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Research Paper

NOVEL NUMERICAL FORMULAS FOR ASYMMETRIC SCALES (17- AND 24-NOTE SYSTEMS) FOR PSYCHOLOGICAL EFFECTS IN MUSIC THERAPY

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Abstract. This research employs numerical rules and algebraic calculations within 17- and 24-note systems to identify the temperaments and zodiac signs associated with musical scales, linking them to specific psychological states and effects. It introduces new computational formulas for three types of asymmetric scales: those with two Shāhed notes, those derived from other modes, and those featuring a variable note. The Āvāzs of Iranian classical music serve as a case study to test the practical use of these formulas. Since these Āvāzs use quarter-tone intervals, the analysis is performed using both the 17- and 24-note numerical systems. The matching results in both systems confirm the strength of the proposed method. Importantly, analyzing the Iranian Radif with these 17- and 24-note systems shows that the common intervals among Dastgāhs and Āvāzs do not necessarily mean they come from a specific main mode. The results indicated that, contrary to the theory of some Iranian theorists, the Āvāzes of Abūatā, Bayāt-e Tork, Afshāri, and Dashti are each independent modes and are not derived from the Dastgāh of Shūr. Instead, their classification and the structure of the Radif is based on their shared connection to fundamental human temperaments. This finding highlights the view of Iranian scholars who focused on using music to support physical and mental health.

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1. Introduction

Scientific music is a mathematical field that studies the harmony and discord of notes, as well as the timing and rhythm of melodies, to understand the principles of composition and musical structure [10]. Ancient Iranian and Greek musicians sought to connect musical modes to the zodiac by drawing on astronomical data and the astrological calculations established by their contemporaries [17]. Throughout history, specific astronomical ideas have greatly influenced the progress of science and culture in different civilizations. The zodiac is a key example. Defined as a celestial band that contains the constellations through which the Moon and planets seem to move, it played a crucial role in the astronomical systems of ancient Mesopotamia, ancient Greece, and the Islamic world [12]. The zodiac is divided into twelve equal parts called signs: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, and Pisces [6]. The zodiac is linked to the four classical elements (water, earth, wind, fire), which are believed to influence bodily temperament and the humoral balance (blood, phlegm, yellow bile, black bile) [7]. The human constitution is understood to be governed by the balance of four humors: blood, phlegm, yellow bile, and black bile, which collectively shape the individual's overall temperament. While two are classified as warm and two as cold, they differ in secondary qualities: one warm humor is dry, the other moist; one cold humor is dry, the other moist [18]. The concept of "temperament," derived from the Iranian meaning of "blending," is a fundamental principle in traditional Iranian medicine. It refers to a composite state or condition that results from the combined and opposing influences of the four elemental qualities, water, earth,wind, and fire [11].

Within the musical framework of humoral theory, specific pitch ranges are posited to possess distinct elemental qualities and corresponding physiological effects. The high-pitched, or treble, the register is characterized as warm and dry. It exerts a warming influence on the individual's temperament and serves to moderate an excessively coarse or harsh nature. Conversely, the low-pitched, or bass, register is categorized as cold and moist. Its primary effect is a cooling of the temperament, rendering it particularly beneficial and pleasing for individuals of a warm and dry constitution. Music that occupies a mid-range frequency, balancing these extremes, is held to promote equilibrium and maintain balance across all four temperaments [6].

In his work Behjat ol-Rooh (Happiness of the Soul), Safi al-Din Ormavi (d. 1294) pointed out the astro-musical correlations between the twelve Iranian Maqāms and zodiac constellations.[1] He related each Maqām to a quote from a past musician in a diagram. Also specifies the seasons and months suitable for the authorities of Iranian music. For example, what should be played in spring, when the sun is in Aries, Taurus, or Gemini, and in summer, when the sun is

[1]. Please see appendices I.

in Cancer, Leo, and Virgo, what tunes are used, and in the same way, he listed other seasons and Maqāms [19]. According to natal astrology, the position of the Sun, Moon, and planets at the time of birth can be used to determine the personality, temperament, and behavior of a subject [20].

A historical analysis of the Dastgāh system's development indicates that it originated as a concept of multimodal performance, rooted in the earlier theoretical framework of modal proximity. This concept was first documented, albeit briefly, in the fourteenth century. By the sixteenth century, musicological texts had evolved to articulate a systematic pedagogy of multimodal performance. This instruction formalized four distinct multimodal suites, each characterized by a cyclic structure that began in a specific mode and concluded by returning to that same mode [15]. Structurally, the Iranian musical repertoire (Radif-e Dastgāhi) is a systematic arrangement of the modal frameworks, melodic models, Āvāzs repertoire, and essential musical vocabulary of the Iranian plateau. This codification was undertaken primarily to create a formal pedagogical apparatus for teaching canonical Iranian music [4].

The current Iranian Radif musical system consists of seven Dastgāhs (Māhur, Shūr, Homāyun, Segāh, Chāhārgāh, Navā, Rāst-Panjgāh) and five Āvāzes (Abūatā, Bayāt-e Tork, Afshāri, Dashti, Isfahān) [9]. A Dastgāh is a musical system comprising multiple Gushehs, starting from a core "mother mode" that forms its basis. The performance then modulates through a series of modes in a set order, with each section usually ending in a Forud (cadence), returning the melody to the mother mode [16]. Iranian musical Dastgāhs are not merely melodic systems; they are traditionally believed to evoke distinct psychological and emotional states in the listener. This presumed affective power suggests a deep, structured relationship between specific Dastgāhs and the human temperament as defined in classical humoralism (the theory of the four bodily humors). To statistically investigate this ancient proposition, a General Linear Model (GLM)[2] analysis was employed. The results confirm a significant relationship between temperament type and the psychological trait of neuroticism. Notably, this relationship is strongest in individuals identified with a black bile (melancholic) temperament and those who express a musical preference for the Dastgāh-e Shūr [5].

1.1. **Introducing the 17- and 24-note number system.**

An examination of numerical sequencing in musical scales and their computational methods will be presented, based on the intervals within different octave divisions, for systems of twelve, seventeen, and twenty-four notes. These systems aim to determine both innate and acquired temperaments[3] and to establish links between each scale and the celestial zodiac constellations and planets. In a computational number system with 17-note, starting with the initial note

[2]. The general linear model (GLM) is a statistical analysis method.

[3]. In this article, the temperament (mizāj) of musical modes is compared to the four human temperaments, described as warm/cold and dry/wet, which are based on the four classical elements.

"Do" and ending with the final note "Si," the ascending process (totaling 17-note or 16 unequal consecutive intervals)[4] is introduced through two models: Makāni and Manzeli (place-value notation and positional), sharing the same order but using different numerical values according to common computational rules. As shown in Table 1, the sequence of notes, Makāni and Manzeli numbers, based on valuation, correspond to the values assigned in the Manzeli system. In the Makāni model, the note Sol Koron equals 10, while in the Manzeli model, it equals 2, with subsequent notes changing based on this foundation. The alignment of note degrees with their numbers in the Makāni table is clear. However, the reason for the number changes in the Manzeli table is to avoid two-digit numbers [2].

TABLE 1. 17-note numerical Makāni and Manzeli Models [2].

Note symbols	C	D♭	D𝄭	D	E♭	E𝄭	E	F	G♭	G𝄭	G	A♭	A𝄭	A	B♭	B𝄭	B
Makāni numbers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Manzeli numbers	1	2	3	4	5	6	7	8	9	2	3	4	5	6	7	8	9

The 24-note numerical system represents a theoretical model in which an octave is subdivided into 24 equal quarter-tone steps. In this framework, every note possesses three defining attributes: (1) a numerical value, (2) an elemental association (earth, water, wind, fire), and (3) a planetary correspondence. The properties and expressive qualities of each note are theoretically determined by the synergistic effects of its designated element and celestial counterpart [3], as shown in Teble 2.

TABLE 2. Manzeli notes numbers in 24-note system [3].

Names	**Saturn**	**Jupiter**	**Mars**	**Sun**	**Venus**	**Mercury**	**Moon**
Fire		Fa 5	Sol 9	La𝄭 5	Si𝄭 9	Do 5	Re 9
Wind	Mi𝄭 2	Fa𝄯 6		**La** 6	Si 2	Do𝄯 6	Re𝄯 2
Water	Mi 3	Sol♭ 7	Sol𝄯 3	La𝄯 7		Re♭ 7	Mi♭ 3
Earth	Mi𝄯 4	Sol𝄭 8	La♭ 4	Si♭ 8	Si𝄯 4	Re𝄭 8	

Note 1.1. A flat note (♭) lowers the sound of the note by a half step (semitone).

Note 1.2. A Koron (𝄭) lowers the sound of the note by approximately a quarter tone.

4. It is theoretically (not practically) assumed that diatonic and chromatic half steps are equal, and that a quarter tone is one-fourth of a whole step. Consequently, the sizes of the comma, the savart, and the cent are not discussed.

Note 1.3. A Sori (𝄯) uppers the sound of the note by approximately a quarter tone.

The order of the note cells table is Makāni numbers and, based on valuation, Manzeli numbers. The logic governing Manzeli numbers in the 24-note system (analogous to the 17-note system) inherently avoids two-digit numbers through numerical reduction.

The Manzeli numbering system operates on a 28-cell matrix partitioned into 9 sections, where each cell's value is derived via a cyclic digit-addition process that handles two-digit emergence systematically. This work proposes a novel mathematical formalization of this algorithm – the Cyclic Boundary Theorem (CBT) – which, to the best of current knowledge, has not been previously described in the literature. CBT[5] provides a rule for handling numbers in repeating 9-count cycles. When reaching 9 (cycle end), it converts $10 \to 2$ (instead of the conventional $10 \to 1$), creating predictable boundary behavior [3].

Cyclic Boundary Theorem (CBT). A mathematical framework for periodic numerical systems defined as:

Notation:

- $k \in \mathbb{Z}^+$: Cycle index (each cycle contains 9 consecutive numbers),
- $P_k = 9$: Base value at cycle boundary Manzeli numbers (constant for all complete cycles),
- M_k: Final computed value after transformations,
- $\mathrm{dr}(x)$: Digital root of x (e.g., $\mathrm{dr}(123) = 1 + 2 + 3 = 6$).

Core Algorithm:

$$M_k = \begin{cases} k, & \text{if } k = 1 \quad \text{(first cycle)}, \\ dr(P_k + 1) + 1 = 2, & \text{if } k \geq 2 \quad \text{(subsequent cycles)}. \end{cases}$$

Key Mechanism: At each cycle boundary (when $P_k = 9$):

$$9 \to 9 + 1 = 10 \to \mathrm{dr}(10) = 1 \to 1 + 1 = 2 \to M_k = 2.$$

In other words, when reaching Manzeli number 9 (cycle boundary), the system calculates the new cycle's starting value through consecutive digit summation: first summing $9 + 1$'s digits (since the tenth cell corresponds to the Manzeli number), then, it adds 10 to 1, resulting in 11, and the sum of its digits $(1 + 1)$ equals 2 [3].

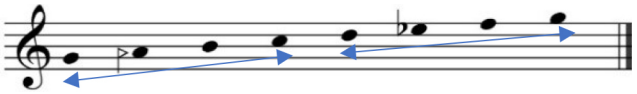
1.2. **Symmetrical and asymmetrical scales based on 24-note numerical systems.** As established in its previous study [2, 3], when analyzing symmetrical and asymmetrical scales, structural similarity between two tetrachords alone proves insufficient for classification.

[5]. CBT has two computational methods. For more information, see reference [3].

Crucially, the sum of note values calculated using the Manzeli numbering system for the 24-note must also be identical for both tetrachords. This requirement accounts for cases where apparent structural similarities may nonetheless yield different numerical values. The classification model is presented in Figure 1 with Table 3 for asymmetrical Homāyun - $A_\flat$ scale, following this methodology [3].[6] In Figure 2, to become familiar with the intervals of songs attributed to passion.

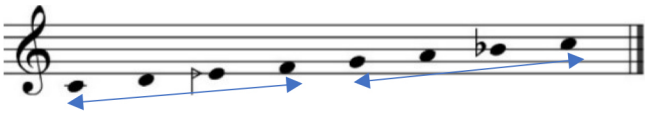
TABLE 3. The extracted number of the innate scale of the Homāyun - $A_\flat$.

G	**$A_\flat$**	**B**	**C**	**Notes**	**D**	**$E_\flat$**	**F**	**G**
9	5	2	5	**Numbers**	9	3	5	9
$9+5+2+5=21$				**Total**	$9+3+5+9=26$			



M, (Q +W), H- W - H, W, W

FIGURE 1. Homāyun - $A_\flat$ & Isfahān - C.



W, M, M- W - W, H, W

FIGURE 2. Innate scale of the Abūatā - $E_\flat$, Bayāt-e-Tork - F, Afshāri - G, Dashti - A.

Note 1.4. "M" is "Mojannab = Half step ± quarter-tone (ascending or descending).

The scope of this article excludes an examination of the internal esoteric qualities of scales and their variables. Nevertheless, it is pertinent to recognize that specific notes can hold particular significance within a scale's structure. In this context, two important observations are made:

Shāhed note: The Shāhed note is the main note around which the melody evolves, the note to which melodic passages constantly return [13]. Also, the Shāheds were introduced based on the ascending degree of the scale.

Variable note: It is a note that changes temporarily during the performance of the Gusheh. (Ascending or descending) [16].

Table 4 shows these two specials in the primary mode.

6. For more information about the intervals of the primary modes of Iranian musical Dastgāhes, refer to the article [2].

Note 1.5. Because the Āvāzs attributed to the Shūr's Dastgāh are introduced as independent scales in the conclusion of this article, their Shāheds of the note degree are different from those of Iranian music theorists.

Table 4. Variable and Shāhed Notes by Mode.

No.	Mode	Shāhed Note	Variable Note
1	Abūatā	Dominant then Mediant	Does not have
2	Bayāt-e-tork	Subdominant	Does not have
3	Afshāri	Dominant	Does have
4	Dashti	Sub- mediant	Does have
5	Bayāt-e Isfahān	Subdominant	Does not have

Note 1.6. The degrees of the modes in Table 4 are based on their innate scale, not practical or acquired scale.

2. Calculation notation

The notations used in this paper are presented in Table 5.

Table 5. Calculation notation of 17- and 24-note numerical systems [3, 2].

Explanation	Notation
Asymmetric scale with a Shāhed note of derivatives	$\ddot{S}t_1$
Asymmetric scale with a Shāhed note	$\dot{S}t_1$
Asymmetric scale with two Shāhed notes	$\ddot{S}t_2$
Asymmetric scale with a variable note	$\ddot{\S}t_1$
Sum of the notes	Sn_7
Sum of the notes	Sn_8
Mother scale	$M\ddot{S}$
First Shāhed note	t_1
Second Shāhed note	t_2
The first tetrachord	x_1
The second tetrachords	x_2
The difference in the sum of the Manzeli numbers of two tetrachords	X
Answers to calculations in the first, second, and third steps	a_1, a_2, a_3
First scale element in computational steps	es_1
Second scale element in computational steps	es_2
Extracted number	e
Answers to calculations in the first, second	a_1, a_2
Obtained number from total calculations	TC
Final innate element of scale (temper notation)	Tes
Scale zodiac constellation	Sz

3. Computational rules of the 17- and 24-note numerical system

To determine the elemental quality and zodiac constellation of a scale, the calculation rules of the 17- and 24-note systems are identical. The only difference lies in the method used to obtain the extracted number.

I. Obtaining the corresponding scale element (innate temper scale): divide the last number in the total calculation by 4, and the remainder will be the scale element, i.e.,

$$TC \overset{4}{\equiv} Tes.$$

In which the notation $a \overset{c}{\equiv} b$ means that b is the remainder of a divided by c.

Each element is also associated with a temperament. The numerical notation of each element and their temperaments are indicated in Table 6 [1].

Table 6. Numerical notations of each element and their temperament [1].

Element	Fire	Wind	Water	Earth
Temper	Warm and dry	Warm and moist	Cold and moist	Cold and dry
Notation	1	2	3	4

Note 3.1. When the remainder is zero,"Tes" and "es" represent earth, but the ancient people considered the number 4 as its symbol.

II. Obtaining the corresponding scale zodiac constellation: the remainder of dividing the total number of calculations by the number 12 will be the corresponding scale zodiac constellation, i.e.,

$$TC \overset{12}{\equiv} Sz.$$

The numerical notation of each zodiac constellation is shown in Table 7 [2].

Table 7. Numerical notations of zodiac constellation [2].

Zodiac	Aries	Taurus	Gemini	Cancer	Leo	Virgo
Notation	1	2	3	4	5	6
Zodiac	Libra	Scorpio	Sagittarius	Capricorn	Aquarius	Pisces
Notation	7	8	9	10	11	12

Note 3.2. When the remainder is zero, "Sz" represents Pisces, but ancient people considered the number 12 as its symbol.

III. Extracted Number

- Extracted number for 17-note numerical system

This number, which is used for asymmetric scales according to Table 10, is one of the numbers 4, 8, or 12 based on computational experience. The extraction method is as follows:

If the note (Shāhed) is part of the first three columns (1, 2, 3), the number is 4. If it is part of the second three columns (4, 5, 6), the number is 8. If it is part of the third three columns (7, 8, 9), the number is 12. However, since adding the number 12 does not affect the result, it can be ignored in calculations (The extracted number is necessary for accurate calculation of the corresponding scale zodiac constellation) [2],as given in Tables 8.

TABLE 8. Table of extracted number [2].

Number	1	2	3	4	5	6	7	8	9
Note	Do	Re♭	Re𝄲	Re	Mi♭	Mi𝄲	Mi	Fa	
Note	Sol♭	Sol𝄲	Sol	La♭	La𝄲	La	Si♭	Si𝄲	Si

- Extracted number for 24-note numerical system

Instructions for the extracted number in the 24-note numerical system for asymmetric scales are given below. Note that the extracted number is required for the accurate calculation of the scale zodiac constellation.

- Asymmetric scale with a Shāhed note or two Shāhed notes and derivatives of Dastgāh: The difference between the sum of the Manzeli numbers of the two tetrachords of a scale (X) is calculated as follows:
  - If $X := |x_2 - x_1| > 4$, reduce X by 4 repeatedly until it falls within the range $1 \leq X \leq 4$ (i.e., X mod 4).
  - The final extracted number (e) is obtained by multiplying the result by 4 [3].

$$X = |x_2 - x_1|, \quad e := 4 \times \begin{cases} X \bmod 4, & \text{if } X > 4, \\ X, & \text{otherwise.} \end{cases}$$

- Asymmetric scale with a variable note: The difference in the sum of the Manzeli numbers of two tetrachords of one scale (X) is multiplied by 4. If the variable note (Vt) is after the Shāhed note, (X) it is added to the number 4, and if it is before the Shāhed note, (X) it is subtracted from the number 4. It is worth mentioning that the variable note in the tetrachord is counted among the notes.

$$X := x_2 - x_1, \qquad e := 4X,$$

$$e := \begin{cases} X + 4 & \text{if } t, \text{ Next, } V_t, \\ X - 4 & \text{if } V_t, \text{ Before, } t, \end{cases}$$

Note 3.3. The difference (X) must be between 1 and 4. If $X > 4$, subtract 4 repeatedly until the result is within this range (e.g., if $X = 11$, then $11 - 4 - 4 = 3$, and $e = 4 \times 3 = 12$).

Note 3.4. To obtain the extracted number, asymmetric scales that contain a single Shāhed note are calculated by examining the first and second tetrachords starting from the initial

note of their innate scale. However, in the case of asymmetric scales with two Shāhed note, asymmetric scales derived from the Dastgāh system, or asymmetric scales that include a variable note, the calculation is performed starting from the Shāhed note.

4. A novel computational method for the 17- and 24-note numerical systems

Before discussing the computational method, it is important to note that the author developed this numerical system and computational approach through trial and error, testing various methods. Although the numerical systems share the same functionality across all ethnic and national music, the examples mentioned in the article illustrate the calculation of Iranian classical music Āvāzs.

4.1. **Asymmetric scales with a Shāhed note.**

- The 17-note numerical system [2]
  - A. The sum of the scale numbers (which are eight notes) is multiplied by the Shāhed note, defined as $a_1 := Sn_8 \times t_1$, where the $:=$ notation represents a definition or an assignment.
  - B. The answer from item A is divided by 4 to determine the remainder of the scale, which is the scale element (es_1). Then, the element number (es_1) is multiplied by 4 because there are four seasons in a year (symbolized by four temperaments), i.e.,
$$a_1 \overset{4}{\equiv} es_1, \quad a_2 := es_1 \times 4.$$
  - C. The results of steps A and B are summed. In the final step for the asymmetric scale, the extracted number e is added: $TC = a_1 + a_2 + e$.
- The 24-note numerical system [3] In asymmetric scales containing a single Shāhed note, the sum of the seven scale notes is multiplied by the index of the Shāhed note; the resulting product is then added to the extracted number, as shown below:

$$TC_{\ddot{S}t_1} := (Sn_7 \times t_1) + e.$$

4.2. **Asymmetric scales with two Shāhed note.**

- The 17-note numerical system
  - A. The sum of the scale numbers (which are eight notes) is multiplied by the first Shāhed note and denoted as $a_1 = Sn_8 \times t_1$.
  - B. The answer to step A is divided by 4 to determine the remainder of the scale, which is the scale element (es_1). Then, the element number (es_1) is multiplied by 4 to obtain a_2, i.e.,
$$a_1 \overset{4}{\equiv} es_1, \qquad a_2 = es_1 \times 4.$$

C. The answers from steps A and B are added together to obtain $a_3 = a_1 + a_2$.

D. (es_2) is multiplied by 4, and the result is multiplied by the second Shāhed note (t_2).

$$a_3 \overset{4}{\equiv} es_2, \qquad a_4 = (es_2 \times 4) \times t_2.$$

E. The answers from steps C and D are added to the position of the first Shāhed note, using the extracted number (since each calculation scale only uses the extracted number once), as follows:

$$TC = a_3 + a_4 + e.$$

- The 24-note numerical system

Its calculation method is common with symmetrical scales with two Shāhed notes; but in the asymmetric scale, the result is added with the extracted number, i.e.,

$$a_1 = Sn_7 \times t_1, \qquad TC = (t_2 \times 4) + a_1 + e.$$

4.3. **Asymmetric scales from derivatives of Dastgāh.**

- The 17-note numerical system

A. The following mathematical calculations are performed to obtain the mother scale:

$$a_1 = Sn_8 \times t_1, \qquad a_1 \overset{4}{\equiv} es_1,$$

$$a_2 = es_1 \times 4, \qquad M\ddot{S} = a_1 + a_2 + e.$$

B. The obtained element number (es_2) is multiplied by 4, and the result is multiplied by the Shāhed note of the dependent scale (t_2), as below:

$$M\ddot{S} \overset{4}{\equiv} es_2, \qquad a_3 = (es_2 \times 4) \times t_2.$$

C. The answers from steps A and B are added together with the extracted number as follows:

$$TC = M\ddot{S} + a_3 + e.$$

- The 24-note numerical system

A. The following mathematical calculations are performed to obtain the mother scale:

$$M\ddot{S} = (Sn_7 \times t_1) + e.$$

B. The corresponding dependent Shāhed note and the number 4 are multiplied together as below, and the result is added to the result from phase A:

$$a_1 = (t_2 \times 4) + M\ddot{S}.$$

C. The answer from step B is added to the extracted number as follows:

$$TC = a_1 + e.$$

4.4. **Asymmetric scales with a variable note.**

- The 17-note numerical system
  A. The notes of a scale (eight notes) and a variable note (V_t) are added together and their sum is subtracted by 1 (because one note is redundant), then multiplied by the Shāhed note as shown below:

$$a_1 = \big((Sn_8 + V_t) - 1\big) \times t_1.$$

  B. The answer to A is divided by 4 to determine the remainder of the scale, which is the scale element (es_1). Then, the element number (es_1) is multiplied by 4 to obtain a_2, i.e.,

$$a_1 \overset{4}{\equiv} es_1, \qquad a_2 = es_1 \times 4.$$

  C. The answers from steps A and B are added together with the extracted number, as follows:

$$TC = a_1 + a_2 + e.$$

- The 24-note numerical system
  A. The notes of a scale (seven notes) and a variable note (V_t) are added together and their sum is subtracted by 1 (because one note is redundant), then multiplied by the Shāhed note as:

$$a_1 = \big((Sn_7 + V_t) - 1\big) \times t_1.$$

  B. The answer a_1 is added to the extracted number as follows:

$$TC = a_1 + e.$$

For better understanding and comparison of the algebraic formulas of the 17- and 24-note systems, as shown in Table 9.

TABLE 9. Formulas of 17- and 24-note numerical systems.

Formula Name	**17-note Formula**	**24-note Formula**
Asymmetric scales with a Shāhed note	$a_1 := Sn_8 \times t_1$ $a_1 \overset{4}{\equiv} es_1$ $a_2 := es_1 \times 4$ $TC = a_1 + a_2 + e$	$TC_{\ddot{S}t_1} := (Sn_7 \times t_1) + e$
Asymmetric scales with two Shāhed note	$a_1 := Sn_8 \times t_1$ $a_1 \overset{4}{\equiv} es_1$ $a_2 := es_1 \times 4$ $a_3 := a_1 + a_2$ $a_3 \overset{4}{\equiv} es_2$ $a_4 := (es_2 \times 4) \times t_2$ $TC = a_3 + a_4 + e$	$a_1 := Sn_7 \times t_1$ $TC := (t_2 \times 4) + a_1 + e$
Asymmetric scales from derivatives of Dastgāh	$a_1 := Sn_8 \times t_1$ $a_1 \overset{4}{\equiv} es_1$ $a_2 := es_1 \times 4$ $M\ddot{S} = a_1 + a_2 + e$ $M\ddot{S} \overset{4}{\equiv} es_2$ $a_3 := (es_2 \times 4) \times t_2$ $TC = M\ddot{S} + a_3 + e$	$M\ddot{S} := (Sn_7 \times t_1) + e$ $a_1 := (t_2 \times 4) + M\ddot{S}$ $TC := a_1 + e$
Asymmetric scales with a variable note	$a_1 := ((Sn_8 + V_t) - 1) \times t_1$ $a_1 \overset{4}{\equiv} es_1$ $a_2 := es_1 \times 4$ $TC = a_1 + a_2 + e$	$a_1 := ((Sn_7 + V_t) - 1) \times t_1$ $TC := a_1 + e$

Note 4.1. the 17-note system is an incomplete version of the 24-note model. Therefore, there are two key differences in their calculations:

- Calculating the octave number in the sum of the notes for each scale: 7 notes are considered in the 24-note system, while 8 notes are calculated in the 17-note system by including the octave number.
- The scale element calculations (es_1 and es_2) are irrelevant in the 24-note system, but become essential for resolving the incomplete cycle inherent to the 17-note system.[7]

[7]. See reference [2].

5. Findings

Isfahān - C:[8] Asymmetric scale with a Shāhed note of derivatives as shown in Tables 10 to 13.

Table 10. 17-note (Makāni) - The Shāhed note is C = 1.

G	A$_\flat$	B	**C**	D	E$_\flat$	F	G	Total
11	13	17	1	4	5	8	11	70
$a_1 = 70 \times 13 = 910, \quad es_1 : 910 \overset{4}{\equiv} 2, \quad a_2 = 2 \times 4 = 8,$ $M\ddot{S}t = 910 + 8 + 8 = 926, \quad es_2 : 926 \overset{4}{\equiv} 2$ $a_3 := (1 \times 2) \times 4 = 8, \quad TC = 926 + 8 + 4 = 938$								

A. [9] The remaining number is 2 and its temperament is warm and moist.

$$938 \overset{4}{\equiv} 2.$$

B. [10] The remaining number 2 and the constellation scale is Taurus.

$$938 \overset{12}{\equiv} 2.$$

Table 11. 17-note (Manzeli) - The Shāhed note is C = 1.

G	A$_\flat$	B	**C**	D	E$_\flat$	F	G	Total
3	5	9	1	4	5	8	3	38
$a_1 = 38 \times 5 = 190, \quad es_1 : 190 \overset{4}{\equiv} 2, \quad a_2 = 2 \times 4 = 8,$ $M\ddot{S}t = 190 + 8 + 8 = 206, \quad es_2 : 206 \overset{4}{\equiv} 2$ $a_3 := (1 \times 2) \times 4 = 8, \quad TC = 206 + 8 + 4 = 218$								

A. The remaining number is 2 and its temperament is warm and moist.

$$218 \overset{4}{\equiv} 2.$$

B. The remaining number 2 and the constellation scale is Taurus.

$$218 \overset{12}{\equiv} 2.$$

8. Researches indicate that the Isfahān is derived from the Homāyun Dastgāh. For further information on the computations of the Dastgāh-e Homāyun, please see references [2, 3].
9. Corresponding scale temper: The intended meaning is the innate temperament of the scale.
10. Corresponding scale zodiac constellation: The intended meaning is zodiac constellation of the scale.

TABLE 12. The extracted number of the scales (24-note).

G	**A♭**	**B**	**C**	**Notes**	**D**	**E♭**	**F**	**G**
9	5	2	5	**Numbers**	9	3	5	9
$9+5+2+5=21$				**Total**	$9+3+5+9=26$			
$26-21=5$,				$5-4=1$,				$e=1\times 4=4$.

TABLE 13. 24-note (Manzeli) - The Shāhed note is C = 5.

G	A♭	B	**C**	D	E♭	F	Sn_7
9	5	2	5	9	3	5	38
$a_1=38\times 5=190, eM\ddot{S}t=4$ $M\ddot{S}t=190+4=194.$ $a_1:=(5\times 4)+194=214$ $TC=214+4=218.$							

A: The remaining number is 2 and its temperament is warm and moist.

$$218 \overset{4}{\equiv} 2.$$

B: The remaining number 2 and the constellation scale is Taurus.

$$218 \overset{12}{\equiv} 2.$$

Abūatā - E♭: Asymmetric scales with two Shāhed note as indicated in Tables 14 to 17.

TABLE 14. 17-note (Makāni) - The Shāheds notes are G = 11, & E♭ = 6.

C	D	**E♭**	F	**G**	A	B♭	C	Total
1	4	6	8	11	14	15	1	60
$a_1=60\times 11=660,\quad es_1:660\overset{4}{\equiv}4,$ $a_2=4\times 4=16,\quad a_3=660+16=676.$ $es_2:676\overset{4}{\equiv}4,\quad a_4:=(6\times 4)\times 4=96,$ $TC=676+96+4=776.$								

A. The remaining number is 0 and its temperament is cold and dry.

$$776 \overset{4}{\equiv} 0.$$

B. The remaining number 8 and the constellation scale is Scorpio.

$$776 \overset{12}{\equiv} 8.$$

TABLE 15. 17-note (Manzeli) - The Shāheds notes are G = 3, & $E_\flat = 6$.

C	D	$\mathbf{E_\flat}$	F	**G**	A	$B_\flat$	C	Total
1	4	6	8	3	6	7	1	36
$a_1 = 36 \times 3 = 108, \quad es_1 : 108 \overset{4}{\equiv} 4,$ $a_2 = 4 \times 4 = 16, \quad a_3 = 108 + 16 = 124.$ $es_2 : 124 \overset{4}{\equiv} 4, \quad a_4 := (6 \times 4) \times 4 = 96,$ $TC = 124 + 96 + 4 = 224$								

A. The remaining number is 0 and its temperament is cold and dry.

$$224 \overset{4}{\equiv} 0.$$

B. The remaining number 8 and the constellation scale is Scorpio.

$$224 \overset{12}{\equiv} 8.$$

TABLE 16. The extracted number of the scale (24-note).

C	**D**	$\mathbf{E_\flat}$	**F**	**Notes**	**G**	**A**	$\mathbf{B_\flat}$	**C**
5	9	2	5	**Numbers**	9	6	8	5
$5 + 9 + 2 + 5 = 21$				**Total**	$9 + 6 + 8 + 5 = 28$			
$28 - 21 = 7\ , \quad 7 - 4 = 3, \quad e = 3 \times 4 = 12.$								

TABLE 17. 24-note - The Shāheds note are G = 9 & $E_\flat = 2$.

C	D	$\mathbf{E_\flat}$	F	**G**	A	$B_\flat$	Sn_7
5	9	2	5	9	6	8	44
$a_1 = 44 \times 9 = 396,$ $TC = (2 \times 4) + 396 + 12 = 416.$							

A: The remaining number is 0 and its temperament is cold and dry.

$$416 \overset{4}{\equiv} 0.$$

B: The remaining number 8 and the constellation scale is Scorpio.

$$416 \overset{12}{\equiv} 8.$$

Bayāt-e-Tork -F (Fa): Asymmetric scale with a Shāhed note as given in Tables 18 to21.

TABLE 18. 17-note (Makāni) - The Shāheds note are F = 8.

C	D	E♭	**F**	G	A	B♭	C	Total
1	4	6	8	11	14	15	1	60
$a_1 = 60 \times 8 = 480, \quad es_1 : 480 \overset{4}{\equiv} 4,$ $a_2 = 4 \times 4 = 16,$ $TC = 480 + 16 + 12 = 508$								

A. The remaining number is 0 and its temperament is cold and dry.

$$508 \overset{4}{\equiv} 0.$$

B. The remaining number 8 and the constellation scale is Cancer.

$$508 \overset{12}{\equiv} 4.$$

TABLE 19. 17-note (Manzeli) - The Shāheds note are F = 8.

C	D	E♭	**F**	G	A	B♭	C	Total
1	4	6	8	3	6	7	1	36
$a_1 = 36 \times 8 = 288, \quad es_1 : 288 \overset{4}{\equiv} 4,$ $a_2 = 4 \times 4 = 16,$ $TC = 288 + 16 + 12 = 316$								

A. The remaining number is 0 and its temperament is cold and dry.

$$316 \overset{4}{\equiv} 0.$$

B. The remaining number 8 and the constellation scale is Cancer.

$$316 \overset{12}{\equiv} 4.$$

TABLE 20. The extracted number of the scale (24-note).

C	**D**	**E♭**	**F**	**Notes**	**G**	**A**	**B♭**	**C**
5	9	2	5	**Numbers**	9	6	8	5
$5 + 9 + 2 + 5 = 21$				**Total**	$9 + 6 + 8 + 5 = 28$			
$28 - 21 = 7, \quad 7 - 4 = 3, \quad e = 3 \times 4 = 12.$								

TABLE 21. 24-note (Manzeli) - The Shāhed note is F = 5.

C	D	E$_\flat$	**F**	G	A	B$_\flat$	Sn_7
5	9	2	5	9	6	8	44
$a_1 = 44 \times 5 = 220,$ $TC = 220 + 12 = 232.$							

A: The remaining number is 0 and its temperament is cold and dry.

$$232 \overset{4}{\equiv} 0.$$

B: The remaining number 8 and the constellation scale is Cancer.

$$232 \overset{12}{\equiv} 4.$$

Afshāri -G (Sol): Asymmetric scale with a variable note as shown in Tables 22 to 25.

TABLE 22. 17-note (Makāni) - The Shāhed note is G = 11 & Variable note is A$_\flat$ = 13.

C	D	E$_\flat$	F	**G**	A$_\flat$	A	B$_\flat$	C	Total
1	4	6	8	11	13	14	15	1	73
$a_1 = (73-1) \times 11 = 792,$ $es_1 : 792 \overset{4}{\equiv} 4,$ $a_2 = 4 \times 4 = 16,$ $TC = 792 + 16 + 4 = 812$									

A. The remaining number is 0 and its temperament is cold and dry.

$$812 \overset{4}{\equiv} 0.$$

B. The remaining number 8 and the constellation scale is Scorpio.

$$812 \overset{12}{\equiv} 8.$$

TABLE 23. 17-note (Manzeli) - The Shāhed note is G = 3 & Variable note is A$_\flat$ = 5.

C	D	E$_\flat$	F	**G**	A$_\flat$	A	B$_\flat$	C	Total
1	4	6	8	3	5	6	7	1	41
$a_1 = (41-1) \times 3 = 120,$ $es_1 : 120 \overset{4}{\equiv} 4,$ $a_2 = 4 \times 4 = 16,$ $TC = 120 + 16 + 4 = 140$									

A. The remaining number is 0 and its temperament is cold and dry.

$$140 \overset{4}{\equiv} 0.$$

B. The remaining number 8 and the constellation scale is Scorpio.

$$140 \overset{12}{\equiv} 8.$$

TABLE 24. The extracted number of the scale (24-note).

C	**D**	**E♭**	**F**	**Notes**	**G**	**A♭**	**A**	**B♭**	**C**
5	9	2	5	**Numbers**	9	5	6	8	5
$5+9+2+5=21$				**Total**	$9+5+6+8+5=33$				
$33-21=12\ ,\quad 12-(2\times 4)=4,$ $4\times 4=16,\quad e=16+4=20.$									

TABLE 25. 24-note (Manzeli) - The Shāhed note is G = 9 & Variable note is $A_♭ = 5$.

C	D	E♭	F	**G**	A♭	A	B♭	Sn_7
5	9	2	5	9	5	6	8	49
$a_1=(49-1)\times 9=432,$ $TC=432+20=452.$								

A: The remaining number is 0 and its temperament is cold and dry.

$$452 \overset{4}{\equiv} 0.$$

B: The remaining number 8 and the constellation scale is Scorpio.

$$452 \overset{12}{\equiv} 8.$$

Dashti -A (LA): Asymmetric scale with a variable note as shown in Tables 26 to 29.

TABLE 26. 17-note (Makāni) - The Shāhed note is A = 14 & Variable note is $A_♭ = 13$.

C	**D**	E♭	F	G	A♭	**A**	B♭	C	Total
1	4	6	8	11	13	14	15	1	73
$a_1=(73-1)\times 14=1008,\quad es_1: 1008 \overset{4}{\equiv} 4,$ $a_2=4\times 4=16,\quad TC=1008+16+8=1032$									

A. The remaining number is 0 and its temperament is cold and dry.

$$1032 \overset{4}{\equiv} 0.$$

B. The remaining number 0 and the constellation scale is Pisces.

$$1032 \overset{12}{\equiv} 0.$$

TABLE 27. 17-note (Manzeli) - The Shāhed note is A = 6 & Variable note is $A_{\flat} = 5$.

C	**D**	$E_{\flat}$	F	G	$A_{\flat}$	**A**	$B_{\flat}$	C	Total
1	4	6	8	3	5	6	7	1	41
$a_1 = (41-1) \times 6 = 240, \quad es_1 : 240 \overset{4}{\equiv} 4,$ $a_2 = 4 \times 4 = 16, \quad TC = 240 + 16 + 8 = 264$									

A. The remaining number is 0 and its temperament is cold and dry.

$$264 \overset{4}{\equiv} 0.$$

B. The remaining number 0 and the constellation scale is Pisces.

$$264 \overset{12}{\equiv} 0.$$

TABLE 28. The extracted number of the scale (24-note).

C	**D**	**$E_{\flat}$**	**F**	**Notes**	**G**	**$A_{\flat}$**	**A**	**$B_{\flat}$**	**C**
5	9	2	5	**Numbers**	9	5	6	8	5
$5+9+2+5=21$				**Total**	$9+5+6+8+5=33$				
$33-21=12\ , \quad 12-(2\times 4)=4,$ $4 \times 4 = 16, \quad e = 16 - 4 = 12.$									

TABLE 29. 24-note (Manzeli) - The Shāhed note is A = 6 & Variable note $A_{\flat}$ = 5.

C	D	$E_{\flat}$	F	G	$A_{\flat}$	**A**	$B_{\flat}$	Sn_7
5	9	2	5	9	5	6	8	49
$a_1 = (49-1) \times 6 = 288,$ $TC = 288 + 12 = 300.$								

A: The remaining number is 0 and its temperament is cold and dry.

$$300 \overset{4}{\equiv} 0.$$

B: The remaining number 0 and the constellation scale is Pisces.

$$300 \overset{12}{\equiv} 0.$$

The results of calculation steps for seven Dastgāhs are shown in Tables 30 to 38 centering

TABLE 30. Symmetrical scale with a Shāheds note of derivatives for Isfahān in 17-note.

Model	$M\ddot{S}t$	es_2	t	a_3	e	TC	Tes	Sz
Makāni	926	2	1	8	4	938	2	2
Manzeli	206	2	1	8	4	218	2	2

TABLE 31. Symmetrical scale with a Shāheds note of derivatives for Isfahān in 24-note.

Model	$M\ddot{S}t$	t_1	a_1	TC	Tes	Sz
Manzeli	194	5	214	218	2	2

TABLE 32. Symmetrical scale with two Shāheds note for Abūatā in 17-note.

Model	Sn_8	t_1	a_1	es_1	a_2	a_3	t_2	es_2	a_4	e	TC	Tes	Sz
Makāni	60	11	660	4	16	676	6	4	96	4	776	0	8
Manzeli	36	3	108	4	124	4	6	4	96	4	224	0	8

TABLE 33. Symmetrical scale with two Shāheds note for Abūatā in 24-note.

Model	Sn_7	t_1	a_1	t_2	e	TC	Tes	Sz
Manzeli	44	9	396	2	12	416	0	8

TABLE 34. Asymmetric scale Makāni model to a Shāhed note for Bayāt-e-Tork in 17-note.

Model	Sn_8	t_1	a_1	es_1	a_2	e	TC	Tes	Sz
Makān	60	8	480	4	16	12	508	0	4
Manzeli	36	8	288	4	16	12	316	0	4

TABLE 35. Asymmetric scale to a Shāhed note for Bayāt-e-Tork in 24-note.

Model	Sn_7	t_1	a_1	e	TC	Tes	Sz
Manzeli	44	5	220	12	232	0	4

TABLE 36. Asymmetric scale to a variable note for Afshāri and Dashti in 17-note.

Mode	Model	Sn_9	t_1	Vt	a_1	es_1	a_2	e	TC	Tes	Sz
Afshāri	Makān	73	11	13	792	4	16	4	812	0	8
Afshāri	Manzeli	41	3	5	120	4	16	4	140	0	8
Dashti	Makān	73	14	13	1008	4	16	8	1032	0	0
Dashti	Manzeli	41	6	5	240	4	16	8	264	0	0

Table 37. Asymmetric scale to a variable note for Afshāri and Dashti in 24-note.

Mode	Model	Sn_8	t_1	a_1	e	TC	Tes	Sz
Afshāri	Manzeli	49	9	432	20	452	0	8
Dashti	Manzeli	49	6	288	12	300	0	0

Table 38. Innate temperament and zodiac constellation table Āvāzs classical Iranian music.

Mode	**Innate temperament**	**Zodiac sign**	**zodiac temperament**
Isfahān	Warm and moist	Taurus	Cold and dry
Abūatā	Cold and dry	Scorpio	Cold and moist
Bayāt-e-Tork	Cold and dry	Cancer	Cold and moist
Afshāri	Cold and dry	Scorpio	Cold and moist
Dashti	Cold and dry	Pisces	Cold and moist

6. Discussion

This paper, representing the fourth installment in studies on 'Numerical Systems of Music', is founded on the principle that a fundamental connection exists between music, numbers, and astronomy, which in turn leads to psychological insights. This principle has been emphasized by scholars and sages from the time of Pythagoras to the present day. However, throughout history, this connection has largely remained at a theoretical and philosophical level, never systematically presented in the form of precise mathematical formulas.

In this time, for the first time, utilizing a novel approach and persistent trial and error, this age-old link has been moved from its theoretical state. Through precise mathematical and logical formulations, various types of asymmetric musical scales have been analyzed. This numerical model not only bridges classical theories with modern therapeutic applications but also provides a foundation for the formulaic understanding of the profound effects of musical scales on the human psyche.

Numerical systems provide a powerful framework for the precise analysis of musical theory, leading to a deeper understanding of the psychological states and effects arising from melodies and scales. As shown in Table 38, the Āvāz-e Dashti, with its innate temperament of "cold and dry," is associated with the element of "Earth," a connection that fosters patience and endurance. In contrast, its acquired temperament - aligned with the sign of Pisces - is "cold and moist," corresponding to the element of "Water," which invites flexibility, composure, and profound tranquility.

Beyond these theoretical foundations, anecdotal evidence and cultural practice have long recognized the effectiveness of Dashti in soothing those experiencing grief. This is arguably due to its capacity to facilitate emotional release (catharsis), allowing individuals to process

pain through expression. From a scientific perspective, this phenomenon can be interpreted as the gradual neutralization of the "fiery" intensity of grief and separation by the balancing elements of Water and Earth inherent in the scale. By harmonizing these elemental energies, Dashti helps in channeling deep emotions, ultimately strengthening resilience and fortitude in the face of hardship.

Each culture's music possesses unique rules and theories that, for accurate analysis and computation, necessitate the application of systematic approaches. In this paper, five renowned Persian musical Āvāz have been examined using 17- and 24-note numerical systems, presenting a comprehensive analysis for understanding Iranian music, accompanied by their respective mathematical formulas.

7. Conclusion

The results from the 17-note numerical system, despite differing valuations in spatial and positional models, have been found to be entirely consistent with the 24-note system. This concordance validates the accuracy of the computational method and lends greater credibility to the novel approach of numerical systems in analyzing musical scales.

Given the novel topics introduced in music theory through numerical systems, studying previous articles listed in the references section is essential for a deeper understanding and easier comprehension of the subject matter.

The accurate detection of the initial note of each scale is of paramount importance for theoretical analysis, and this is achieved through numerical computational rules. For instance, some Iranian musicologists have attributed the scales such as Abūatā, Bayāt-e Tork, Afshāri, and Dashti to the Dastgāh-e Shūr due to their common scale and the precedence of the Dastgāh-e Shūr's Shāhed note over the Shāhed note of other Maqāms. However, the fundamental scale (innate scale) of these Maqāms begins one degree lower than the Shāhed note of the Shūr dastgah. It is noteworthy that the calculations have been initially performed on various foundations. Since the correctness of each calculation is contingent upon obtaining identical results in the 17-note (Makaāi and Manzeli) and 24-note methods, If evaluate the aforementioned Āvāzes based on the perspective of those musicologists, the Dastgāh-e Shūr, identical results would not be achieved. This research indicates that this categorization, from the viewpoint of ancient musicians, is rooted in the common temperaments of these Maqāms with the Dastgāh-e Shūr.

Before the Dastgāh system, Iranian music was organized by Maqām and has evolved into its current form through logical development. Even the Āvāz-e Abūatā, previously considered a form of Forud (cadence) within Dastgāh-e Shūr,[11]later gained recognition as an independent

[11]. In the old tradition of this art, Abūatā is usually referred to as Sāranj, and in the past it was a part of Forud (cadence) of Shūr [14].

Iranian Āvāz due to its potential and scope. Numerical system calculations indicated that Abūatā is independent and not derived from Shūr.

It is noteworthy that regarding the Shāheds note of Āvāz-e Abūatā, two perspectives exist: some describe it with one Shāheds note, while others identify two Shāheds notes. Calculations using the numerical systems of scales demonstrated that this Maqām possesses two Shāheds note, and using one Shāheds note would result in inconsistent outcomes across three tables.

Calculations and analyses have shown that Āvāz–e Isfahān is derived from Dastgāh-e Homāyun and is the only Āvāz- among the seven Dastgāhs and five renowned Āvāz of Iranian music that belongs to a primary Dastgāh.

References

[1] M. Aslani, *An innovative numerical system model to identify the effects and psychological states of musical rhythm and tempo through four temperaments*. Mathematics and Computational Sciences, (2025) Articles in Press. https://doi.org/10.30511/mcs.2025.2044621.1252

[2] M. Aslani, *Innovative 17-note number system and its computational method to identify the effects and psychological states of music scale*. Transactions on Combinatorics (TOC), (2025). https://doi.org/10.22108/toc.2025.145560.2290

[3] M. Aslani, *Innovative 24-Note Numerical System: Computational Method and Relationship Model Between Notes, Elements, and Planets*, Mathematics and Computational Sciences, (2026) Articles in Press. https://doi.org/10.30511/mcs.2026.2075170.1514

[4] M. Aslani, *Familiarity with Iranian Music Radif (Tar and Setar)*, Author publisher copyright, Tehran, 2012. [Persian]

[5] M. Aslani, (2018). The relationship between four temperaments with the preference mediation of Traditional Iranian music in people ranging 30–45 years Old: providing model, Islamic Azad university, Science and Research Brach, Master thesis. [Persian]

[6] Anonymous. *Manuscript book of Ekhwān al-Safā or Mojmal al-Hekmah (Brothers of the same heart)*, 1912. [Persian]

[7] A. R. M. A. Al-Biruni, *Al-Tafhim li-Awáil Sanáat al-Tanjim (Instruction in the Elements of the Art of Astrology)*, English translation, Reprint of the Edition London, 1934. [Persian]

[8] M. Barkeshley, *Description of Iran's Music Radif, The seven systems traditional of Iranian Music (Radif)*, Writing: M. Maroufi, Tehran : Fine arts of the country, First edition, 1963. [Persian]

[9] F. Fakhreddini, *Analysis and Description of Iranian Musical (Radif)*, Seventh edition, Moein Publications, Tehran, 2021. [Persian]

[10] H. B. A. Ibn Sina, *Shafā, javāme elme riyāziyat (Musicology societies from healing mathematics)*, First edition, Translation S. A. Anwar, Hamedān, Boali Sinā Scientific and Cultural Foundation Publication, 1972. [Persian]

[11] G. R. KordafShari, H. M. Kenari, S. S. Esmaili and M. Moghimi, *Taghziyeh dar tebbe irane islami (Nutrition in Islamic Iranian medicine)*, Ninety Editions, Nesl Nikan Publishing, Tehran, 2019. [Persian]

[12] G. E. Kurtik, . *On the origin of the* 12 *zodiac constellation system in ancient Mesopotamia*, Journal for the History of Astronomy, **52** No. 1 (2021) 53-66.

[13] L. Miller, *Music and Song in Persia (RLE Iran B) : The Art of Avaz*, Routledge, 2012.

[14] S. H. Mashhun, *History of Iranian Music*, First edition, Farhang-e Nashr-e Now Publications, 2001. [Persian]

[15] M. Mohammadi, *Modal Modernities: Formations of Iranian Classical Music and the Recording of a National Tradition*, PhD thesis, Utrecht University, 2017.

[16] H. Moradian, *Receive music* 2*, Theoretical Iranian music*, First edition, Tehran, 2017. [Persian]

[17] J. S. Peters, *Music Therapy: An Introduction*, Second edition, Charles C. Thomas Pub Ltd, 2000.

[18] A. B. M. Reza, *Al-Rasāla-tol al-Zahabiyyeh (A writing containing medical and health teachings)*, Translator M. R. Daryāei, Publisher Safīr Ardehal-Tehran-Iran, 2022. [Persian]

[19] A. Zadeh Mohamadi, *New perspectives on Iranian music*, First edition, Sorod Publications, Tehran, 2000. [Persian]

[20] P. Zarka, *Astronomy and astrology*, Proceedings of the International Astronomical Union, **5** No. S260 (2009) 420-425.

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Appendices

I. Zodiac Constellation and corresponding planet: Each of the twelve regions of the zodiac Constellation, where the Sun is located in one of them each month, is roughly equivalent to solar months. Each of the zodiac signs has a constellation (planet) as follows: 1- Aries: Mars 2- Taurus: Venus 3- Gemini: Mercury 4- Cancer: Moon 5- Leo: Sun 6- Virgo: Mercury 7- Libra: Venus 8- Scorpio: Mars 9- Sagittarius: Jupiter 10- Capricorn: Saturn 11- Aquarius: Saturn 12- Pisces: Jupitercite [2, 3].

II. Innate scale: A scale whose interval structure, from a specific degree to its octave, is identical to that of other scales of the same type, thereby defining it as the foundational (innate) form relative to them. Consequently, in a 17-note system, a single scale can be expressed in 17 different modal rotations, yet only one of these 17 interval patterns serves as the reference (innate) scale [2, 3].

III. Acquired scale: A scale is a scale that begins on a different degree but retains the identical interval structure of the innate scale. For example, the acquired scale Māhūr on Fa is obtained by playing the distance pattern of the innate Māhūr on Do starting from Fa [2, 3].

IV. Practical scale: It is a scale whose performance from a given note defines the personality and emotional state of that Māqam. For instance, the natural minor scale (A minor) originates from the natural major scale (C major), but the practical scale of A minor - the one actually performed - starts on the ascending sixth degree of C major, and it is this specific starting point that establishes the Māqam's identity [2, 3].