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Research Paper

ON INTERSECTION MINIMAL IDEAL GRAPH OF A RING

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Abstract. For a ring R, the intersection minimal ideal graph, denoted by $\wedge(R)$, is a simple undirected graph whose vertices are proper non-zero (right) ideals of R and any two distinct vertices I_1 and I_2 are adjacent if and only if $I_1 \cap I_2$ is a minimal ideal of R. In this article, we explore connectedness, clique number, split character, planarity, independence number, domination number of $\wedge(R)$.

1. Introduction

The interdisciplinary study of graph associated with the algebraic structure ring has been studied by many authors. This enchanting perception was started by Istvan Beck [7]. After that introduction of Beck, many researchers have studied such types of interdisciplinary aspects. Some of them can be found in [1, 2, 3, 4, 8, 9, 18, 19, 20]. In [1], F. H. Abdulquadr introduced and studied the notion of maximal ideal graph of a commutative ring R, where the vertex set contains all the non-trivial ideals of R, and any two distinct vertices are adjacent

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if their sum is a maximal ideal of R. In this article, we discuss the intersection minimal ideal graph of a ring R which is denoted by $\wedge(R)$. The vertex set of $\wedge(R)$ contains all non-trivial (right) ideals of R, and any two distinct vertices are adjacent if and only if their intersection is a minimal ideal of R. We characterize the rings for which the intersection minimal ideal graphs are connected, complete bipartite, star. The concepts of planarity, clique number, independence number, domination number and split character are also studied. Most of the results in this article are observed in Artinian rings.

Now we recollect some definitions and notations which are needed in this sequel.

Let $\mathcal{G}$ be a simple undirected graph with vertex set $V(\mathcal{G})$ and edge set $E(\mathcal{G})$. If $\mathcal{G}$ does not contain any edge, then $\mathcal{G}$ is empty. The neighborhood of $p \in V(\mathcal{G})$ is denoted by $n(p)$. By K_n, we mean the complete graph with n vertices. If the vertices of $\mathcal{G}$ can be partitioned into two disjoint sets W_1 and W_2 with every vertex of W_1 is adjacent to any vertex of W_2 and no two vertices belonging to same set are adjacent, then $\mathcal{G}$ is called a complete bipartite graph. If $|W_1| = m$, $|W_2| = n$, then the complete bipartite graph is denoted by $K_{m,n}$. If one of the partite sets contains exactly one element, then the graph becomes a star graph. If $\mathcal{G}$ graph does not have K_5 or $K_{3,3}$ as its subgraph, then $\mathcal{G}$ is planar [8]. The girth of $\mathcal{G}$, denoted by $girth(\mathcal{G})$, is the length of the smallest cycle in $\mathcal{G}$. If there exists a path between any two distinct vertices, then $\mathcal{G}$ is connected. If P and Q are two distinct vertices of $\mathcal{G}$, then $d(P, Q)$ is the length of the shortest path from P to Q and $d(P, Q) = \infty$, if there does not exist a path between P and Q. The maximum distance among all the distances between every pair of vertices of $\mathcal{G}$ is called the diameter of $\mathcal{G}$, denoted by $diam(\mathcal{G})$. A clique is a complete subgraph in $\mathcal{G}$. The number of vertices in the largest clique of $\mathcal{G}$ is called the clique number of $\mathcal{G}$ and is denoted by $\omega(\mathcal{G})$. A subset S of $V(\mathcal{G})$ is said to be an independent set if no two vertices of S are adjacent. The cardinality of the largest independent set is called independence number and is denoted by $\alpha(\mathcal{G})$. If $V(\mathcal{G})$ can be partitioned in an independent set and a clique then $\mathcal{G}$ is said to be split. A set $D \subset V(\mathcal{G})$ is said to be a dominating set if every vertex not in S is adjacent to at least one of the members of S. The cardinality of smallest dominating set is the domination number of the graph $\mathcal{G}$ and is denoted by $\gamma(\mathcal{G})$.

Let R be a ring with unity. The collection of all minimal ideals of R is denoted by $\min(R)$ and the collection of all maximal ideals of R is denoted by $\max(R)$, respectively. The sum of all minimal ideals of R is called the socle of R which is denoted by $Soc(R)$. An ideal E of R is said to be essential if intersection of E with any non-zero ideal of R is non-zero. The socle of R is always contained in any essential ideal of R. For any two ideals I and J of R, we have $\frac{I+J}{I} \cong \frac{J}{I \cap J}$. If no infinite strictly descending chain of ideals of R exist, then R is an Artinian ring. In an Artinian ring, every ideal contains a minimal ideal. Any undefined terminology can be found in [5, 6, 10, 11, 12, 13, 14, 15, 16, 17].

Unless otherwise specified, R is a ring with unity.

2. Connectedness of $\wedge(R)$

Lemma 2.1. *The following hold in $\wedge(R)$:*
(i) Every non-minimal ideal of R is adjacent to at least one of the minimal ideals of R.
(ii) If $Soc(R) \neq R$, then every member of $\min(R)$ is adjacent to $Soc(R)$.

Remark 2.2. If $m, n \in \min(R)$, then it is easy to observe that m and n are not adjacent in $\wedge(R)$. Thus the subgraph induced by the minimal ideals of R is empty.

Proposition 2.3. *If P, Q, n are distinct vertices in $\wedge(R)$ with $n \in \min(R)$ and $P \cap Q \neq n$, then the following hold:*
(i) $n \in n(P \cap Q)$ if and only if $n \in n(P) \cap n(Q)$.
(ii) If $Soc(R) \subsetneqq P$, then $n \in n(P)$.
(iii) If $P \notin \min(R)$ and $P \subsetneqq Q$, then $P \notin n(Q)$.

Proof. (i) If $n \in n(P) \cap n(Q)$, then $n \cap P = n = n \cap Q$. Clearly, $n \subsetneqq P \cap Q$, which infers that $n \in n(P \cap Q)$. In the same way, the opposite direction can be proved.
(ii) Since $n \subset Soc(R)$ and $Soc(R) \subsetneqq P$, so $n \subsetneqq P$. This gives that $n \cap P = n$. Hence $n \in n(P)$.
(iii) If $P \subsetneqq Q$, then $P \cap Q = P$. Since $P \notin \min(R)$, we get $P \notin n(Q)$. □

Proposition 2.4. *If $M, N \notin \min(R)$ and $\{M, N\} \in E(\wedge(R))$, then there exists a unique $m \in \min(R)$ with $n \in n(M) \cap n(N)$.*

Proof. If $\{M, N\} \in E(\wedge(R))$, then $M \cap N \in \min(R)$. Clearly, $M \cap N$ is adjacent to both M and N. If possible, assume that there exists an $n \in \min(R)$ with $n \neq M \cap N$ and n is adjacent to both M and N. By Proposition 2.3, it is clear that $n \in n(M \cap N)$. So, $n \subsetneqq M, N$. This gives $n \subset M \cap N$. Since $M \cap N$ is minimal, $n = M \cap N$. This completes the proof. □

Proposition 2.5. *Every non-zero proper ideal of R is minimal if and only if $\wedge(R)$ is empty.*

Proof. Assume that every non-zero proper ideal of R is minimal. Consider two vertices M and N in $\wedge(R)$. Clearly, $M \cap N = 0$. So, M and N are not adjacent in $\wedge(R)$. Since M and N are arbitrary, we assert that $\wedge(R)$ is empty. Conversely, assume that $\wedge(R)$ is empty and $S \in V(\wedge(R))$. Suppose that $S \notin \min(R)$. Since R is Artinian, there exists some $s \in \min(R)$ with $s \subsetneqq S$. This gives that s and S are adjacent, a contradiction to the empty character of $\wedge(R)$. Thus every ideal of R is minimal. Hence the proposition. □

Proposition 2.6. *The graph $\wedge(R)$ is connected if and only if the sum of any two distinct minimal ideals of R is not R, or $|\min(R)| = 1$.*

Proof. If $|\min(R)| = 1$, then it is obvious that $\wedge(R)$ is connected. Suppose that $|\min(R)| \neq 1$, and the sum of any two distinct minimal ideals of R is not R. Consider two vertices I and J of $\wedge(R)$. If $\{I, J\} \in E(\wedge(R))$, then $I - J$ is a path. Suppose $\{I, J\} \notin E(\wedge(R))$. Then either $m \subsetneqq I \cap J$ for some $m \in \min(R)$, or $I \cap J = 0$. If $m \subsetneqq I \cap J$, then $I - m - J$ is a path in $\wedge(R)$. If $I \cap J = 0$, then following three cases arise.
Case 1: Suppose I and J are both minimal. Then $I - (I + J) - J$ is a path in $\wedge(R)$.
Case 2: If exactly one of I and J is minimal, then without loss of generality, assume that $I \in \min(R)$ and $J \notin \min(R)$. Since R is Artinian, there exists some $m \in \min(R)$ such that $m \subsetneqq J$. Thus, we get the path $I - (m + I) - m - J$.
Case 3: If both I and J are not minimal, then there exist $m_1, m_2 \in \min(R)$ such that $m_1 \subsetneqq I$ and $m_2 \subsetneqq J$, respectively. If $m_1 = m_2$, then $I - m_1 - J$ is a path. If $m_1 \neq m_2$, then $I - m_1 - (m_1 + m_2) - m_2 - J$ is a path. Hence we conclude that $\wedge(R)$ is connected.

Conversely, consider that $\wedge(R)$ is connected. If possible, assume that there exist two minimal ideals m_1 and m_2 such that $m_1 + m_2 = R$. Clearly, $R = m_1 \bigoplus m_2$. Also, $\frac{R}{m_1} \cong m_2$ and $\frac{R}{m_2} \cong m_1$. Since R is a commutative Artinian ring, m_1 and m_2 are minimal as well as maximal ideals of R. Assume that m_1 is adjacent to some $K \in V(\wedge(R))$. Then $m_1 \cap K = m_1$, which implies that $m_1 \subsetneqq K$. Since m_1 is maximal, we get $m_1 = K$. This asserts that m_1 is an isolated vertex, a contradiction. This completes the proof. □

Proposition 2.7. *If $\wedge(R)$ is a connnected graph, then $diam(\wedge(R)) \leq 4$.*

Proof. Consider that $\wedge(R)$ is connnected. If $\min(R) = 1$, then obviously $diam(\wedge(R)) = 2$. Assume that $|\min(R)| \neq 1$. Suppose $\{S, T\} \notin E(\wedge(R))$. Then either $m \subsetneqq S \cap T$ for some $m \in \min(R)$ or $S \cap T = 0$. Similarly, as in Proposition 2.6, we can also establish that $d(S, T) = 2$ *or* 4. Hence $diam(\wedge(R)) \leq 4$. □

Proposition 2.8. *If $F = F_1 \times F_2 \times ... \times F_n$, then $diam(\wedge(F)) = 2$, where F_i is a field, for $i = 1, 2, ..., n$.*

Proof. Let $F = F_1 \times F_2 \times ... \times F_n$, where F_i is a field, for $i = 1, 2, ..., n$. Any ideal of F is of the form $A = \prod_{i=1}^{n} G_i$, where $G_i = 0$ *or* F_i, and the minimal ideals of F is of the form $m_k = \prod_{i=1}^{n} G_i$, where $G_i = 0\, for\ i \neq k$ and $G_k = F_k$. Thus F has n minimal ideals. Consider two non adjacent vertices L and M in $\wedge(F)$. If L and M both contain a same minimal ideal, then $d(M,N)=2$. If not, then there exist m_i and m_j with $m_i \subset L$, $m_j \subset M$, $m_i \not\subset M$ and $m_j \not\subset L$. Now we consider the ideal $X=\prod_{l=1}^{n} G_l$, where $G_l=F_l$, for $l=i,j$ and 0 otherwise. This gives the path $L-X-M$. Therefore, $diam(\wedge(F))=2$. □

Proposition 2.9. *If $Soc(R)\neq R$, then $girth(\wedge(R))=3,4$ whenever $\wedge(R)$ contains a cycle.*

Proof. Let $Soc(R)\neq R$. Suppose that $\{S,T\}\in E(\wedge(R))$. Clearly, at least one of S or T does not belong to $\min(R)$. If $S,T\notin \min(R)$, then $S-S\cap T-T-S$ is a cycle. In this case, $girth(\wedge(R))=3$. Assume that one of S or T is minimal. Without loss of generality, take $S\in\min(R)$ and $T\notin\min(R)$. Then there exists some $p\in\min(R)$ such that $p\subsetneqq T$. Thus we get the cycle $S-T-p-Soc(R)-S$. In this case, $girth(\wedge(R))=4$. The proof is complete. □

Proposition 2.10. *If $F=F_1\times F_2\times ...\times F_n$, where F_i is a field, for $i=1,2,...,n$, then $girth(\wedge(R))=3$.*

Proof. Since $F=F_1\times F_2\times ...\times F_n$, where F_i is a field, for $i=1,2,...,n$, any ideal of F is of the form $A=\prod_{i=1}^{n} G_i$, where $G_i=0$ or F_i. Let us consider the ideal $X=\prod_{i=1}^{n} G_i$, where $G_i=F_i$, for $i=1,2$ and otherwise $G_i=0$; $Y=\prod_{i=1}^{n} G_i$, where $G_i=F_i$, for $i=1,3$ and otherwise $G_i=0$; $Z=\prod_{i=1}^{n} G_i$, where $G_i=F_i$, for $i=2,3$ and otherwise $G_i=0$. Also, every minimal ideal of F is of the form $m_k=\prod_{i=1}^{n} G_i$, where $G_i=0,\ for\ i\neq k$ and $G_k=F_k$. So, F has n minimal ideals. Since $X\cap Y=m_1$, $Y\cap Z=m_3$ and $X\cap Z=m_2$, thus we get the cycle $X-Y-Z-X$. This concludes that $girth(\wedge(R))=3$. □

Proposition 2.11. *If $\wedge(R)$ is complete, then R is a ring with $|\min(R)|=1$.*

Proof. Suppose $\wedge(R)$ is complete. If $p,q\in\min(R)$ and $p\neq q$, then $p\cap q=0$. This implies that $|\min(R)|=1$. □

Remark 2.12. We observe that Z_{p^n} has exactly one minimal ideal, but $\wedge(Z_{p^n})$ is not complete. Hence the converse of Proposition 2.11 does not hold.

Proposition 2.13. *If a chain is formed by the ideals of R, then $\wedge(R)$ is star.*

Proof. If a chain is formed by the ideals of R, then there exists a $p\in\min(R)$ such that $\{p,Q\}\in E(\wedge(R))$, for every $Q\in V(\wedge(R))$. If $S,T\in V(\wedge(R))$ and $S\neq p,T\neq p$, then it is clear that S and T are not adjacent. Hence $\wedge(R)$ is star. □

Proposition 2.14. *Let $Soc(R) \neq R$. Then $\wedge(R)$ is compete bipartite if and only if every ideal of R is either essential or minimal.*

Proof. Let V_1 and V_2 be the set of minimal ideals and essential ideals of R, respectively. If $p, q \in V_1$, then $p \cap q = 0$. Thus any two vertices of V_1 are not adjacent. Also, if $S, T \in V_2$, then $Soc(R) \subset S \cap T$. So any two vertices of V_2 are also not adjacent. Again, using Proposition 2.3, we get that every vertex in V_1 is adjacent to each vertex in V_2. Thus $\wedge(R)$ is a complete bipartite graph. For the opposite direction, assume that $\wedge(R)$ is a complete bipartite graph. It is easy to prove that the vertex set $V(\wedge(R))$ can be partitioned into the two disjoint subsets $\min(R)$ and $\{P \in V(\wedge(R)) : Soc(R) \subset P\}$. This completes the proof. □

Proposition 2.15. *If the sum of any two distinct minimal ideals of R is not R, and K is a cut vertex of $\wedge(R)$, then $K = S + T$, for some $S, T \in \min(R)$.*

Proof. If $K \in \min(R)$, then the result is obvious. Let $K \notin \min(R)$. Suppose P and Q are two vertices in the distinct components C_1 and C_2 of $V(\wedge(R) \setminus \{K\}$, respectively. We have the following cases:

Case 1: If $P, Q \in \min(R)$, then $P + Q \in n(P) \cap n(Q)$. Thus $K = P + Q$, as K is a cut vertex.

Case 2: If $P \in \min(R)$ and $Q \notin \min(R)$, then there exists some $S \in \min(R)$ with $S \subsetneqq Q$. Thus S and Q belong to the same component C_2. As $S + P \in n(S) \cap n(P)$, and each of S and P belongs to two different components, so $K = P + S$.

Case 3: If $P, Q \notin \min(R)$, then there exist some $S, T \in \min(R)$ with $S \subsetneqq P$ and $T \subsetneqq Q$. Here, P and S belong to the component C_1 and Q and T belong to the other component C_2. As $S + T \in n(S) \cap n(T)$, and S, T belong to C_2, therefore $K = S + T$. The proof is complete. □

3. Clique number, independence number, planarity of $\wedge(R)$

Proposition 3.1. *In $\wedge(R)$, a clique is contained in the subgraph induced by $\{P \in V(\wedge(R)) : Q \subset P\}$, for some $Q \in \min(R)$.*

Proof. Assume that $\mathcal{C}$ is a clique in $\wedge(R)$. As no two distinct minimal ideals are adjacent in $\wedge(R)$, so $\mathcal{C}$ has at most one minimal ideal. The completeness of $\mathcal{C}$ and Proposition 2.4 give that there exists a unique $Q \in \min(R)$ such that $\mathcal{C}$ is a subgraph induced by $\{P \in V(\wedge(R)) : Q \subset P\}$. Hence the proposition. □

Proposition 3.2. *If $\wedge(R)$ is not empty and $V(\wedge(R)) = \min(R) \cup \max(R)$, then $\wedge(R)$ is split.*

Proof. Consider the subgraph induced by $\max(R)$ of $\wedge(R)$. Let $P, Q \in \max(R)$ with $P \neq Q$. If possible, assume that $P \cap Q = 0$. Then $\frac{R}{P} \cong Q$ and $\frac{R}{Q} \cong P$. This implies that P and Q are simple rings [16] and so P and Q are minimal. By Proposition 2.5, $\wedge(R)$ is non-empty, a contradiction. Thus $P \cap Q \neq 0$. It is clear that $P \cap Q \notin \max(R)$. Therefore $P \cap Q \in \min(R)$. From this, the subgraph induced by $\max(R)$ is complete. Again, by Remark 2.2, the subgraph induced by $\min(R)$ is empty. Hence $\wedge(R)$ is split. □

Proposition 3.3. *If $V(\wedge(R)) = \min(R) \cup \max(R)$ and $|\max(R)| \leq 3$, then $\wedge(R)$ is planar.*

Proof. If $V(\wedge(R)) = \min(R) \cup \max(R)$, then, as in Proposition 3.2, $\wedge(R)$ is a split graph. Since $|\max(R)| \leq 3$, any subgraph induced by five vertices is not complete. Therefore, K_5 is not contained in $\wedge(R)$. If possible, assume that $K_{3,3}$ is contained in $\wedge(R)$ with partite sets $W_1 = \{P_1, P_2, P_3\}$ and $W_1 = \{Q_1, Q_2, Q_3\}$. It is clear that either $W_1 \subset \min(R)$ or $W_2 \subset \min(R)$. If we take $W_1 \subset \min(R)$, then $W_2 \subset \max(R)$, a contradiction to the fact that any two maximal ideals are adjacent. Hence, $\wedge(R)$ is a planar graph. □

Proposition 3.4. *If $|\min(R)|$ is finite for an Artinian ring R, then $\alpha(\wedge(R)) = |\min(R)|$.*

Proof. Suppose $\min(R) = \{m_1, m_2, ..., m_n\}$. Clearly, $\min(R)$ is an independent set, by Remark 2.2. Therefore, $n \leq \alpha(\wedge(R))$. Suppose $S = \{p_1, p_2, ..., p_l\}$ is a maximal independent set. So, $\alpha(\wedge(R)) = l$. For each $I \in S$, there exists some $m_i \in \min(R)$ such that $m_i \subset I$. If $l > n$, then by Pigeonhole principle, there exist at least two vertices $p_i, p_j \in S$ which contain the same minimal ideal. This implies that p_i and p_j are adjacent, a contradiction to the fact that S is an independent set. Therefore $l = n$, that is $\alpha(\wedge(R)) = n$. □

Proposition 3.5. *If R_1 and R_2 are two Artinian rings with unique minimal ideals, then $\gamma(\wedge(R_1 \times R_2)) = 2$.*

Proof. Any ideal of $R_1 \times R_2$ is of the form $K_1 \times K_2$, where K_1 and K_2 are ideals of R_1 and R_2, respectively. If the minimal ideals of R_1 and R_2 are p_1 and p_2, respectively, then the minimal ideals of $R_1 \times R_2$ are $p_1 \times (0)$ and $(0) \times p_2$. So, any vertex of the graph is adjacent to at least one of the elements of the set $\{p_1 \times (0), (0) \times p_2\}$. This implies that $\gamma(\wedge(R_1 \times R_2)) = 2$. □

Proposition 3.6. *If R is an Artinian ring with a unique minimal ideal and F is a field, then $\gamma(\wedge(R \times F) = 1$.*

Proof. It is clear. □

Proposition 3.7. *Let F_1 and F_2 be two fields, then $\gamma(\wedge(F_1 \times F_2)) = 2$.*

Proof. It is clear. □

Proposition 3.8. *Let $F = F_1 \times F_2 \times ... \times F_n$, then $\gamma(\wedge(F) \leq n$, where F_i is a field for $i = 1, 2, ..., n$.*

Proof. Let $F = F_1 \times F_2 \times ... \times F_n$, where F_i is a field for $i = 1, 2, ..., n$. Any ideal of F is of the form $A = \prod_{i=1}^{n} G_i$, where $G_i = 0$ *or* F_i, and a minimal ideal of F is of the form $m_k = \prod_{i=1}^{n} G_i$, where $G_i = 0 \ for \ i \neq k$ and $G_k = F_k$. So, F has n minimal ideals. Consider the set $S = \{m_i : i = 1, 2, ..., n\}$. The set S dominates all the vertices of the graph. So, $\gamma(\wedge(R \times F) \leq n$. □

Example 3.9. If $F = F_1 \times F_2 \times F_3$, where F_i is a field for $i = 1, 2, ..., n$, then $V(\wedge(F)) = \{F_1 \times 0 \times 0,\ F_1 \times F_2 \times 0,\ 0 \times F_2 \times 0,\ 0 \times F_2 \times F_3,\ F_1 \times 0 \times F_3,\ 0 \times 0 \times F_3\}$. Now consider the set $S = \{F_1 \times F_2 \times 0,\ 0 \times F_2 \times F_3\}$. Every vertex of $\wedge(F)$ is adjacent at least one of the vertices of S. Hence $\gamma(\wedge(F)) = 2(< 3)$.

This example provides that the equality does not hold necessarily in Proposition 3.8.

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References

[1] F. H. Abdulqadr, *Maximal ideal graph of commutative rings*, Iraqi J. Sci., **61** No. 8 (2020) 2070–2076.

[2] S. Akbari, R. Nikandish, and M. J. Nikmehr, *Some results on the intersection graphs of ideals of rings*, J. Algebra Its Appl., **12** No. 04 (2013) 1250200.

[3] D. F. Anderson and A. Badawi, *The total graph of a commutative ring*, J. Algebra, **320** (2020) 2706-2719.

[4] S. E. Atani, S. Hesari and M. Khoramdel, *A graph associated to proper non-small ideals of a commutative ring*, Comment. Math. Univ. Carol., **58** No. 1 1-12.

[5] M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra*, Second edition, Addison-Wesley, London, 1969.

[6] R. Balkrishnan and K. Ranganathan, *A Text Book of Graph Theory*, Springer-verlag New York, Inc., 2008.

[7] I. Beck, *Coloring of commutative rings*, J. Algebra, **116** (1988) 208-226.

[8] I. Chakrabarty and J. V. Kureethara, *A survey on the intersection graphs of ideals of rings*, Commun. comb. optim., **7** No. 2 (2022) 121-167.

[9] I. Chakrabarty, S. Ghosh, T. K. Mukherjee and M. K. Sen, *Intersection graphs of ideals of rings*, Discrete Math., **309** No. 17 (2009) 5381-5392.

[10] K. R. Goodearl, *Ring Theory*, Second edition, Marcel Dekker, 1976.

[11] F. Harary, *Graph Theory*, Addison-Wesley Publishing Company, New York, 1969.

[12] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc., New York, 1998.

[13] J. A. Huckaba, *Commutative Rings with Zero-Divisors*, Marcel-Dekker, New York, Basel, 1988.

[14] I. Kaplansky, *Commutative Rings*, Revised edition, University of Chicago Press, Chicago, 1974.

[15] F. Kasch, *Modules and Rings*, Academic Press Inc., London Ltd., 1982.

[16] T. Lam, *Lectures on Nodules and Rings*, Graduate Texts in Mathematics, Volume 189, Springer, New York, 1999.

[17] J. Lambeck, *Lectures on Rings and Modules*, Blaisdell Publishing Company, Waltham, Toronto, London, 1966.

[18] K. K. Rajkhowa and H. K. Saikia, *Center of intersection graph of submodules of a module*, AKCE Int. J. Graphs Comb., **16** No. 2 (2019) 198-203.

[19] K. K. Rajkhowa and H. K. Saikia, *Prime intersection graph of ideals of a ring*, Proc. Math. Sci., **130** (2020) 0973-7685.

[20] P. K. Sharma and S. M. Bhatwadekar, *A note on graphical representation of rings*, J. Algebra, **176** No. 1 (1995) 124-127.

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