \cap D = D,$$

$$N_\Gamma[v_{i1}] \cap D = D \setminus \{v_{i2}\}, \text{ for every } i, 2 \leq i \leq t-1,$$

$$N_\Gamma[v_{i2}] \cap D = D \setminus \{v_{i1}\}, \text{ for every } i, 2 \leq i \leq t-1,$$

$$N_\Gamma[v_{t1}] \cap D = D \setminus X_t,$$

$$N_\Gamma[v_{ti}] \cap D = (D \setminus X_t) \cup \{v_{ti}\}, \text{ for every } i, 2 \leq i \leq r.$$

Hence $N_\Gamma[x] \cap D \neq N_\Gamma[y] \cap D$, for every $x, y \in \Gamma$. So D is an identifying code of Γ . Thus $\gamma^{ID}(\Gamma) \leq |D| = n-2$. Therefore, $\gamma^{ID}(\Gamma) = n-2$. $\square$

3. IDENTIFYING CODE NUMBER OF NON-COMMUTING GRAPH OF A GROUP

In this section, we obtain a lower bound for the identifying code number of non-commuting graph of a group by using its center. If G is a group of order n and H is a subgroup of G of order $\ell > \frac{n}{2}$, then $G = H$.

Theorem 3.1. *Let G be a non-abelian group.*

- i) *If a and b are twins in Γ_G , then $o(a) = o(b) = 2$.*
- ii) *If Γ_G has twin vertices, then $|Z(G)| = 1$.*

Proof. i) Let two vertices a and b be twin in Γ_G . Then a is adjacent to b in Γ_G . Thus $ab \neq ba$. Let $a \neq a^{-1}$. Since $a \notin Z(G)$, so $a^{-1} \notin Z(G)$. From $aa^{-1} = a^{-1}a$, we imply that a is not adjacent to a^{-1} in Γ_G . Thus $a^{-1} \notin N_{\Gamma_G}[a]$. Since $N_{\Gamma_G}[a] = N_{\Gamma_G}[b]$, so $a^{-1} \notin N_{\Gamma_G}[b]$. Hence $a^{-1}b = ba^{-1}$. It contradicts with this fact that $ab \neq ba$. Therefore, $o(a) = 2$. Similarly, we have $o(b) = 2$.

- ii) Let x and y be two distinct vertices of Γ_G such that $N_{\Gamma_G}[x] = N_{\Gamma_G}[y]$. Then $xy \neq yx$.
 By the definition of the non-commuting graph a group, $C_G(x) \setminus \{x\} = C_G(y) \setminus \{y\}$.
 Let $|Z(G)| \neq 1$ and $L = C_G(x) \cap C_G(y)$. Then $|L| \geq 2$ and $|C_G(x)| = |L| + 1 = |C_G(y)|$.
 Since $|L| > \frac{|L|+1}{2}$, $|C_G(x) \cap C_G(y)| > \frac{|C_G(x)|}{2}$. From $C_G(x) \cap C_G(y) \leq C_G(x)$ and $|C_G(x) \cap C_G(y)| > \frac{|C_G(x)|}{2}$, imply $C_G(x) \cap C_G(y) = C_G(x)$. Thus $x \in C_G(y)$ and so $xy = yx$. Which is a contradiction. Therefore, $|Z(G)| = 1$.

□

From Theorem 3.1, we conclude that if the group G is non abelian and $|Z(G)| \geq 2$, then Γ_G is an identifiable graph.

Theorem 3.2. *Let G be a non-abelian group of order n and $|Z(G)| = t \geq 2$. Then*

$$\gamma^{ID}(\Gamma_G) \geq \frac{|\Gamma_G|(|Z(G)| - 1)}{|Z(G)|}.$$

Proof. Let $[G : Z(G)] = \frac{n}{t} = d$ and $\frac{G}{Z(G)} = \{a_i Z(G) \mid 1 \leq i \leq d, a_1 = 1\}$. For every $z_1, z_2 \in Z(G)$, $(a_i z_1)(a_i z_2) = (a_i z_2)(a_i z_1)$. Thus the induced subgraphs on $a_i Z(G)$ in Γ_G are isomorphic to $\overline{K_t}$, for every $2 \leq i \leq d$.

For every i and j ($2 \leq i, j \leq d$), $(a_i z_1)(a_j z_2) = (a_j z_2)(a_i z_1)$ if and only if $a_i a_j = a_j a_i$, where z_1 and z_2 are two arbitrary elements of $Z(G)$. Thus for every $z \in Z(G)$, we have $N_{\Gamma_G}(a_i z) = N_{\Gamma_G}(a_i)$.

Let C be an identifying code of Γ_G with $\gamma^{ID}(\Gamma_G) = |C|$. By Lemma 2.7, $|C \cap a_i Z(G)| \geq t - 1$. Thus $|C| \geq (d - 1)(t - 1) = (\frac{n}{t} - 1)(t - 1) = \frac{(n-t)(t-1)}{t}$. Therefore, $\gamma^{ID}(\Gamma_G) \geq \frac{|\Gamma_G|(|Z(G)| - 1)}{|Z(G)|}$. □

Proposition 3.3. *Let G be isomorphic to Q_8 or D_8 . Then Γ_G is shown in Figure 2 and $\gamma^{ID}(\Gamma_G) = 5$.*

Proof. Since $\Gamma_{Q_8} \cong \Gamma_{D_8} \cong A_1 \bowtie A_1 \bowtie A_1$ and by Theorem 2.4, the proof is straightforward.

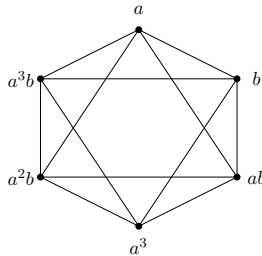


FIGURE 2. Γ_{Q_8} .

□

Corollary 3.4. *Let G be a non-abelian group of order n .*

- i) *If $|Z(G)| = 2$, then $\gamma^{ID}(\Gamma_G) \geq 5$.*
- ii) *If $|Z(G)| \geq 3$, then $\gamma^{ID}(\Gamma_G) \geq 6$.*

Proof. i) Let $|Z(G)| = 2$.

If $|\frac{G}{Z(G)}| \in \{2, 3, 5\}$, then by [11, page 83], G is an abelian group. Which is a contradiction.

If $|\frac{G}{Z(G)}| = 4$, then $|G| = 8$. Thus $G \cong Q_8$ or $G \cong D_8$. By Proposition 3.3, $\gamma^{ID}(\Gamma_G) = 5$.

If $|\frac{G}{Z(G)}| \geq 6$, then by Theorem 3.2, $\gamma^{ID}(\Gamma_G) \geq 5$.

ii) Since G is a non-abelian group, $|\frac{G}{Z(G)}| \geq 4$. By Theorem 3.2, $\gamma^{ID}(\Gamma_G) \geq 6$.

□

4. IDENTIFYING CODE NUMBER OF SPECIAL GROUPS

In this section, we obtain identifying code number of special groups and also we prove D_{2n} is an identifiable graph if n is even.

Theorem 4.1. *Let G be a non-abelian group of order p^3 (p is an odd prime number). Then $\gamma^{ID}(\Gamma_G) = p^3 - 2p - 1$.*

Proof. By [11, page 103], $|Z(G)| \neq 1$ and since G is non-abelian group $|Z(G)| \neq p^3$. By Theorem 3.1, Γ_G is an identifiable graph. If $|Z(G)| = p^2$, then $|\frac{G}{Z(G)}| = p$ and so G is an abelian group, which is a contradiction. Hence $|Z(G)| = p$ and $\frac{G}{Z(G)} \cong \mathbb{Z}_P \times \mathbb{Z}_P$.

Let $g_1 \in G \setminus Z(G)$, $A_1 = \langle g_1 Z(G) \rangle$ and $\hat{A}_1 = \bigcup_{\ell=1}^{p-1} g_1^\ell Z(G)$. Then $\frac{G}{Z(G)} \neq A_1$ and $G \setminus Z(G) \neq \hat{A}_1$.

Thus there is $g_2 \in G \setminus (\hat{A}_1 \cup Z(G))$.

Let $A_2 = \langle g_2 Z(G) \rangle$ and $\hat{A}_2 = \bigcup_{\ell=1}^{p-1} g_2^\ell Z(G)$. Then $\frac{G}{Z(G)} \neq A_1 \cup A_2$, $A_1 \cap A_2 = \{Z(G)\}$ and $G \setminus Z(G) \neq \hat{A}_1 \cup \hat{A}_2$. So there is $g_3 \in G \setminus (\hat{A}_1 \cup \hat{A}_2 \cup Z(G))$.

Let $A_3 = \langle g_3 Z(G) \rangle$ and $\hat{A}_3 = \bigcup_{\ell=1}^{p-1} g_3^\ell Z(G)$. Then $\frac{G}{Z(G)} \neq A_1 \cup A_2 \cup A_3$, $A_1 \cap A_2 = A_1 \cap A_3 = A_2 \cap A_3 = \{Z(G)\}$ and $G \setminus Z(G) \neq \hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3$.

Since $|\frac{G}{Z(G)}| = p^2$, we have $\frac{G}{Z(G)} = \bigcup_{i=1}^{p+1} A_i$ and $G \setminus Z(G) = \bigcup_{i=1}^{p+1} \hat{A}_i$, where $A_i = \langle g_i Z(G) \rangle$,

$\hat{A}_i = \bigcup_{\ell=1}^{p-1} g_i^\ell Z(G)$, $A_i \cap A_j = \{Z(G)\}$ and $g_j \in G \setminus (\bigcup_{\ell=1}^{j-1} \hat{A}_\ell \cup Z(G))$ ($1 \leq j \leq p+1$).

For every $x, y \in \hat{A}_i$, we have $xy = yx$. Hence the induced subgraphs on $\hat{A}_i$ is empty graph.

On the other hand for every $x \in \hat{A}_i$, we have $\hat{A}_i \cup Z(G) \subseteq C_G(x)$ and so $|C_G(x)| \geq p^2$. Since $x \notin Z(G)$, $|C_G(x)| = p^2$. Thus $C_G(x) \setminus Z(G) = \hat{A}_i$. Hence for every $x \in \hat{A}_i$ and $y \in \hat{A}_j$, we have

$xy \neq yx$. Hence x is adjacent to y in Γ_G . Therefore, Γ_G is isomorphic to complete $p+1$ -partite graph $K_{p^2-p, \dots, p^2-p}$. By Theorem 2.6, $\gamma^{ID}(\Gamma_G) = (p^3 - p) - (p + 1) = p^3 - 2p - 1$. $\square$

Theorem 4.2. *Let G be a non-abelian group of order pq , where $3 \leq p < q$ and p, q are prime number. Then $p \mid (q - 1)$ and $\gamma^{ID}(\Gamma_G) = \begin{cases} 3q - 3, & p = 3, \\ q(p - 1) - 2, & p \neq 3. \end{cases}$*

Proof. If $p \nmid (q - 1)$, then G is isomorphic to cyclic group $\mathbb{Z}_{pq}$, (see [11, page 91]). Which is not true. Let $H = \langle a \rangle$ and $K = \langle b \rangle$ be a Sylow p -subgroup and a Sylow q -subgroup of G , respectively. We have $n_q(G) = 1 + kq \mid p$ and so $n_q(G) = 1$. Thus $K \triangleleft G$.

If $|Z(G)| \neq 1$, then $|Z(G)| = p$ or $|Z(G)| = q$. However $\frac{G}{Z(G)}$ is a cyclic group and so G is an abelian group, which is a contradiction. Thus $|Z(G)| = 1$.

Also for every $x \in G \setminus K$, $|C_G(x)| \mid pq$. If $|C_G(x)| = q$, then $C_G(x) = K$ and so $x \in K$, which is a contradiction. Hence $|C_G(x)| = p$. It is easy to see that for every $x, y \in G \setminus K$, we have $C_G(x) = C_G(y)$ or $C_G(x) \cap C_G(y) = \{1\}$.

Let $X = \{x_1, x_2, \dots, x_t\}$ be the largest set such that $C_G(x_i) \neq C_G(x_j)$ for every $1 \leq i, j \leq t$.

Then $t = q$ and $G = (\bigcup_{i=1}^q C_G(x_i)) \cup K$.

Also for every $1 \leq i \leq q$, the induced subgraphs on $C_G(x_i) \setminus \{1\}$ are isomorphic to $\overline{K_{p-1}}$ and the induced subgraph on $K \setminus \{1\}$ is isomorphic to $\overline{K_{q-1}}$. On the other hand for every $1 \leq i, j \leq q$, since $C_G(x_i) \cap C_G(x_j) = \{1\}$ and $C_G(x_i) \cap K = \{1\}$, so Γ_G is isomorphic to complete $q+1$ -partite graph $K_{p-1, p-1, \dots, p-1, q-1}$.

If $p = 3$, then $\Gamma_G \cong K_{2, 2, \dots, 2, q-1}$. By Lemma 2.10, $\gamma^{ID}(\Gamma_G) = 3q - 3$.

If $p \neq 3$, then $\Gamma_G \cong K_{p-1, p-1, \dots, p-1, q-1}$. By Theorem 2.6, $\gamma^{ID}(\Gamma_G) = pq - (q + 2)$. $\square$

Theorem 4.3. *The non-commuting graph of Dihedral group D_{2n} is not an identifiable graph if and only if n is an odd number.*

Proof. Let n be an odd number. Then $C_{D_{2n}}(b) = \{1, b\}$ and $C_{D_{2n}}(ab) = \{1, ab\}$. Hence $N_{\Gamma_{D_{2n}}}[b] = N_{\Gamma_{D_{2n}}}[ab]$. Therefore, D_{2n} is not an identifiable graph.

Conversely, let n be even. Then $Z(G) \neq \{1\}$ and by Theorem 3.1, $\Gamma_{D_{2n}}$ is an identifiable graph. $\square$

Theorem 4.4. *Let $G \cong D_{2n}$, $n = 2m$ and $m \geq 2$.*

- i) *If $n = 4$, then $\gamma^{ID}(\Gamma_G) = 5$.*
- ii) *If $n \geq 6$, then $\gamma^{ID}(\Gamma_G) = 2n - 4$.*

Proof. We know if n is even, then by [11, page 47], $|Z(D_{2n})| = 2$. By Theorem 3.1 (ii), Γ_G is an identifiable graph.

i) By Proposition 3.3, $\gamma^{ID}(\Gamma_{D_8}) = 5$.

ii) For $i < m$, we have $C_G(a^i) = \langle a \rangle$ and for every $i, 1 \leq i \leq m$, $C_G(a^i b) = \{1, a^m, a^i b, a^{m+i} b\}$.

Suppose that $A = \langle a \rangle \setminus \{1, a^m\}$ and for every $i, 1 \leq i \leq m$, $B_i = \{a^i b, a^{m+i} b\}$.

Then $V(\Gamma_G) = A \cup (\bigcup_{i=1}^m B_i)$. For every $i, 1 \leq i \leq m$, the induced subgraphs on A and B_i are $\overline{K_{n-2}}$ and $\overline{K_2}$, respectively.

For every $i, 1 \leq i \leq m$, $N_{\Gamma_G}(a^i b) = G \setminus \{1, a^m, a^i b, a^{m+i} b\}$ and $N_{\Gamma_G}(a^{m+i} b) = G \setminus \{1, a^m, a^i b, a^{m+i} b\}$. Also for every $j, 1 \leq j < n, j \neq m$, $N_{\Gamma_G}(a^j) = G \setminus \langle a \rangle$. So Γ_G is isomorphic to $K_{2,2,\dots,2,n-2}$. (See Fig 3). By Lemma 2.10, $\gamma^{ID}(\Gamma_G) = |\Gamma_G| - 2 = (2n - 2) - 2 = 2n - 4$.

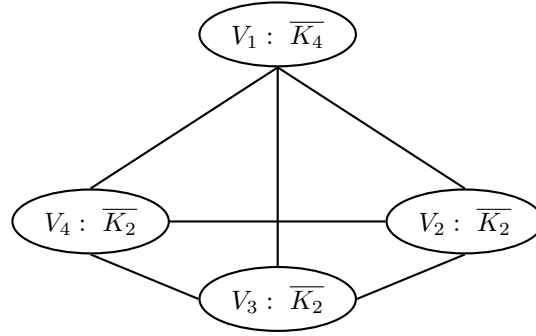


FIGURE 3. $\Gamma_{D_{12}}$.

Bold line: Every vertex of the set is adjacent to every vertex of the other set.

□

For study about symmetric group S_n , see [11, page 108].

Theorem 4.5. Γ_{S_n} is a nonempty identifiable graph if and only if $n \geq 4$.

Proof. If $n = 3$, then $N_{\Gamma_{S_3}}[(1, 2)] = S_3 \setminus \{(1)\} = N_{\Gamma_{S_3}}[(1, 3)]$. Thus Γ_{S_3} is not an identifiable graph.

Let $n \geq 4$ and assume that Γ_{S_n} is not an identifiable graph. Then there are two permutation σ and δ in S_n such that $N_{\Gamma_{S_n}}[\sigma] = N_{\Gamma_{S_n}}[\delta]$. Thus $C_{S_n}(\sigma) \setminus \{1, \sigma\} = C_{S_n}(\delta) \setminus \{1, \delta\}$. By Theorem 3.1, we have $\sigma = (i_1, i_2)(i_3, i_4) \dots (i_{r-1}, i_r)$ and $\delta = (j_1, j_2)(j_3, j_4) \dots (j_{s-1}, j_s)$, where $|\{i_k \mid 1 \leq$

$k \leq r\} = r$ and $|\{j_k \mid 1 \leq k \leq s\}| = s$. Since $\sigma\delta \neq \delta\sigma$, so $\{i_k \mid 1 \leq k \leq r\} \cap \{j_k \mid 1 \leq k \leq s\} \neq \emptyset$. Thus there exist $2 \leq \ell \leq r$ and $2 \leq t \leq s$ such that $|\{i_{\ell-1}, i_\ell\} \cap \{j_{t-1}, j_t\}| = 1$. Without loss of generality we can assume that $i_{\ell-1} = j_{t-1}$. It is easy to see that $(i_{\ell-1}, i_\ell)\sigma = \sigma(i_{\ell-1}, i_\ell)$ and $(i_{\ell-1}, i_\ell)\delta \neq \delta(i_{\ell-1}, i_\ell)$. Thus $(i_{\ell-1}, i_\ell) \in C_{S_n}(\sigma) \setminus \{1, \sigma\}$ and $(i_{\ell-1}, i_\ell) \notin C_{S_n}(\delta) \setminus \{1, \delta\}$. It is in contradiction with the fact that $C_{S_n}(\sigma) \setminus \{1, \sigma\} = C_{S_n}(\delta) \setminus \{1, \delta\}$. $\square$

Let $T = \langle a, b \mid a^6 = 1, a^3 = b^2, b^{-1}ab = a^{-1} \rangle$. It is well known that if G is a non-abelian group of order 12, then $G \cong D_{12}$ or $G \cong A_4$ or $G \cong T$.

Also every non-abelian group of order $2p$ is isomorphic to D_{2p} , where p is an odd prime number.

Proposition 4.6. *If G is isomorphic to the group A_4 , then $\gamma^{ID}(\Gamma_G) = 9$.*

Proof. We know that $Z(A_4) = (1)$ and

$$A_4 = \{(1), (12)(34), (13)(24), (14)(23), (123), (132), (124), (142), (134), (143), (234), (243)\}.$$

Then

$$C_{A_4}((12)(34)) = \{(1), (12)(34), (13)(24), (14)(23)\} = V_1,$$

$$C_{A_4}((123)) = \{(1), (132), (123)\} = V_2,$$

$$C_{A_4}((124)) = \{(1), (124), (142)\} = V_3,$$

$$C_{A_4}((134)) = \{(1), (134), (143)\} = V_4,$$

$$C_{A_4}((234)) = \{(1), (234), (243)\} = V_5.$$

By the definition of non-commuting graph, $\Gamma_{A_4}[V_1] \cong \overline{K_3}$ and $\Gamma_{A_4}[V_i] \cong \overline{K_2}$ for every $2 \leq i \leq 5$. Also, every $x \in V_i$ is adjacent to every $y \in V_j$, for every $1 \leq i \neq j \leq 5$. Hence, Γ_{A_4} is isomorphic to complete 5-partite graph $K_{2,2,2,2,3}$. By Lemma 2.10, $\gamma^{ID}(\Gamma_{A_4}) = 9$.

$\square$

Theorem 4.7. *Let G be a finite non-abelian group and Γ_G be an identifiable graph. Then $\gamma^{ID}(\Gamma_G) \geq 5$.*

Proof. Let $\gamma^{ID}(\Gamma_G) = 1$. By Theorem 2.5, $|\Gamma_G| = 1$. Hence $|G| = 2$. Which is impossible.

Let $\gamma^{ID}(\Gamma_G) = 2$. By Theorem 2.5, $|\Gamma_G| \leq 3$. By Corollary 3.4, $|Z(G)| = 1$ and so $|G| \in \{3, 4\}$.

Thus G is an abelian group, which is false.

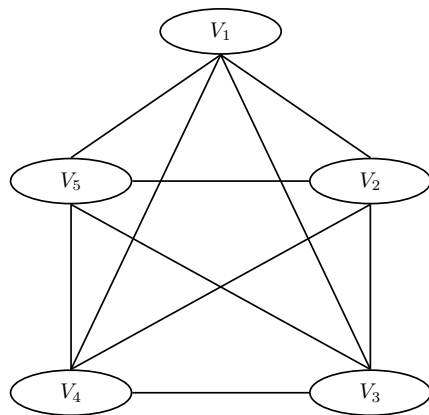
Let $\gamma^{ID}(\Gamma_G) = 3$. By Theorem 2.5, $|\Gamma_G| \leq 7$. By Corollary 3.4, $|Z(G)| = 1$. Thus $|G| \leq 8$.

Since G is not an abelian group, so $|G| \in \{6, 8\}$.

If $|G| = 6$. By Theorem 4.3, Γ_G is not an identifiable graph.

If $|G| = 8$, then $G \cong Q_8$ or $G \cong D_8$ and so $|Z(G)| = 2$. It is not true.

Let $\gamma^{ID}(\Gamma_G) = 4$. By Theorem 2.5, $|\Gamma_G| \leq 15$. By Corollary 3.4, $|Z(G)| = 1$. Thus $|G| \leq 16$.

FIGURE 4. Γ_{A_4} .

Bold line: Every vertex of the set is adjacent to every vertex of the other set.

Since G is not an abelian group, so $|G| \in \{6, 8, 10, 12, 14\}$.

If $|G| = 8$, then $G \cong Q_8$ or $G \cong D_8$ and so $|Z(G)| = 2$. It is not true.

If $|G| \in \{6, 10, 14\}$. By Theorem 4.3, Γ_G is not an identifiable graph.

If $|G| = 12$, then $G \cong D_{12}$ or $G \cong A_4$ or $G \cong T$. Since $|Z(G)| = 1$, $|Z(D_{12})| \neq 1$ and $|Z(T)| \neq 1$, so G just isomorphic to A_4 . By Proposition 4.6, $\gamma^{ID}(\Gamma_{A_4}) = 9$. Which is a contradiction. Therefore, $\gamma^{ID}(\Gamma_G) \geq 5$. $\square$

5. CONCLUSION

In this paper, we have studied the identifying code number of the non-commuting graph of a non-abelian group. In Theorem 3.1, we have shown that twin vertices occur only among the elements of the group of order two. Also we have proved that if $|Z(G)| \neq 1$, then Γ_G is not an identifiable graph. In Theorem 3.2, we have established a lower bound for Γ_G . In Theorems 4.3 and 4.4, we have investigated the identifiability and the identifying code number of $\Gamma_{D_{2n}}$. In Theorem 4.5, we have proved that if $n \geq 4$, then Γ_{S_n} is an identifiable graph. Finally, in Theorem 4.7, we have shown that $\gamma^{ID}(\Gamma_G) \geq 5$.

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