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Research Paper

CONGRUENCES IN SEMINEARRINGS AND THEIR CORRESPONDENCE WITH STRONG IDEALS

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Abstract. In this paper, we define the notion of strong ideal of a seminearring S. If S is a nearring or a ring then the concept of a strong ideal of S coincides with the usual ideal of S. We show that there is one-one correspondence between strong ideals of S and strong congruences on S. Using the concept of strong ideals, we prove classical isomorphism theorems on S. We study insertion of factors property and obtain basic results on equisemiprime ideals.

1. Introduction

Seminearrings have several applications in various domains of mathematics. Hoorn and Rootselaar [26] introduced the concept of seminearring combining the vices of semiring and nearring. A right (resp. left) seminearring S is an algebraic structure with two binary operations such that S forms a semigroup with respect to these two binary operations and the right

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(resp. left) distributivity law holds. The set $M(S)$ of all mappings on an additive semigroup S with respect to addition and composition of mappings form a right seminearring.

Hoorn and Rootselaar [26] considered the kernel of a seminearring homomorphism as an ideal of a seminearring. Then Ahsan [2] generalized this definition and discussed the properties of regular seminearrings, a subclass of strongly idempotent seminearrings. Subsequently, Ashan [3] showed that a subset containing all minimal prime ideals forms a Hausdorff space in a distributively generated strongly idempotent seminearring.

Weinert [27] provided results on multiplicatively cancellative seminearrings. Kornthorng and Iampan [16] extended the k-ideals of semirings to seminearrings and obtained related results. Gilbert and Samman [6, 7] discussed the results on endomosphism seminearring classes over Clifford and Brandt semigroups. Sardar and Mukherjee [25] studied various types of congruences on different types of additively regular seminearrings. This work was continued in Mukherjee et al. [22], Mukherjee et al. [21]. Krishna and Chatterjee [17] utilized the idea of S-semigroup that gives an algebraic representation of seminearrings to prove a categorical representation. Further, they have classified seminearrings and provided approximate categories of seminearrings in which a given seminearring is primitive.

Krishna and Chatterjee [18] used Eilenberg's technique to study the structure of transformation semigroups. In addition, the Holcombe's holonomy decomposition of nearrings is extended to seminearrings.

Khachorncharoenkul, Laipaporn and Wananiyakul [10] introduced left almost seminearrings which are the generalization of left almost rings, near left almost semirings and left almost semirings and investigated some properties on left almost seminearrings. Khana et al. [11] introduced gamma seminearrings and gave characterization results. Then they introduced different types of ideals of group seminearrings and proved the isomorphism theorems. Subsequently, the notion of weakly prime and weakly primary ideals of gamma seminearring and their characterizations have been presented in Khan et al. [12].

Manikandan and Perumal [19] introduced the concept of mid-units in seminearrings and studied the relationship between the idempotents and mid-units of seminearrings. In addition, they derived a condition for seminearrings to have a mid-unit. Koppula et al. [14] defined the perfect ideal of a seminearring and proved isomorphism theorems by using the concept tame morphism. Koppula et al. [15] defined various prime ideals of seminearrings and proved results on radicals.

In the case of rings or nearrings, the explicit definition of an ideal exists, and ideals are characterized by the kernel of homomorphism. However, the corresponding problem in seminearrings is still unresolved. It is an interesting problem from the point of view of universal algebra to find a definition of an ideal of a seminearring S that produces an order preserving one-one correspondence between congruences on S and ideals of S. To resolve this problem, we introduce the concept of a strong ideal of a seminearring. Strong ideal generalizes the notion of ideal in nearring. We also define an equisemiprime strong ideal of a seminearring S. Equisemiprime strong ideal generalizes the notion of equisemiprime ideal in nearrings. Then we prove some results related to commutator equisemiprime strong ideals.

2. Preliminaries

Definition 2.1. [26] A non-empty set S with respect to $+$ and $.$ is said to be a right seminearring if the following conditions are satisfied:

(1) $(S, +)$ is a semigroup.
(2) There exists $0 \in S$ such that $0 + a = a + 0 = a$ for all $a \in S$.
(3) $(S, \cdot)$ is a semigroup.
(4) For all $a, b, c \in S, (a + b)c = ac + bc$.
(5) For all $a \in S$, $0a = 0$.

Definition 2.2. [26] A map ψ from a seminearring S to be a seminearring $S^{'}$ is said to be a seminearring homomorphism if $\psi(mn) = \psi(m)\psi(n)$ and $\psi(m + n) = \psi(m) + \psi(n)$ for all $m, n \in S$.

Definition 2.3. Let $(S, +)$ be a semigroup and A be a non-empty subset S. Then A is said to be a subsemigroup of S, if $a_1, a_2 \in A$ then $a_1 + a_2 \in A$.

Definition 2.4. A subsemigroup A of a seminearing S is said to be left invariant (resp. right invariant) if $SA \subseteq A$ (resp. $AS \subseteq A$) and A is invariant if it is both right and left invariant.

Definition 2.5. An equivalence relation R on S is said to be a congruence relation on S if the following conditions hold.

(1) aRb and cRd then $(a + c)\ R\ (b + d)$.
(2) aRb and cRd then $(ac)\ R\ (bd),\ a, b, c, d \in S$.

Definition 2.6. Let N be a nearring and I be an ideal of $N,\ x, y \in N$. Then I is said be an equisemiprime ideal of N, if $(x - y)rx - (x - y)ry \in I\ \forall r \in N$, then $x - y \in I$.

Throughout this work, all seminearrings are considered as right seminearrings.

The isomorphism theorems on semirings are proved by using the notions of tame morphism of semirngs and partitioning ideal of a semiring. For the isomorphism theorems on semirings, we refer to Golan [8]. For the results and isomorphism theorems on nearrings, we refer to Pilz [23], Bhavanari and Kuncham [5]. For the results on semigroups, nearrings, rings and modules, we refer to Sahami et al. [24], Aishwarya et al. [1], Mouhssine and Boua [20] Koppula et al. [13] and Amouzegar [4].

3. Strong ideal of a seminearring

In this section, we give the definition of strong ideal of a seminearring S. Then we define the concept of strong congruence relation on S and prove that there exists a one-one correspondence between strong ideals of S and strong congruences. Then we prove isomorphism theorems on seminearrings.

Throughout this paper, S denotes a right seminearring.

Definition 3.1. Let K be any subset of S, $a, b \in S$. Then $a \equiv_K b$ iff there exist $k_1, k_2 \in K$ such that $k_1 + a = k_2 + b$.

Definition 3.2. A non-empty subset P of S is said to be a strong ideal of S if the following conditions are satisfied.

(1) For $a, b \in P$, $a + b \in P$ $(P + P \subseteq P)$.
(2) For $a \in S$, $a + P \subseteq P + a$.
(3) For $a, b \in S$, if $a \equiv_P b$ then $a \in P + b$.
(4) $a(P + b) \subseteq P + ab$ for all $a, b \in S$.
(5) $Pa \subseteq P$ for all $a \in S$.

P is a left strong ideal of S, if P satisfies (1), (2), (3) and (4) whereas P is a right strong ideal of S, if P satisfies (1), (2), (3) and (5) of the Definition 3.2.

Note 3.3. *From (3) of Definition 3.2, as* $0 \equiv_P 0$, *we get* $0 \in P$.

Proposition 3.4. *Let P be a strong ideal of S.*

(1) *If $x \in S$ and $y \in P$ such that $x + y \in P$ then $x \equiv_P y$.*
(2) *If $x \in S$ and $y \in P$ such that $y + x \in P$ then $x \equiv_P y$.*
(3) *If $x, y \in P$ then $x \equiv_P y$.*

Proof. (1) There exists $p_1 \in P$ such that $x + y = p_1$ (Because $x + y \in P$). Now, $(x + y) + y = p_1 + y$. This implies $x + p_2 = p_1 + y$ [$y \in P$ implies $y + y = p_2 \in P$]. As $x + P \subseteq P + x$, there exists $p_3 \in P$ such that $x + p_2 = p_3 + x$. This gives $p_3 + x = p_1 + y$. Hence $x \equiv_P y$.

(2) There exists $i_1 \in P$ such that $y + x = i_1$ (Because $y + x \in P$). Now, $y + (y + x) = y + i_1$. This implies $i_2 + x = y + i_1$ [$y \in P$ implies $y + y = i_2 \in P$]. We have $y + P \subseteq P + y$. Then there exists $i_3 \in P$ such that $y + i_1 = i_3 + y$. This gives $i_2 + x = i_3 + y$. Hence $x \equiv_P y$.

(3) The proof follows from (1). □

Example 3.5. Let $S = \{0, p, q, r\}$ be a set with respect to $+$ and $\cdot$ defined as follows:

$+$	0	p	q	r
0	0	p	q	r
p	p	0	r	q
q	q	q	q	q
r	r	r	r	r

$.$	0	p	q	r
0	0	0	0	0
p	0	p	0	p
q	q	q	q	q
r	q	r	q	r

Then $P = \{0, p\}$ is strong ideal of a seminearring S.

Proposition 3.6. *If P and Q are strong ideals of S, then $P + Q$ is a strong ideal of S.*

Proof. If $a, b \in P + Q$, then clearly we get $a + b \in P + Q$.

Now, take $x \equiv_{P+Q} y$. Then there exist $k_1, k_2 \in P + Q$ such that $k_1 + x = k_2 + y$. As $k_1, k_2 \in P+Q$,we have $k_1 = p_1+q_1$ and $k_2 = p_2+q_2$, for some $p_1, p_2 \in P$ and $q_1, q_2 \in Q$. Then $p_1+(q_1+x) = p_2+(q_2+y)$. This gives $q_1+x \equiv_P q_2+y$. This implies $q_1+x \in P+(q_2+y)$. Then there exists $p_3 \in P$ such that $q_1 + x = (p_3 + q_2) + y = (q_3 + p_3) + y$, for some $q_3 \in Q$. Therefore $q_1+x = q_3+(p_3+y)$. This gives $x \equiv_Q p_3+y$. This implies $x \in Q+p_3+y \subseteq Q+P+y \subseteq P+Q+y$.

Let $y \in s+(P+Q)$. Then $\exists\ k_1 \in P+Q$ such that $y = s+k_1 = s+(p_1+q_1)$, for some $p_1 \in P$ and $q_1 \in Q$. Then there exists $p_2 \in P$ such that $(s + p_1) + q_1 = (p_2 + s) + q_1 = p_2 + (s + q_1) = p_2 + (q_2 + s) \in P + Q + s$, for some $q_2 \in Q$. Hence $s + P + Q \subseteq P + Q + s$.

Let $y \in s((P + Q) + s')$. Then $\exists\ k \in P + Q$ such that $y = s(k + s') = s((p_1 + q_1) + s')$, for some $p_1 \in P$ and $q_1 \in Q$. Then $\exists\ p_2 \in P$ such that $s(p_1 + (q_1 + s')) = p_2 + s(q_1 + s') = p_2 + (q_2 + ss') \in P + Q + ss'$, for some $q_2 \in Q$. Hence $s((P + Q) + s') \subseteq P + Q + ss'$. Clearly, we get $(P + Q)s \subseteq P + Q$. □

Proposition 3.7. *If P and Q are strong ideals of S, then $P \cap Q$ is a strong ideal of Q.*

Proof. As $0 \in P \cap Q$, we have $P \cap Q$ is non-empty. Now, take $k_1, k_2 \in P \cap Q$. As $k_1, k_2 \in P$, we get $k_1 + k_2 \in P$ and as $k_1, k_2 \in Q$, we get $k_1 + k_2 \in Q$. This implies $k_1 + k_2 \in P \cap Q$.

Let $z \in s + P \cap Q$. Then $z = s + k_1$, for some $k_1 \in P \cap Q$. As $k_1 \in P$, there exists $p_1 \in P$ such that $z = s + k_1 = p_1 + s$. Then $p_1 + s = q_2 + p_1$, for some $q_2 \in Q$. Now, as $k_1 \in Q$ and $s \in Q$, $z = s + k_1 = q_1 \in Q$. Therefore $q_2 + p_1 = q_1 + 0$. This also can be written as $p_1 \equiv_Q 0$. This implies $p_1 \in Q$. Hence $z = p_1 + s \in P \cap Q + s$. Therefore $s + P \cap Q \subseteq P \cap Q + s$.

Now, take $x \equiv_{P\cap Q} y$. Then there exist $k_1, k_2 \in P \cap Q$ such that $k_1 + x = k_2 + y$. As $k_1, k_2 \in P$, we have $x \equiv_P y$. This implies $x \in P + y$. Then $x = p_1 + y$, for some $p_1 \in P$. As $y \in Q$, there exists $q_1 \in Q$ such that $p_1 + y = q_1 + p_1$. Therefore $x + 0 = q_1 + p_1$. This gives $0 \equiv_Q p_1$. This implies $p_1 \in Q$. Therefore $x = p_1 + y \in P \cap Q + y$.

Let $z \in s(P \cap Q + s')$. Then $\exists\ k_1 \in P \cap Q$ such that $z = s(k_1 + s')$. As $k_1 \in P$, we have $s(k_1 + s') = p_1 + ss'$, for some $p_1 \in P$. As $ss' \in Q$, there exists $q_1 \in Q$ such that

$p_1 + ss^{'} = q_1 + p_1$. Similarly, as $k_1, k_2 \in Q$, $s(k_1 + s^{'}) = q_2 \in Q$. Therefore $q_1 + p_1 = q_2 + 0$. This also can be written as $p_1 \equiv_Q 0$. This implies $p_1 \in Q$. Therefore $z = p_1 + ss^{'} \in P \cap Q + ss^{'}$. Hence $s(P \cap Q + s^{'}) \subseteq P \cap Q + ss^{'}$.

Let $z \in (P \cap Q)s$. Then $\exists\ k \in P \cap Q$ such that $z = ks$. As $k \in P$ and $k \in Q$, we get $ks \in P$ and $ks \in Q$. This implies $ks \in P \cap Q$. Hence $(P \cap Q)s \subseteq (P \cap Q)$. Thus $P \cap Q$ is a strong ideal of Q. □

In rest of the paper, we consider P as a strong ideal of S.

Proposition 3.8. *$\equiv_P$ is an equivalence relation on S.*

Proof. Let $a \in S$. As $0 \in P$, we have $0 + a = 0 + a$. This gives $a \equiv_P a$. Hence $\equiv_P$ is reflexive.

Now, take $a \equiv_P b$. Then $p_1 + a = p_2 + b$, for some $p_1, p_2 \in P$. This also can be written as $p_2 + b = p_1 + a$. This implies $b \equiv_P a$. Hence $\equiv_P$ is symmetric.

Let $a \equiv_P b$ and $b \equiv_P c$. Then there exist $p_1, p_2, p_3, p_4 \in P$ such that $p_1 + a = p_2 + b$ and $p_3 + b = p_4 + c$. Now, take $p_3 + (p_1 + a) = p_3 + (p_2 + b)$. Then there exists $p_5 \in P$ such that $(p_3 + p_2) + b = (p_5 + p_3) + b$ [By condition (2) of Definition 3.2] $= p_5 + (p_3 + b) = p_5 + (p_4 + c)$. Therefore $p_3 + p_1 + a = p_5 + p_4 + c$. This implies $p_6 + a = p_7 + c$. $[p_3 + p_1 = p_6 \in P; p_5 + p_4 = p_7 \in P]$. This gives $a \equiv_P c$. Hence $\equiv_P$ is transitive. □

Remark 3.9. An equivalence class containing $s \in S$ is denoted by

$$[s]_{\equiv_P} = \{s^{'} \in S \mid s \equiv_P s^{'}\} = s/P$$

.

Definition 3.10. A congruence relation R on S is said to be a strong congruence relation on S if $[s]_R \subseteq [0]_R + s, \forall s \in S$.

Proposition 3.11. *$\equiv_P$ is a strong congruence relation on S.*

Proof. By Proposition 3.8, we have $\equiv_P$ is an equivalence relation on S. Now, take $r \equiv_P r^{'}$ and $s \equiv_P s^{'}$. Then there exist $p_1,\ p_2,\ p_3,\ p_4 \in P$ such that $p_1 + r = p_2 + r^{'}$ and $p_3 + s = p_4 + s^{'}$. Now, we have

$$(p_1 + r) + (p_3 + s) = (p_2 + r^{'}) + (p_4 + s^{'}),$$
$$\Rightarrow p_1 + (r + p_3) + s = p_2 + (r^{'} + p_4) + s^{'}.$$

Then there exist $p_5, p_6 \in P$ such that

$$p_1 + (p_5 + r) + s = p_2 + (p_6 + r^{'}) + s^{'},$$
$$\Rightarrow p_7 + r + s = p_8 + r^{'} + s^{'}[p_1 + p_5 = p_7 \in P;\ p_2 + p_6 = p_8 \in P].$$

This gives $r+s \equiv_P r^{'}+s^{'}$. Now, we have

$$(p_1+r)(p_3+s) = (p_2+r^{'})(p_4+s^{'}),$$

$$\Rightarrow p_1(p_3+s)+r(p_3+s) = p_2(p_4+s^{'})+r^{'}(p_4+s^{'}),$$

$$\Rightarrow p_5+r(p_3+s) = p_6+r^{'}(p_4+s^{'})[p_1(p_3+s)=p_5 \in P;\ p_2(p_4+s^{'})=p_6 \in P].$$

Then, there exist $p_7, p_8 \in P$ such that

$$p_5+(p_7+rs) = p_6+(p_8+r^{'}s^{'}),$$

$$\Rightarrow p_9+rs = p_{10}+r^{'}s^{'}[p_5+p_7=p_9 \in P; p_6+p_8=p_{10} \in P].$$

This gives $rs \equiv_P r^{'}s^{'}$.

Now, take $y \in [s]_{\equiv_P}$. This gives $y \equiv_P s$. This implies $y \in P+s = [0]_{\equiv_P}+s$. Hence

$$[s]_{\equiv_P} \subseteq [0]_{\equiv_P}+s.$$

Thus $\equiv_P$ is a strong congruence relation on S. □

Note 3.12. *If S is a ring or nearring, then the strong congruence relation coincides with the usual congruence relation on S.*

The following theorem gives the connection between the strong congruences on S and strong ideals of S.

Theorem 3.13. *The following statements hold.*

(1) *The binary relation θ defined on S as $(s,s^{'}) \in \theta$ if and only if $s \equiv_P s^{'}$ ($s \equiv_P s^{'}$ implies there exist $p_1, p_2 \in P$ such that $p_1+s = p_2+s^{'}$) is a strong congruence relation on S with $(0/\theta) = P$.*

(2) *If θ is a strong congruence relation on S, then $[0]_\theta = (0/\theta) = \{x \in S \mid x\ \theta\ 0\}$ is a strong ideal of S and for $s, s^{'} \in S$ we have $(s,s^{'}) \in \theta$ if and only if $s \equiv_{[0]_\theta} s^{'}$.*

Proof. (1) By Proposition 3.11, we have $\equiv_P$ is a strong congruence relation on S. Now, take $z \in (0/\theta)$. This implies $(z,0) \in \theta$. This gives $z \equiv_P 0$. This implies $z \in P$. Hence $(0/\theta) \subseteq P$. As $0 \in P$, clearly we get $P \subseteq (0/\theta)$. Therefore $(0/\theta) = P$.

(2) Suppose θ is a strong congruence relation on S. As $0\ \theta\ 0$, we have $[0]_\theta$ is non-empty. Now, take $x, y \in [0]_\theta$. Then $x\ \theta\ 0$ and $y\ \theta\ 0$. As θ is a congruence relation, $(x+y)\ \theta\ 0$. This implies $x+y \in [0]_\theta$.

Now, take $z \in s+[0]_\theta$. Then there exists $p_1 \in [0]_\theta$ such that $z = s+p_1$. As $p_1\ \theta\ 0$ and θ is a congruence relation, we have $(s+p_1)\ \theta\ (0+s)$. Therefore $z\ \theta\ s$. This implies $z \in [s]_\theta \subseteq [0]_\theta+s$. Hence $(s+[0]_\theta) \subseteq ([0]_\theta+s)$.

Now, take $z \in s([0]_\theta + s')$. Then there exists $p_2 \in [0]_\theta$ such that $z = s(p_2 + s')$. As $p_2\ \theta\ 0$, we get $[s(p_2 + s')]\ \theta\ (ss')$. This implies $z\ \theta\ ss'$. Hence $z \in [ss']_\theta \subseteq [0]_\theta + ss'$. Therefore $s([0]_\theta + s') \subseteq [0]_\theta + ss'$.

Let $x \equiv_{[0]_\theta} y$. Then $p_1 + x = p_2 + y$, for some $p_1, p_2 \in [0]_\theta$. As $p_1\ \theta\ 0$ and $p_2\ \theta\ 0$, we get $(p_1 + x)\ \theta\ x$ and $(p_2 + y)\ \theta\ y$ respectively. Therefore $x\ \theta\ y$. This implies $x \in [y]_\theta \subseteq [0]_\theta + y$.

Let $z \in [0]_\theta s$. Then $\exists\ p \in [0]_\theta$ such that $z = ps$. As $p\ \theta\ 0$, we get $(ps)\ \theta\ (0s)$. This implies $(ps)\ \theta\ 0$. Hence $ps \in [0]_\theta$. Therefore $[0]_\theta s \subseteq [0]_\theta$. Thus $[0]_\theta$ is a strong ideal of S.

Now, take $(s, s') \in \theta$. This implies $s \in [s']_\theta \subseteq [0]_\theta + s'$. This gives $s \in [0]_\theta + s'$. This implies $\exists\ i \in [0]_\theta$ such that $0 + s = i + s'$. This gives $s \equiv_{[0]_\theta} s'$.

Now, consider $s \equiv_{[0]_\theta} s'$. This implies there exist $i_1, i_2 \in [0]_\theta$ such that $i_1 + s = i_2 + s'$. As $i_1, i_2 \in [0]_\theta$ and θ is a congruence relation on S, we get $(i_1 + s)\ \theta\ s$ and $(i_2 + s')\ \theta\ s'$. Hence $s\ \theta\ s'$. Thus $(s, s') \in \theta$. □

Corollary 3.14. *The mapping $\theta \to (0/\theta)$ is an order preserving one-one correspondence between the strong congruences on S and strong ideals of S.*

Theorem 3.15. *Define $+$ and $\cdot$ on S/P as*

$$(s/P) + (s'/P) = (s + s')/P,$$

$$(s/P) \cdot (s'/P) = (ss')/P.$$

Then $(S/P, +, \cdot)$ is a seminearring.

Definition 3.16. Let $\phi : S \to R$ be a seminearring homomorphism. Then

$$ker\ \phi = \{s \in S \mid \phi(s) = \phi(0)\}.$$

Definition 3.17. Let S and R be seminearrings. Then a homomorphism $\phi : S \to R$ is said to be a strong homomorphism if $\phi(x) = \phi(y)$ then $x \in ker\ \phi + y$.

Theorem 3.18. *The following statements hold.*

(1) *The projection map $\pi : S \to S/P$ is an onto seminearring strong homomorphism.*
(2) *If $\phi : S \to R$ is an onto seminearring strong homomorphism then $ker\ \phi$ is a strong ideal of S and $S/ker\ \phi \cong R$.*

Proof. (1) We have $\pi : S \to S/P$ defined by $\pi(s) = s/P$. Then clearly π is an onto seminearring homomorphism.

Now,

$$\begin{aligned} ker\ \pi &= \{s \in S \mid \pi(s) = 0/P\} \\ &= \{s \in S \mid s/P = 0/P\} \\ &= \{s \in S \mid s \equiv_P 0\} \\ &= \{s \in S \mid s \in P\}. \end{aligned}$$

This implies $ker\ \pi = P$. Now, take $\pi(x) = \pi(y)$. Then $x/P = y/P$. Because $ker\ \pi = P$ is a strong ideal, we have $x \in ker\ \pi + y$. Hence π is an onto seminearring strong homomorphism.

(2) Suppose $\phi : S \to R$ is an onto strong homomorphism. As $0 \in S$, we have $0 \in ker\ \phi$. Hence $ker\ \phi$ is non-empty. Now, take $a, b \in ker\ \phi$. Then $\phi(a) = \phi(0) = \phi(b)$. Consider $\phi(a+b) = \phi(a) + \phi(b) = \phi(0) + \phi(0) = \phi(0)$. This implies $a + b \in ker\ \phi$.

Let $z \in s + ker\ \phi$. Then $z = s + k_1$, for some $k_1 \in ker\ \phi$. Now, $\phi(z) = \phi(s + k_1) = \phi(s) + \phi(k_1) = \phi(s) + \phi(0) = \phi(s + 0) = \phi(s)$. As ϕ is a strong homomorphism, we get $z \in ker\ \phi + s$. Therefore $s + ker\ \phi \subseteq ker\ \phi + s$.

Let $x \equiv_{ker\ \phi} y$. Then there exist $k_3, k_4 \in ker\ \phi$ such that $k_3 + x = k_4 + y$. Then

$$\begin{aligned} & \phi(k_3 + x) = \phi(k_4 + y) \\ \Rightarrow\ & \phi(k_3) + \phi(x) = \phi(k_4) + \phi(y) \\ \Rightarrow\ & \phi(0) + \phi(x) = \phi(0) + \phi(y) \\ \Rightarrow\ & \phi(0 + x) = \phi(0 + y) \\ \Rightarrow\ & \phi(x) = \phi(y) \\ \Rightarrow\ & x \in ker\ \phi + y. \end{aligned}$$

Let $z \in s(ker\ \phi + s^{'})$. Then $z = s(k_6 + s^{'})$, for some $k_6 \in ker\ \phi$. Now, Let $x \equiv_{ker\ \phi} y$. Then there exist $k_3, k_4 \in ker\ \phi$ such that $k_3 + x = k_4 + y$. Then

$$\begin{aligned} \phi(z) &= \phi(s(k_6 + s^{'})) \\ &= \phi(s)\phi(k_6 + s^{'}) \\ &= \phi(s)[\phi(k_6) + \phi(s^{'})] \\ &= \phi(s)[\phi(0) + \phi(s^{'})] \\ &= \phi(s)\phi(0 + s^{'}) \\ &= \phi(s)\phi(s^{'}) = \phi(ss^{'}). \end{aligned}$$

As ϕ is a strong homomorphism, we get $z \in ker\ \phi + ss^{'}$. Therefore $s(ker\ \phi + s^{'}) \subseteq ker\ \phi + ss^{'}$.

Now, take $z \in (ker\ \phi)s$. Then $z = ks$, for some $k \in ker\ \phi$. Now, $\phi(z) = \phi(ks) = \phi(k)\phi(s) = \phi(0)\phi(s) = \phi(0s) = \phi(0)$. This gives $z \in ker\ \phi$. Therefore $ker\ \phi$ is a strong ideal of S.

Now, define a map $\psi : S/ker\ \phi \to R$ as $\psi(s/ker\ \phi) = \phi(s)$. Let $s_1/ker\ \phi = s_2/ker\ \phi$. Then $s_1 \equiv_{ker\ \phi} s_2$. This implies there exist $k_1, k_2 \in ker\ \phi$ such that

$$\begin{aligned} & k_1 + s_1 = k_2 + s_2 \\ \Rightarrow\ & \phi(k_1 + s_1) = \phi(k_2 + s_2) \\ \Rightarrow\ & \phi(k_1) + \phi(s_1) = \phi(k_2) + \phi(s_2) \\ \Rightarrow\ & \phi(0) + \phi(s_1) = \phi(0) + \phi(s_2) \\ \Rightarrow\ & \phi(s_1) = \phi(s_2) \\ \Rightarrow\ & \psi(s_1/ker\ \phi) = \psi(s_2/ker\ \phi). \end{aligned}$$

Hence ψ is well-defined.

Now, take

$$\begin{aligned} \psi(s_1/ker\ \phi + s_2/ker\ \phi) &= \psi((s_1 + s_2)/ker\ \phi) \\ &= \phi(s_1 + s_2) = \phi(s_1) + \phi(s_2) \\ &= \psi(s_1/ker\ \phi) + \psi(s_2/ker\ \phi), \\ \psi((s_1/ker\ \phi)(s_2/ker\ \phi)) &= \psi((s_1 s_2)/ker\ \phi) \\ &= \phi(s_1 s_2) = \phi(s_1)\phi(s_2) \\ &= \psi(s_1/ker\ \phi)\psi(s_2/ker\ \phi). \end{aligned}$$

Let $r \in R$. Then $\phi(s) = r$, for some $s \in S$. As $s \in S$, we have $\psi(s/P) = \phi(s) = r$. Hence ψ is onto. Let $\psi(s_1/ker\ \phi) = \psi(s_2/ker\ \phi)$. This implies $\phi(s_1) = \phi(s_2)$.

As ϕ is a strong homomorphism, we get $s_1 \in ker\ \phi + s_2$. This implies there exists $k \in ker\ \phi$ such that $0 + s_1 = k + s_2$. This gives $s_1 \equiv_{ker\ \phi} s_2$. This implies $s_1/ker\ \phi = s_2/ker\ \phi$. Hence ψ is one-one. Thus $S/ker\ \phi \cong R$. □

Now, we illustrate Theorem 3.18 with an example.

Example 3.19. Let $S = \{0, a, b, 1\}$ be a set with respect to $+$ and $\cdot$ defined as follows:

$+$	0	a	b	1
0	0	a	b	1
a	a	0	b	1
b	b	b	b	1
1	1	1	1	1

$.$	0	a	b	1
0	0	0	0	0
a	0	0	a	a
b	0	0	b	b
1	0	0	b	1

Then S is a seminearring. Now, take $R = \{0, c, 1\}(0 < c < 1)$. Then (R, max, min) is a seminearring. Define $\phi : S \to R$ as

$$\phi(z) = \begin{cases} 0, & \text{if} \quad z \in \{0, a\}, \\ c, & \text{if} \quad z = b, \\ 1, & \text{if} \quad z = 1. \end{cases}$$

Hence ϕ is an onto seminearring strong homomorphism, $ker\ \phi = \{0, a\}$. Thus

$$S/ker\ \phi = \{0/ker\ \phi,\ b/ker\ \phi,\ 1/ker\ \phi\} \cong R.$$

Theorem 3.20. *If P and Q are strong ideals of S then*

$$(P + Q)/P \cong Q/(P \cap Q).$$

Proof. By Proposition 3.7, we have $P \cap Q$ is a strong ideal of Q. Define a map$\phi : P + Q \to Q/(P \cap Q)$ as $\phi(x + y) = y/(P \cap Q)$. Then we will show that ϕ is well-defined. Let $x_1, x_2 \in P$ and $y_1, y_2 \in Q$ such that $x_1 + y_1 = x_2 + y_2$. This gives $y_1 \equiv_P y_2$. As $y_1, y_2 \in Q$, we have $y_1 \equiv_Q y_2$. Now, we show that $y_1/(P \cap Q) = y_2/(P \cap Q)$.

Let $z \in y_1/(P \cap Q)$. Then there exist $k_1, k_2 \in P \cap Q$ such that $k_1 + z = k_2 + y_1$. As $k_1, k_2 \in P$, we have $z \equiv_P y_1$ and as $k_1, k_2 \in Q$, we have $z \equiv_Q y_1$. Because $z \equiv_P y_1$ and $y_1 \equiv_P y_2$, we get $z \equiv_P y_2$. This implies $z \in P + y_2$. Similarly, we get $z \in Q + y_2$.

Therefore $z \in (P + y_2) \cap (Q + y_2) \subseteq (P \cap Q) + y_2$. This implies $0 + z = k + y_2$, for some $k \in P \cap Q$. This gives $z \equiv_{P \cap Q} y_2$. This implies $z \in y_2/(P \cap Q)$. Therefore $y_1/(P \cap Q) \subseteq y_2/(P \cap Q)$. Similarly, we get $y_2/(P \cap Q) \subseteq y_1/(P \cap Q)$. Hence $y_1/(P \cap Q) = y_2/(P \cap Q)$.

Now, take $x, y \in P + Q$. Then there exist $x_1, x_2 \in P$ and $y_1, y_2 \in Q$ such that

$$\begin{aligned} \phi(x + y) &= \phi((x_1 + y_1) + (x_2 + y_2)) \\ &= \phi(x_1 + (y_1 + x_2) + y_2) \\ &= \phi(x_1 + (x_3 + y_1) + y_2), \end{aligned}$$

for some $x_3 \in P$. Then

$$\begin{aligned} \phi((x_1 + x_3) + (y_1 + y_2)) &= \phi(x_4 + y_4) = y_4/(P \cap Q) \\ &= (y_1 + y_2)/(P \cap Q) = y_1/(P \cap Q) + y_2/(P \cap Q) \\ &= \phi(x_1 + y_1) + \phi(x_2 + y_2) = \phi(x) + \phi(y). \end{aligned}$$

Similarly, we can show that $\phi(xy) = \phi(x)\phi(y)$. Hence ϕ is a homorphism and clearly ϕ is onto. Now,

$$\begin{aligned} ker\ \phi &= \{z \in P+Q | \phi(z) = \phi(0)\} \\ &= \{z = p+q \in P+Q, \text{ for some } p \in P \text{ and } q \in Q \mid \phi(p+q) = 0/(P \cap Q)\} \\ &= \{p+q \in P+Q | q/(P \cap Q) = 0/(P \cap Q)\} \\ &= \{p+q \in P+Q | q \in (P \cap Q)\} = P. \end{aligned}$$

Now, take $x, y \in P+Q$ such that $\phi(x) = \phi(y)$. This implies $\phi(p_1+q_1) = \phi(p_2+q_2)$, for some $p_1, p_2 \in P$ and $q_1, q_2 \in Q$. Then we get $q_1/(P \cap Q) = q_2/(P \cap Q)$. This gives $q_1 \equiv_{P \cap Q} q_2$. This implies $q_1 \in (P \cap Q) + q_2$.

Now, consider $p_2 + x = p_2 + (p_1 + q_1) = (p_2 + p_1) + q_1 = (p_2 + p_1) + (k_1 + q_2)$, for some $k_1 \in (P \cap Q)$. As $k_1 \in P$, we have $p_2 + (p_1 + k_1) + q_2 = (p_2 + p_3) + q_2$ $[p_1 + k_1 = p_3 \in P]$. Then there exists $p_4 \in P$ such that $(p_2 + p_3) + q_2 = (p_4 + p_2) + q_2 = p_4 + (p_2 + q_2) = p_4 + y$. Therefore $p_2 + x = p_4 + y$. This gives $x \equiv_P y$. This implies $x \in ker\ \phi + y$. Hence ϕ is an onto strong homomorphism. Then by Theroem 3.18, we get $(P+Q)/P \cong Q/(P \cap Q)$. □

Theorem 3.21. *If P and Q are strong ideals of S and $P \subseteq Q$ then*

$$S/Q \cong (S/P)/(Q/P).$$

Proof. Define a map $\phi : S/P \to S/Q$ as $\phi(s/P) = s/Q$. Then ϕ is well-defined and an onto homomrphism.

Now,

$$\begin{aligned} ker\phi &= \{s/P \in S/P | \phi(s/P) = \phi(0)\} \\ &= \{s/P \in S/P | s/Q = 0/Q\} \\ &= \{s/P \in S/P | s \equiv_Q 0\} \\ &= \{s/P \in S/P | s \in Q\} = Q/P. \end{aligned}$$

Let $s_1/P, s_2/P \in S/P$ such that $\phi(s_1/P) = \phi(s_2/P)$. This implies $s_1/Q = s_2/Q$. This gives $s_1 \equiv_Q s_2$. This implies $s_1 \in Q + s_2$ and $s_2 \in Q + s_1$. Then there exist $q_1, q_2 \in Q$ such that $s_1 = q_1 + s_2$ and $s_2 = q_2 + s_1$. Hence we get $s_1/P = q_1/P + s_2/P \in Q/P + s_2/P$ and $s_2/P = q_2/P + s_1/P \in Q/P + s_2/P$. Therefore ϕ is strong. Hence ϕ is an onto seminearring strong homomorphism. Thus by Theorem 3.18, we get $S/Q \cong (S/P)/(Q/P)$. □

Now, we illustrate Theorem 3.20 and Theorem 3.21 with the following example.

Example 3.22. Let $S = \{0, a, b, 1\}$ be a set with respect to $+$ and $\cdot$ defined as follows:

$+$	0	a	b	1
0	0	a	b	1
a	a	0	1	b
b	b	1	a	0
1	1	b	0	a

$.$	0	a	b	1
0	0	0	0	0
a	0	0	a	a
b	0	0	1	b
1	0	0	b	1

Then S is a seminearring and $P = \{0, a\}$, $Q = \{0, a, b, 1\}$ are strong ideals of S. P partitions S into the equivalence classes as $\{0/P, b/P\}$, where $0/P = \{0, a\}$ and $b/P = \{b, 1\}$. Similarly, Q partitions S into a single equivalence class as $0/Q = \{0, a, b, 1\}$.

Now, $P + Q = \{0, a, b, 1\}$ and $P \cap Q = \{0, a\}$. Then $(P + Q)/P = \{0/P, b/P\}$ and $Q/(P \cap Q) = Q/P = \{0/P, b/P\}$. Hence $(P + Q)/P \cong Q/(P \cap Q)$. Now, $(S/P)/(Q/P) = \{0/P, b/P\}/\{0/P, b/P\}$ and $S/Q = \{0/Q\}$. Hence $(S/P)/(Q/P) \cong S/Q$.

Note 3.23. *Suppose P satisfies conditions (1), (2), (4), (5) of Definition 3.2 and for every $a \in S$, $a \equiv_P 0$ implies $a \in P$. Then $(S/P, +, \cdot)$ forms a seminearring and all the isomorphism theorems hold. However, to get the proper one-one correspondence between ideals and congruences, we require additional conditions imposed on the definition of ideal, as illustrated in this paper.*

4. Results on prime strong ideals

In this section, we provide results related to equiprime strong, 3-prime strong and c-prime strong ideals. These ideals are defined in Koppula, Kedukodi and Kuncham[15].

The following Definition 4.1 is actually defined on nearrings and is taken from Pilz[23], which can also be used for seminearirngs.

Definition 4.1. Let P be a strong ideal of S. Then P is said to have an IFP (insertion of factors property) for $x, y \in S$, if $xy \in P$ then $xsy \in P$, $\forall\ s \in S$.

Proposition 4.2. *Let P be an equiprime strong ideal of S. If P has IFP then P is a c-prime strong ideal of S.*

Proof. Let $x, y \in S$ such that $xy \in P$. Suppose $x \notin P$. As P has IFP, we get $xsy \in P, \forall s \in S$. From Koppula, Kedukodi and Kuncham[15], we have $S_c \subseteq P$. Then $xs0 \in S_c \subseteq P, \forall s \in S$. Hence we have $xsy \equiv_P xs0, \forall s \in S$. As P is an equiprime strong ideal and $x \notin P$, we get $y \equiv_P 0$. This implies $y \in P$. □

Proposition 4.3. *Let P be a strong ideal of S. Then P is left invariant in S if and only if $s0 \in P$, $\forall\ s \in S$.*

Proof. As P is left invariant in S, we have $SP \subseteq P$. Since $0 \in P$, we get $s0 \in P, \forall s \in S$. Suppose $s0 \in P, \forall s \in S$. Let $s \in S$ be fix and $a \in P$. Then $sa = s(a+0) \subseteq P + s0 \subseteq P$. As $s \in S$ is arbitrary, we have $SP \subseteq P$. Thus P is left invariant in S. □

Proposition 4.4. *Let P be an equiprime strong ideal of S. Then P is left invariant in S.*

Proof. Let $m, r \in S$ such that $(m0)r(m0) = m0$ and $(m0)r(0) = m0$. Then $(m0)r(m0) \equiv_P (m0)r(0), \forall\ r \in S$. As I is an equiprime strong ideal, we get $m0 \in P$. Then by Proposition 4.3, P is left invariant in S. □

Proposition 4.5. *Let P be an equiprime strong ideal of S and J be an invariant subsemigroup of S. Then $J \cap P$ is an equiprime strong ideal of J.*

Proof. Let $x \in J \backslash (J \cap P)$ and $a, b \in J$ such that $xra \equiv_{(J \cap P)} xrb$ forall $r \in J$. Suppose $a \not\equiv_{J \cap P} b$. As $a, b \in J$, we have $a \equiv_J b$. Hence $a \not\equiv_P b$. Since P is an equiprime strong ideal of S, there exists $t \in S$ such that $xta \not\equiv_P xtb$. Similarly, as $xta \not\equiv_P xtb$ and $x \notin P$, there exists $n \in S$ such that $xn(xta) \not\equiv_P xn(xtb)$. Since J is an invariant subsemigroup of S, we have $nxt \in J$. Therefore $x(nxt)a \equiv_{(J \cap P)} x(nxt)b$. Which is a contradiction to $xn(xta) \not\equiv_P xn(xtb)$. Therefore our assumption $a \not\equiv_{J \cap P} b$ is wrong. Hence $a \equiv_{(J \cap P)} b$. Thus $J \cap P$ is an equiprime strong ideal of J. □

Definition 4.6. A strong ideal P of S is said to be a commutator equisemiprime strong ideal, if the following conditions hold.

(1) $(x + y) \equiv_P (y + x),\ \forall\ x, y \in S$.
(2) For $x, y \in S$, if $(yry + xrx) \equiv_P (xry + yrx)\ \forall\ r \in S$, then $x \equiv_P y$.

Proposition 4.7. *If S is a nearring and P is a commutator equisemiprime strong ideal of S, then $[x, y] \in P,\ \forall\ x, y \in S$.*

Proof. Let $x, y \in S$. As P is a commutator equisemiprime strong ideal, from Definition 4.6(1), we have $(x + y) \equiv_P (y + x)$. This implies there exist $p_1, p_2 \in P$ such that

$$\begin{aligned} p_1 + x + y &= p_2 + y + x \\ &\Rightarrow p_1 + x + y - x - y = p_2 \\ &\Rightarrow x + y - x - y = -p_1 + p_2 \in P \\ &\Rightarrow [x, y] \in P. \end{aligned}$$

□

Proposition 4.8. *If S is a nearring then every commutator equisemiprime strong ideal of S is an equisemiprime ideal.*

Proof. Let $x, y \in S$ be such that $yry + xrx \equiv_P xry + yrx, \ \forall \ r \in S$, then $x \equiv_P y$. Clearly, $x \equiv_P y$ implies $x - y \in P$. Now, fix $r \in S$. Then there exist $p_1, p_2 \in P$ such that

$$\begin{aligned} p_1 + yry + xrx &= p_2 + xry + yrx \\ &\Rightarrow p_1 + yry + xrx - yrx = p_2 + xry \\ &\Rightarrow p_1 + yry + (x-y)rx = p_2 + xry \\ &\Rightarrow yry + (x-y)rx = -p_1 + p_2 + xry \\ &\Rightarrow yry + (x-y)rx = p_3 + xry, \text{ for some } p_3 = -p_1 + p_2 \in P. \\ &\Rightarrow (x-y)rx = -yry + p_3 + xry. \\ &\Rightarrow (x-y)rx = (-yry + p_3 + yry) - yry + xry. \end{aligned}$$

Then there exists $p_4 \in P$ such that $(x-y)rx = p_4 + (-y+x)ry$. Now, consider

$$(x-y)rx - (x-y)ry = p_4 + (-y+x)ry - (x-y)ry,$$

for some $p_5 \in P$, (From Proposition 4.7)

$$= p_4 + (-y + x + y - x)ry = p_4 + p_5 ry,$$

for some $p_6 = p_5 ry \in P$,

$$= p_4 + p_6 \in P.$$

As $r \in S$ is arbitrary, we have $(x-y)rx - (x-y)ry \in P, \ \forall \ r \in S$ implies $x - y \in P$. Thus P is an equisemiprime ideal of S. □

Proposition 4.9. *If P is a commutator equisemiprime strong ideal of S, then $S_c \subseteq I$.*

Proof. Let $a \in S_c$. Then $ar \in S, \forall \ r \in S$. Let $r \in S$ be arbitrarily fix. Then $ara + 0r0 = 0ra + ar0$. This gives $ara + 0r0 \equiv_P 0ra + ar0$. As $r \in S$ is arbitrary, we have $ara + 0r0 \equiv_P 0ra + ar0, \ \forall \ r \in S$. Because P is an equisemiprime storng ideal, we get $a \equiv_P 0$. This implies $a \in P$. Thus $S_c \subseteq I$. □

Proposition 4.10. *If P is a commutator equisemiprime strong ideal of S then P is a 3-semiprime strong ideal.*

Proof. Let P be an equisemiprime strong ideal of S and $a \in S$ such that $ara \in P, \forall\ r \in S$. Now, fix $r \in S$. As $S_c \subseteq P$, we have $ar0 \in P$. Because $r \in S$ is an arbitrary, we have $ara + 0r0 \equiv_P 0ra + ar0,\ \forall\ r \in S$. As P is an equisemiprime, we get $a \equiv_P 0$. This implies $a \in P$. □

5. Conclusion

We have defined the concept of strong ideal of a seminearring with desirable properties from the point of view of universal algebras. The ideas presented in this paper will help to extend various results on seminearrings. We have defined a commutator equisemiprime strong ideal of a seminearring and obtained related results.

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References

[1] S. Aishwarya, B. S. Kedukodi and S. P. Kuncham, *Commutativity in 3-prime nearrings through permutation identities*, Asian-Eur. J. Math., **15** No. 06 (2022) 2250109.

[2] J. Ahsan, *Seminear-rings characterized by their S-ideals. I.*, Proc. Japan Acad. Ser. A, **71** (1995) 101-103.

[3] J. Ahsan, *Seminear-rings characterized by their S-ideals. II.*, Proc. Japan Acad. Ser. A, **71** (1995) 111-113.

[4] T. Amouzegar, *An approach to extending modules via homomorphisms*, Alg. Struc. Appl., **9** No. 1 (2022) 31-39.

[5] S. Bhavanari and S. P. Kuncham, *Near rings, Fuzzy ideals and Graph theory*, CRC Press, London, 2013.

[6] N. D. Gilbert and M. Samman, *Clifford semigroups and seminear-rings of endomorphisms*, Int. Electron. J. Algebra, **7** (2010) 110-119.

[7] N. D. Gilbert and M. Samman, *Endomorphism seminear-rings of brandt semigroups*, Commun. Algebra, **38** (2010) 4028-4041.

[8] J. S. Golan, *Semirings and their Applications*, Kluwer Acadamic Publishers, 1999.

[9] Y. B. Jun and K. Kim, *On structures of* Γ*-seminear-rings*, Int. Math. J., **1** (2002) 97-103.

[10] P. Khachorncharoenkul, P, K. Laipaporn and S. Wananiyakul, *Left Almost Seminearrings*, Lobachevskii J. Math., **41** (2020) 349-361.

[11] W. A. Khan, W. Arif and A. Taouti, *Group seminearrings*, J. Math. Computer Sci., **24** (2022) 338-344.

[12] W. A. Khan, A. Taouti, S. Karkain, A. Salami and W. Arif, *Weakly prime and weakly primary ideals in gamma seminearrings*, Eur. J. Appl. Math., **12** (2019) 544-552.

[13] K. Koppula, B. S. Kedukodi and S. P. Kuncham, *Markov chains and rough sets*, Soft comput., **23** (2019) 6441-6453.

[14] K. Koppula, B. S. Kedukodi and S. P. Kuncham, *On perfect ideals of seminearrings*, Beitr. Algebra Geom., **62** (2021) 823-842.

[15] K. Koppula, B. S. Kedukodi and S. P. Kuncham, *On prime strong ideals of a seminearring*, Mat. Vesn.,**72** (2020) 243-256.

[16] N. Kornthorng and A. Iampan, *A note on right full k-ideals of seminearrings*, J. infor. math. sci., **3** (2012) 255-261.

[17] K. V. Krishna and N. Chatterjee, *Representation of Near-Semirings and approximation of their categories*, Southeast Asian Bull. Math., **31** (2007) 903-914.

[18] K. V. Krishna and N. Chatterjee, *Holonomy decomposition of seminearrings*, Southeast Asian Bull. Math., **31** (2007) 1113-1122.

[19] G. Manikandan and R. Perumal, *Mid-units in duo seminearrings*, J. Phys. Conf. Ser., **1850** (2021), 012037.

[20] S. Mouhssine and A. Boua, *Homoderivations and semigroup ideals in 3-prime near-rings*, Alg. Struc. Appl., **8** No. 2 (2021) 177-194.

[21] R. Mukherjee, P. Pal, T. Manna and S. K. Sardar, *On additively completely regular seminearrings-ii*, Commun. Algebra, **47** No. 5 (2019) 1954-1963.

[22] R. Mukherjee, P. Pal and S. K. Sardar, *On additively completelyn regular seminearrings*, Commun. Algebra, **45** (2017) 5111-5122.

[23] G. Pilz, *Near-Rings: the Theory and its Applications*, Revised Edition, NorthHollond Publishing Company, Amsterdam, 1983.

[24] A. Sahami, M. Rostami and S. Kalantari, *On pseudo-contractibility of certain algebras related to a discrete semigroup*, Alg. Struc. Appl., **8** No. 2 (2021) 145-155.

[25] S. K. Sardar and R. Mukherjee, *On additively regular seminearrings*, Semigr. Forum, **88** (2014) 541-554.

[26] W. G. Van Hoorn and B. Van Rootselaar, *Fundamental notions in the theory of seminearrings*, Compos. Math., **18** (1967) 65-78.

[27] H. J. Weinert, *Seminearrings, seminearfields and their semigroup-theoretical background*, Semigr. Forum, **24** (1982) 231-254.

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