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Research Paper

RIGHT-LEFT INDUCED HYPERLATTICES AND THE GENETIC CODE HYPERLATTICES

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ABSTRACT. In this paper first we introduce right(resp. left) induced hyperlattices and investigate some of their properties. Especially a characterization of the smallest strongly regular relation for the class of distributive right/left induced hyperlattice is investigated. Next we propose and study the generated hyperlattices from hyperlattices. Finally, the right induced hyperlattices of two Boolean lattices of four DNA bases and physico-chemical properties of amino acids of four DNA bases are investigated.

1. INTRODUCTION

The concept of a hypergroup which is based on the notion of hyperoperation was introduced by Marty in [18] and studied extensively by many mathematicians. Hypergroup theory extends some well-known group results and introduces new topics leading thus to a wide variety of applications, as well as to a broadening of the fields of investigation. Surveys of the theory can

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be found in the books of Corsini [4], Davvaz and Leoreanu-Fotea [7], Corsini and Leoreanu [5] and Vougiouklis [28]. Using the notion of hyperoperation, Mittas and Konstantinidou have introduced and investigated the concept of hyperlattice in [12] and superlattices in [19].

Study on the research topic started in [23] about lattices derived from hyperlattices. For studing more connections between hyperstructures and lattices one can see [1, 13, 14, 16, 21] and [22, 23, 27]. Rasouli and Davvaz in [23] investigated derived lattices from hyperlattices. In [20] Nakano studied ring and partly ordered systems and defined Nakano hyperoperations. In this paper, first using ρ-ordered lattice $\mathcal{L}$, we introduce right induced hyperlattice $\mathcal{RH}(\mathcal{L},\rho)$, left induced hyperlattice $\mathcal{LH}(\mathcal{L},\rho)$) and the hyperlattice generated by a hyperlattice. Then some properties of these classes of hyperlattices are investigated. Finally, the right induced hyperlattices of two Boolean lattices of the four DNA bases which are called Primal Boolean Algebra and Dual Boolean Algebra, (see [26]) and two order relations ρ_p and ρ_d which is associated from physico-chemical properties of amino acids of four DNA bases are investigated. Basically, the motivation of this research is proposing diverse examples of hyperlattices which are induced from lattices. Specially distributive hyperlattices and s-distributive hyperlattices. Moreover, a connection between four DNA bases and hyperlattices valids. In the following we give some notions about lattice theory and hypergroup theory. Survey of the theory of lattices and the theory of hyperstructures can be found in the books of [3, 24] and [4], respectively.

2. Preliminaries

Before we study Right-left induced hyperlattice and some result of it, let us state some terminologies. We recall some definitions and properties on hyperlattices.

Definition 2.1. ([3]) A lattice is an algebra $\mathcal{L} = (L, \wedge, \vee)$ satisfying the following conditions

(i) $a = a \wedge a$ and $a = a \vee a$,

(ii) $a \wedge b = b \wedge a$ and $a \vee b = b \vee a$,

(iii) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ and $(a \vee b) \vee c = a \vee (b \vee c)$,

(iv) $a = a \wedge (a \vee b)$ and $a = a \vee (a \wedge b)$,

for all $(a, b, c) \in L^3$.

Theorem 2.2. *([3]) In a lattice $\mathcal{L}$, define $x \leq y$ if and only if $x \wedge y = x$. Then $(L, \leq)$ is an ordered set in which every pair of elements has a greatest lower bound and a least upper bound. Conversely, given an ordered set P with that property, define $x \wedge y = g.l.b.(x, y)$ and $x \vee y = l.u.b.(x, y)$. Then $(P, \wedge, \vee)$ is a lattice.*

Definition 2.3. ([3]) A lattice $\mathcal{L}$ is complete when each of its non-empty subsets $X \subseteq L$ has a $l.u.b$ and a $g.l.b$ in $\mathcal{L}$.

Definition 2.4. ([3]) A lattice $\mathcal{L}$ is distributive satisfying, for all $(a,b,c) \in L^3$, $(a \wedge b) \vee (a \wedge c) = a \wedge (b \vee c)$ and $(a \vee b) \wedge (a \vee c) = a \vee (b \wedge c)$.

Definition 2.5. ([3]) A lattice $\mathcal{L}$ is said to be bounded from below if there is an element $0 \in L$ such that $0 \leq x$ for all $x \in L$. Dually, L is bounded from above if there exists an element $1 \in L$ such that $x \leq 1$ for all $x \in L$. A bounded lattice is one that is bounded both from above and below.

Definition 2.6. ([3]) By a complement of x in a lattice $\mathcal{L}$ with 0 and 1 is meant an element $y \in L$ such that $x \wedge y = 0$ and $x \vee y = 1$; $\mathcal{L}$ is called complemented if all its elements have complements.

Definition 2.7. ([4]) A non-empty set H, endowed with a mapping, called hyperoperation, $\circ : H^2 \longrightarrow \mathcal{P}^*(H)$ is named hypergroupoid. If A and B are non-empty subsets of H, then $A \circ B = \cup_{a \in A, b \in B} a \circ b$.

Definition 2.8. ([4]) A hypergroupoid $(H, \circ)$ is called hypergroup if:

$$\forall (x,y,z) \in H^3, (x \circ y) \circ z = x \circ (y \circ z), \quad and \quad H \circ x = x \circ H = H.$$

Definition 2.9. ([25]) The Rosenberg partial hypergroupoid $H_\rho = (H, \circ_\rho)$ assotiated with a binary relation ρ defined on a non-empty set H is constructed as follows. For any $x, y \in H$,

$$x \circ_\rho x = \{z \in H \mid (x,z) \in \rho\}, x \circ_\rho y = x \circ_\rho x \cup y \circ_\rho y.$$

The set $\mathbb{D}(\rho) = \{x \in H \mid \exists y \in H : (x,y) \in \rho\}$ is called the domain of ρ, while $\mathbb{R}(\rho) = \{y \in H \mid \exists x \in H : (x,y) \in \rho\}$ is the range of the relation ρ. An element $x \in H$ is called outer element of ρ if there exists $h \in H$ such that $(h,x) \notin \rho^2$. The next theorem gives necessary and sufficient conditions, obtained by Rosenberg, under which the partial hypergroupoid H_ρ is a hypergroup.

Theorem 2.10. *([25]) H_ρ is a hypergroup if and only if*

(i) *ρ has full domain: $\mathbb{D}(\rho) = H$,*
(ii) *ρ has full range: $\mathbb{R}(\rho) = H$,*
(iii) *$\rho \subset \rho^2$,*
(iv) *If $(a,x) \in \rho^2$ then $(a,x) \in \rho$, whenever x is an outer element of ρ.*

Definition 2.11. ([12]) A hyperlattice $\mathcal{L} = (L, \otimes, \oplus)$ is a non-empty set with two hyperoperations $\otimes$ and $\oplus$ satisfying

(i) $a \in a \otimes a$ and $a \in a \oplus a$,
(ii) $a \otimes b = b \otimes a$ and $a \oplus b = b \oplus a$,
(iii) $(a \otimes b) \otimes c = a \otimes (b \otimes c)$ and $(a \oplus b) \oplus c = a \oplus (b \oplus c)$,

(iv) $a \in a \otimes (a \oplus b)$ and $a \in a \oplus (a \otimes b)$,

for all $(a, b, c) \in L^3$.

According to [23] a hyperlattice $\mathcal{L}$ is called s-distributive if $a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$ and $a \oplus (b \otimes c) = (a \oplus b) \otimes (a \oplus c)$, for all $(a, b, c) \in L^3$.

Example 2.12. $(L, \otimes, \oplus)$ is a hyperlattice for which $\otimes$ and $\oplus$ are defined on $L = \{a, b\}$ as follows:

$\otimes$	a	b
a	$\{a, b\}$	b
b	b	b

$\oplus$	a	b
a	$\{a, b\}$	$\{a, b\}$
b	$\{a, b\}$	b

Definition 2.13. ([23]). Let $\mathcal{L} = (L, \otimes, \oplus)$ be a hyperlattice. An equivalence relation, θ on L is called strongly regular if $(L/\theta, \wedge, \vee)$ is a lattice, where $\wedge$ and $\vee$ are defined as follow:

$$\bar{a} \wedge \bar{b} = \bar{c}, \quad c \in a \otimes b,$$

and

$$\bar{a} \vee \bar{b} = \bar{d}, \quad d \in a \oplus b.$$

Let $(L, \wedge, \vee)$ be a hyperlattice and X be a non-empty subset of L. $\mathrm{Im}(X)$ denote the set of all finite combinations respect to $\vee$ and $\wedge$. For example, if $X = \{x, y\}$, then $\mathrm{Im}(X) = \{x \vee y, x \wedge y, (x \wedge y) \vee x, (x \wedge (y \vee x)) \vee x, ...\}$. According to [23], if $(L, \wedge, \vee)$ is a hyperlattice, then we set

$$\upsilon_1^L = \{(x, x) | x \in L\},$$

and for every integer $n > 1$, υ_n^L is the relation defined as follows:

$$x \upsilon_n^L y \Leftrightarrow \exists (z_1, z_2, ..., z_n) \in L^n, \exists z \in \mathrm{Im}(\{z_1, z_2, ..., z_n\}) : \{x, y\} \subseteq z.$$

Obviously, for every $n \geq 1$, the relation υ_n^L is symmetric, and the relation $\upsilon^L = \cup_{n \geq 1} \upsilon_n^L$ is reflexive and symmetric. Let υ_*^L be the transitive closure of υ^L. (When it is clear from the context which hyperlattice is being considered, $\upsilon_n, \upsilon, \upsilon^*$ will be written in place $\upsilon_n^L, \upsilon^L, \upsilon_*^L$).

Theorem 2.14. *([23]) Let $(L, \wedge, \vee)$ be a hyperlattice. Then, υ^* is a strong regular relation on L.*

Theorem 2.15. *([23]) The relation υ^* is the smallest equivalence relation such that the quotient L/υ^* is a lattice.*

Corollary 2.16. *([23]) The fundamental relation υ^* is the smallest strong regular relation on a hyperlattice.*

Definition 2.17. ([27]) Let $\mathcal{L} = (L, \otimes, \oplus)$ be a hyperlattice. Then, $\mathcal{L}$ is a complete hyperlattice if for every $S \subseteq L$ and subsets $S^u_{\mathcal{L}} = \{x \in L \mid \forall s \in S, s \in s \otimes x, x \in s \oplus x\}$, $S^L_{\mathcal{L}} = \{x \in L \mid \forall s \in S, s \in s \oplus x, x \in s \otimes x\}$, $S^u_{\mathcal{L}}$ has a least element and $S^L_{\mathcal{L}}$ has a greatest element with the order relation $\leq$ on L.

Proposition 2.18. *Let $L_\rho = (L, \otimes)$ and $L_{\rho'} = (L, \oplus)$ be two partial Rosenberg hypergroups assotiated with binary relations ρ and ρ' defined on the non-empty set L, respectively (see Def. 2.10 and Th. 2.11). Then, $(L, \otimes, \oplus)$ is a hyperlattice if and only if $\otimes$ and $\oplus$ are reflexive relations.*

3. On induced hyperlattices from lattices

In this section first we define ρ-ordered lattices and then a construction of hyperlattices by ρ-ordered lattices is introduced. Also we investigate some properties of the class of induced hyperlattices from ρ-ordered lattices.

Suppose that L is a non-empty set and ρ is a binary relation on L. Then we say ρ is an order relation on L, if it is reflexive, antisymmetric, and transitive.

Definition 3.1. Let $\mathcal{L} = (L, \wedge, \vee)$ be a lattice and ρ be an order relation on L. We say $\mathcal{L}$ is ρ-ordered if for all $(a, b, c) \in L^3$ the following implication holds.

$$a\rho b \Rightarrow [(a \wedge c)\rho(b \wedge c) \ \text{ and } \ (a \vee c)\rho(b \vee c)].$$

Example 3.2. Suppose lattice $\mathcal{L} = (L, \wedge, \vee)$ be as follows:

$\wedge$	e	a	b	c
e	e	e	e	e
a	e	a	e	a
b	e	e	b	b
c	e	a	b	c

$\vee$	e	a	b	c
e	e	a	b	c
a	a	a	c	c
b	b	c	b	c
c	c	c	c	c

Now let $\rho = \{(e, e), (a, a), (b, b), (c, c), (a, b), (a, e), (e, b), (a, c), (c, b)\}$. In this case we have $\mathcal{L} = (L, \wedge, \vee)$ is a ρ-ordered lattice. Indeed

$$x\rho y \Rightarrow [(x \wedge z)\rho(y \wedge z) \ \text{ and } \ (x \vee z)\rho(y \vee z)].$$

Suppose that $\mathcal{L} = (L, \wedge, \vee)$ be a lattice and ρ be an order relation on L, for $a \in L$ we define:

$$L_\rho(a) :\overset{\text{def}}{=} \{h \in L |\ (h, a) \in \rho\} \text{ and } R_\rho(a) :\overset{\text{def}}{=} \{h \in L |\ (a, h) \in \rho\}.$$

Theorem 3.3. *Let $\mathcal{L} = (L, \wedge, \vee)$ be a ρ-ordered lattice. Then $\mathcal{RH}(\mathcal{L}, \rho) = (L, \otimes, \oplus)$ is a hyperlattice, where $a \otimes b = R_\rho(a \wedge b)$ and $a \oplus b = R_\rho(a \vee b)$.*

Proof. Suppose that $(a, b, c) \in L^3$ and $u \in (a \otimes b) \otimes c$. Hence there exists $v \in a \otimes b$ such that $u \in v \otimes c$ therefore $(a \wedge b, v) \in \rho$ and $(v \wedge c, u) \in \rho$. Because $\mathcal{L} = (L, \wedge, \vee)$ is a ρ-ordered lattice we have $((a \wedge b) \wedge c, u) \in \rho$ and so $(a \wedge t, u) \in \rho$, where $t = b \wedge c$ thus $u \in a \otimes t \subseteq a \otimes (b \otimes c)$. Consequently, $(a \otimes b) \otimes c \subseteq a \otimes (b \otimes c)$. Similarly we have $a \otimes (b \otimes c) \subseteq (a \otimes b) \otimes c$ and so associativity for the hyperoperation $\otimes$ holds. Similarly associativity for the hyperoperation $\oplus$ also holds. Hence the condition (iii) of Definition 2.11 valids. It is not hard to see that the conditions (i), (ii) and (iv) of Definition 2.11 also hold. □

Corollary 3.4. *Let $\mathcal{L} = (L, \wedge, \vee)$ be a ρ-ordered lattice. Then $\mathcal{LH}(\mathcal{L}, \rho) = (L, \otimes, \oplus)$ is a hyperlattice, where $a \otimes b = L_\rho(a \wedge b)$ and $a \oplus b = L_\rho(a \vee b)$.*

From now on we call $\mathcal{RH}(\mathcal{L}, \rho)$ the right induced hyperlattice of $\mathcal{L}$ and ρ and $\mathcal{LH}(\mathcal{L}, \rho)$ the left induced hyperlattice of $\mathcal{L}$ and ρ.

Example 3.5. The right induced hyperlattice of the ρ-ordered lattice in Example 3.2 has the following tables.

$\otimes$	e	a	b	c
e	$\{e, b\}$	$\{e, b\}$	$\{e, b\}$	$\{e, b\}$
a	$\{e, b\}$	$\{e, a, b, c\}$	$\{e, b\}$	$\{e, a, b, c\}$
b	$\{e, b\}$	$\{e, b\}$	b	b
c	$\{e, b\}$	$\{e, a, b, c\}$	b	$\{b, c\}$

$\oplus$	e	a	b	c
e	$\{e, b\}$	$\{e, a, b, c\}$	b	$\{b, c\}$
a	$\{e, a, b, c\}$	$\{e, a, b, c\}$	$\{b, c\}$	$\{b, c\}$
b	b	$\{b, c\}$	b	$\{b, c\}$
c	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$

Example 3.6. The left induced hyperlattice of the ρ-ordered lattice in Example 3.2 has the following tables.

$\otimes$	e	a	b	c
e	$\{e,a\}$	$\{e,a\}$	$\{e,a\}$	$\{e,a\}$
a	$\{e,a\}$	a	$\{e,a\}$	a
b	$\{e,a\}$	a	$\{e,a,b,c\}$	$\{e,a,b,c\}$
c	$\{e,a\}$	a	$\{e,a,b,c\}$	$\{c,a\}$

$\oplus$	e	a	b	c
e	$\{e,a\}$	a	$\{e,a,b,c\}$	$\{c,a\}$
a	a	a	$\{c,a\}$	$\{c,a\}$
b	$\{e,a,b,c\}$	$\{c,a\}$	$\{e,a,b,c\}$	$\{c,a\}$
c	$\{c,a\}$	$\{c,a\}$	$\{c,a\}$	$\{c,a\}$

Rasouli and Davvaz in [23] introduced the notion of distributive and s-distributive for the classes of hyperlattices, we say a hyperlattice $(L,\otimes,\oplus)$ is called s-distributive if we have:

$$a\otimes(b\oplus c)=(a\otimes b)\oplus(a\otimes c),$$

and

$$a\oplus(b\otimes c)=(a\oplus b\otimes(a\oplus c).$$

Theorem 3.7. *Let $\mathcal{L}=(L,\wedge,\vee)$ be a lattice. Then, $\mathcal{RH}(\mathcal{L},\rho)$ is s-distributive if and only if $\mathcal{L}$ is a distributive lattice.*

Proof. Let $\mathcal{RH}(\mathcal{L},\rho)$ be a s-distributive hyperlattice. We prove $\mathcal{L}$ is a distributive lattice. Let $a\otimes(b\oplus c)=(a\otimes b)\oplus(a\otimes c)$, for all $(a,b,c)\in\mathcal{RH}(\mathcal{L},\rho)^3$. If $x\in a\otimes(b\oplus c)$ then there exists $u\in(b\oplus c)$ such that $x\in a\otimes u$, therefore

$$\begin{aligned}(b\vee c,u)\in\rho,(a\wedge u,x)\in\rho &\Rightarrow (a\wedge(b\vee c),(a\wedge u))\in\rho\\ &\Rightarrow (a\wedge(b\vee c),x)\in\rho.\end{aligned}$$

On the other hand

$$\begin{aligned}x\in(a\otimes b)\oplus(a\otimes c) &\Rightarrow \exists g\in a\otimes b,\exists h\in a\otimes c: x\in g\oplus h\\ &\Rightarrow ((a\wedge b),g)\in\rho,((a\wedge c),h)\in\rho,((g\vee h),x)\in\rho\\ &\Rightarrow ((a\wedge b)\vee(a\wedge c),g\vee(a\wedge c))\in\rho,(g\vee(a\wedge c),(g\vee h))\in\rho\\ &\Rightarrow ((a\wedge b)\vee(a\wedge c),x)\in\rho.\end{aligned}$$

Now let $x_1 = a\wedge(b\vee c) \in a\otimes(b\oplus c) = (a\otimes b)\oplus(a\otimes c)$, and $x_2 = (a\wedge b)\vee(a\wedge c) \in a\otimes(b\oplus c) = (a\otimes b)\oplus(a\otimes c)$. Then, we have

$$(a \wedge (b \vee c), x_2) \in \rho \Rightarrow (a \wedge (b \vee c), (a \wedge b) \vee (a \wedge c)) \in \rho,$$

and

$$((a \wedge b) \vee (a \wedge c), x_1) \in \rho \Rightarrow ((a \wedge b) \vee (a \wedge c), a \wedge (b \vee c)) \in \rho.$$

Because ρ is anti-symmetric we conclude that $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ and so $\mathcal{L}$ is a distributive lattice. Conversely, let $\mathcal{L}$ be a distributive lattice

$$\begin{aligned} x \in a \otimes (b \oplus c) &\Rightarrow \exists u \in b \oplus c : x \in a \otimes u \\ &\Rightarrow (b \vee c, u) \in \rho, (a \wedge u, x) \in \rho \\ &\Rightarrow (a \wedge (b \vee c), (a \wedge u)) \in \rho \\ &\Rightarrow (a \wedge (b \vee c), x) \in \rho \\ &\Rightarrow ((a \wedge b) \vee (a \wedge c), x) \in \rho. \end{aligned}$$

Suppose that $(d = a \wedge b, f = a \wedge c)$ then

$$x \in d \oplus f \subseteq (a \otimes b) \oplus (a \otimes c) \Rightarrow a \otimes (b \oplus c) \subseteq (a \otimes b) \oplus (a \otimes c).$$

Now we let

$$\begin{aligned} y \in (a \otimes b) \oplus (a \otimes c) &\Rightarrow \exists v \in a \otimes b, t \in a \otimes c, y \in v \oplus t \\ &\Rightarrow (a \wedge b, v) \in \rho, (a \wedge c, t) \in \rho, (v \vee t, y) \in \rho \\ &\Rightarrow (v \vee (a \wedge c), (v \vee t)) \in \rho, ((a \wedge b) \vee (a \wedge c), v \vee (a \wedge c)) \in \rho \\ &\Rightarrow ((a \wedge b) \vee (a \wedge c), (v \vee t)) \in \rho \\ &\Rightarrow ((a \wedge b) \vee (a \wedge c), y) \in \rho \\ &\Rightarrow (a \wedge (b \vee c), y) \in \rho \\ &\Rightarrow y \in (a \otimes (b \vee c)) \subseteq (a \otimes (b \oplus c)). \end{aligned}$$

Thus $(a \otimes b) \oplus (a \otimes c) \subseteq a \otimes (b \oplus c)$ and so

$$a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c).$$

Now we want to prove that $a \oplus (b \otimes c) = (a \oplus b) \otimes (a \oplus c)$. Let

$$\begin{aligned} x \in a \oplus (b \otimes c) &\Rightarrow \exists u \in b \otimes c, x \in a \oplus u \\ &\Rightarrow ((b \wedge c), u) \in \rho, ((a \vee u), x) \in \rho \\ &\Rightarrow ((b \wedge c), u) \in \rho, ((a \vee u, x)) \in \rho \\ &\Rightarrow ((a \vee (b \wedge c)), (a \vee u)) \in \rho \\ &\Rightarrow (a \vee (b \wedge c), x) \in \rho \\ &\Rightarrow ((a \vee b) \wedge (a \vee c), x) \in \rho. \end{aligned}$$

If $a \vee b = d, a \vee c = f$ then

$$x \in d \otimes f \subseteq (a \oplus b) \otimes (a \oplus c) \Rightarrow a \oplus (b \otimes c) \subseteq (a \oplus b) \otimes (a \oplus c).$$

Conversely let

$$\begin{aligned} x \in (a \oplus b) \otimes (a \oplus c) &\Rightarrow \exists g \in a \oplus b, h \in a \oplus c, x \in g \otimes h \\ &\Rightarrow ((g \wedge h), x) \in \rho, ((a \vee b), g) \in \rho, ((a \vee c), h) \in \rho \\ &\Rightarrow ((g \wedge (a \vee c)), (g \wedge h)) \in \rho, (((a \vee b) \wedge (a \vee c)), (g \wedge (a \vee c))) \in \rho \\ &\Rightarrow (((a \vee b) \wedge (a \vee c)), x) \in \rho \\ &\Rightarrow ((a \vee (b \wedge c)), x) \in \rho. \end{aligned}$$

Let $s = b \wedge c$ therefore

$$x \in a \oplus s \subseteq a \oplus (b \otimes c) \Rightarrow (a \oplus b) \otimes (a \oplus c) \subseteq a \oplus (b \otimes c),$$

so

$$a \oplus (b \otimes c) = (a \oplus b) \otimes (a \oplus c).$$

□

Example 3.8. Suppose lattice $\mathcal{L} = (L, \wedge, \vee)$ is defined on $L = \{e, a\}$ as follows:

$\wedge$	e	a
e	e	e
a	e	a

$\vee$	e	a
e	e	a
a	a	a

In this case $\mathcal{L}$ is a distributive lattice. Now let $\rho = \{(e,e),(a,a),(e,a)\}$. Then, $\mathcal{RH}(\mathcal{L},\rho)$ is a s-distributive hyperlattice as bellow:

$\otimes$	e	a
e	$\{e,a\}$	$\{e,a\}$
a	$\{e,a\}$	a

$\oplus$	e	a
e	$\{e,a\}$	a
a	a	a

In a hyperlattice $\mathcal{L}$, define $x \leq y$ if and only if $y \in (x \oplus y)$ and $x \in (x \otimes y)$. In the following examples we show that $\leq$ is not an order relation on $\mathcal{L}$.

Example 3.9. $(L,\otimes,\oplus)$ is a hyperlattice for which $\otimes$ and $\oplus$ are defined on $L = \{a,b\}$ as follow:

$\otimes$	a	b
a	a	$\{a,b\}$
b	$\{a,b\}$	b

$\oplus$	a	b
a	$\{a,b\}$	$\{a,b\}$
b	$\{a,b\}$	b

The relation $a \leq b \Leftrightarrow a \in (a \otimes b), b \in (a \oplus b)$ is not anti-symmetric because $a \leq b$, $b \leq a$, $a \neq b$.

Example 3.10. $(L,\otimes,\oplus)$ is a hyperlattice for which $\otimes$ and $\oplus$ are defined on $L = \{a,b,c\}$ as follow:

$\otimes$	a	b	c
a	a	$\{a,b\}$	c
b	$\{a,b\}$	b	$\{a,b,c\}$
c	c	$\{a,b,c\}$	c

$\oplus$	a	b	c
a	a	$\{a,b,c\}$	a
b	$\{a,b,c\}$	b	$\{b,c\}$
c	a	$\{b,c\}$	c

The relation $a \leq b \Leftrightarrow a \in a \otimes b, b \in a \oplus b$ is not transitive because $a \leq b$, $b \leq c$ but $a \nleq c$. For this example

$$a \leq b \Leftrightarrow a \in a \otimes b = \{a,b\}, b \in a \oplus b = \{a,b,c\},$$

$$b \leq c \Leftrightarrow b \in b \otimes c = \{a,b,c\}, c \in b \oplus c = \{b,c\}.$$

But $a \nleq c$ because $a \notin a \otimes c = \{c\}$, $c \notin a \oplus c = \{a\}$.

Theorem 3.11. *Let L be a non-empty set and ρ be an order relation on L. If $\mathcal{L} = (L,\wedge,\vee)$ is a ρ-ordered lattice, then the following relation*

$$a \leq b \Leftrightarrow a \in a \otimes b, b \in a \oplus b,$$

is an order relation on $\mathcal{RH}(\mathcal{L},\rho)$.

Proof. Clearly $\leq$ is a reflexive relation. First we prove that $\leq$ is an anti-symmetric relation.

$$\begin{aligned} a \leq b &\Leftrightarrow a \in a \otimes b, b \in a \oplus b \\ &\Leftrightarrow ((a \wedge b), a) \in \rho, ((a \vee b), b) \in \rho \\ &\Leftrightarrow (b, (a \vee b)) \in \rho, ((a \vee b), b) \in \rho \\ &\Leftrightarrow b = a \vee b. \end{aligned}$$

$$\begin{aligned} b \leq a &\Leftrightarrow b \in a \otimes b, a \in a \oplus b \\ &\Leftrightarrow ((a \wedge b), b) \in \rho, ((a \vee b), a) \in \rho \\ &\Leftrightarrow (a, (a \vee b)) \in \rho, ((a \vee b), a) \in \rho \\ &\Leftrightarrow a = a \vee b. \end{aligned}$$

Therefore $a = b$ thus $\leq$ is an anti-symmetric relation. Now we should prove that $\leq$ is a transitive relation. Let $a \leq b$, $b \leq c$.

$$\begin{aligned} a \leq b, b \leq c &\Rightarrow a \in a \otimes b, b \in a \oplus b, b \in b \otimes c, c \in b \oplus c \\ &\Rightarrow ((a \wedge b), a) \in \rho, ((a \vee b), b) \in \rho, ((b \wedge c), b) \in \rho, ((b \vee c), c) \in \rho \\ &\Rightarrow (((a \vee b) \vee c), (b \vee c)) \in \rho, (c, (b \vee c)) \in \rho \\ &\Rightarrow ((a \vee c), (a \vee (b \vee c))) \in \rho, ((a \vee (b \vee c)), (b \vee c)) \in \rho, ((b \vee c), c) \in \rho \\ &\Rightarrow ((a \vee c), c) \in \rho \\ &\Rightarrow c \in a \oplus c. \end{aligned}$$

Also we have

$$\begin{aligned} a \leq b, b \leq c &\Rightarrow a \in a \otimes b, b \in a \oplus b, b \in b \otimes c, c \in b \oplus c \\ &\Rightarrow ((a \wedge b), a) \in \rho, ((a \vee b), b) \in \rho, ((b \wedge c), b) \in \rho, ((b \vee c), c) \in \rho \\ &\Rightarrow ((a \wedge c), ((a \wedge b) \wedge c)) \in \rho, (((a \wedge b) \wedge c), (a \wedge b)) \in \rho \\ &\Rightarrow ((a \wedge c), a) \in \rho \\ &\Rightarrow a \in a \otimes c. \end{aligned}$$

Therefore $a \leq c$. Hence $\leq$ is a transitive relation. $\square$

Example 3.12. Consider the s-distributive hyperlattice $\mathcal{RH}(\mathcal{L},\rho)$ in Example 3.8,

$\otimes$	e	a
e	$\{e,a\}$	$\{e,a\}$
a	$\{e,a\}$	a

$\oplus$	e	a
e	$\{e,a\}$	a
a	a	a

We can get the relation

$$x \leq y \Leftrightarrow x \in x \otimes y, y \in x \oplus y,$$

as bellow:

$$\leq = \{(e,e),(a,a),(e,a)\},$$

which is an order relation on $\mathcal{RH}(\mathcal{L},\rho)$.

Proposition 3.13. *Let $\mathcal{L} = (L, \wedge, \vee)$ be a lattice. Then, $\mathcal{RH}(\mathcal{L},\rho)$ is complete if and only if $\mathcal{L}$ is complete.*

Proof. Let S be an arbitrary subset of $\mathcal{RH}(\mathcal{L},\rho)$ therefore

$$\begin{aligned} S^u_{\mathcal{RH}(\mathcal{L},\rho)} &= \{x \in L \mid \forall s \in S, s \in s \otimes x, x \in s \oplus x\} \\ &= \{x \in L \mid \forall s \in S, ((s \wedge x), s) \in \rho, ((s \vee x), x) \in \rho\} \\ &= \{x \in L \mid \forall s \in S, (x, (x \vee s)) \in \rho, ((s \vee x), x) \in \rho\} \\ &= \{x \in L \mid \forall s \in S, x = s \vee x\} \\ &= S^u_{\mathcal{L}}. \end{aligned}$$

With a same argument we have $S^L_{\mathcal{RH}(\mathcal{L},\rho)} = S^L_L$. Therefore we have $\mathcal{RH}(\mathcal{L},\rho)$ is complete if and only if $\mathcal{L}$ is complete. □

Example 3.14. Let $\mathbb{Q}$ be the rational numbers. We define the hyperlattice $(\mathbb{Q}, \oplus, \otimes)$, where $a \oplus b = \{x \in \mathbb{Q} \mid a \leq x, b \leq x\}$ and $a \otimes b = \{x \in \mathbb{Q} \mid x \leq a, x \leq b\}$, for all $a, b \in \mathbb{Q}$. $(\mathbb{Q}, \oplus, \otimes)$ is a hyperlattice because

(i) $a \in a \otimes a$ and $a \in a \oplus a$,
(ii) $a \otimes b = b \otimes a$ and $a \oplus b = b \oplus a$,
(iii) $(a \otimes b) \otimes c = a \otimes (b \otimes c) = \{x \in \mathbb{Q} \mid x \leq min\{a,b,c\}\}$ and $(a \oplus b) \oplus c = a \oplus (b \oplus c) = \{x \in \mathbb{Q} \mid x \geq max\{a,b,c\}\}$,
(iv) $a \in a \otimes (a \oplus b)$ and $a \in a \oplus (a \otimes b)$,

for all $(a,b,c) \in \mathbb{Q}^3$. Moreover, according Proposition 3.12, the hyperlattice $(\mathbb{Q}, \oplus, \otimes)$ is not complete.

Proposition 3.15. *Let $\mathcal{L} = (L, \wedge, \vee)$ be a lattice. Then, $\mathcal{L}$ is bounded if and only if $\mathcal{RH}(\mathcal{L}, \rho)$ is bounded.*

Proof. Suppose that $\mathcal{L}$ is a bounded lattice. Then exist $0, 1 \in L$ such that, for all $x \in L$ we have $0 \leq x, x \leq 1$.

$$\begin{aligned} 0 \leq x, x \leq 1 &\Leftrightarrow x \wedge 0 = 0, x \vee 0 = x, x \vee 1 = 1, x \wedge 1 = x \\ &\Leftrightarrow ((x \wedge 0), 0) \in \rho, ((x \vee 0), x) \in \rho, ((x \vee 1), 1) \in \rho, ((x \wedge 1), x) \in \rho \\ &\Leftrightarrow 0 \in x \otimes 0, x \in x \oplus 0, x \in x \otimes 1, 1 \in x \oplus 1 \\ &\Leftrightarrow 0 \leq_{\mathcal{RH}(\mathcal{L},\rho)} x, x \leq_{\mathcal{RH}(\mathcal{L},\rho)} 1. \end{aligned}$$

Therefore $\mathcal{L}$ is bounded if and only if $\mathcal{RH}(\mathcal{L}, \rho)$ is bounded. □

Proposition 3.16. *Let $\mathcal{L} = (L, \wedge, \vee)$ be a lattice. Then, $\mathcal{L}$ is complemented if and only if $\mathcal{RH}(\mathcal{L}, \rho)$ is complemented.*

Proof. Let $\mathcal{L}$ be a complemented. Then, for all $x \in L$ exists $y \in L$ such that $x \wedge y = 0, x \vee y = 1$.

$$\begin{aligned} x \wedge y = 0, x \vee y = 1 &\Leftrightarrow ((x \wedge y), 0) \in \rho, ((x \vee y), 1) \in \rho \\ &\Leftrightarrow 0 \in x \otimes y, 1 \in x \oplus y. \end{aligned}$$

So $\mathcal{L}$ is complemented if and only if $\mathcal{RH}(\mathcal{L}, \rho)$ is complemented. □

Proposition 3.17. *Let $\mathcal{P}$ be a ρ-ordered lattice and $\mathcal{Q}$ be a ρ'-ordered lattice. If $f : P \to Q$ is an isomorphism such that $a\rho b$ if and only if $f(a)\rho' f(b)$, then $f : \mathcal{RH}(\mathcal{P}, \rho) \to \mathcal{RH}(\mathcal{Q}, \rho')$ is an isomorphism of induced hyperlattices.*

Proof. Suppose $(a, b) \in P^2$ then we have

$$\begin{aligned} f(a \otimes b) &= \{f(x) : x \in a \otimes b\} \\ &= \{f(x) : (a \wedge b)\rho x\} \\ &= \{f(x) : (f(a) \wedge f(b))\rho' f(x)\} \\ &= f(a) \otimes f(b). \end{aligned}$$

With the same argument we have $f(a \oplus b) = f(a) \oplus f(b)$ and hence f is an isomorphism between two hyperlattices, $\mathcal{RH}(\mathcal{P}, \rho)$ and $\mathcal{RH}(\mathcal{Q}, \rho')$. □

Remark 3.18. We have utilized $\mathcal{RH}(\mathcal{L}, \rho)$ in the text of 3.7, 3.11, 3.13, 3.15, 3.16 and 3.17 and get some results. Obviously, if we use $\mathcal{LH}(\mathcal{L}, \rho)$ instead of $\mathcal{RH}(\mathcal{L}, \rho)$ the results hold.

In the following we give a characterization of the relation υ^* for the class of distributive right induced hyperlattice of $\mathcal{L}$ and ρ.

Theorem 3.19. *Let $\mathcal{RH}(\mathcal{L},\rho)=(L,\otimes,\oplus)$ be a distributive hyperlattice. Now let the relation $\theta=\rho^{-1}\circ\rho$ on L. Then, the transitive closure of θ which is denoted by θ^*, is smallest strong regular relation on L and the quotient L/θ^* is a lattice.*

Proof. First of all we should prove that

$$x\theta y \Leftrightarrow \exists(z_1,z_2,...,z_n)\in L^n, \exists z\in \mathrm{Im}(\{z_1,z_2,...,z_n\}) : \{x,y\}\subseteq z.$$

Let x,y be arbitrary elements of L. Then, we have

$$\begin{aligned} x\theta y &\Leftrightarrow x\rho^{-1}\circ\rho y \\ &\Leftrightarrow \exists a\in L : x\rho^{-1}a, a\rho y \\ &\Leftrightarrow a\rho x, a\rho y \\ &\Leftrightarrow (a\wedge a)\rho x, (a\wedge a)\rho y \\ &\Leftrightarrow \{x,y\}\subseteq a\otimes a. \end{aligned}$$

Consequently $\theta\subseteq\upsilon$. Because L/θ^* is a lattice, by Corollary 2.16 we have θ^* is the smallest strong regular relation on L and the proof is completed. □

Remark 3.20. If we use $\mathcal{LH}(\mathcal{L},\rho)$=$(L,\otimes,\oplus)$ and $\theta'=\rho\circ\rho^{-1}$ then by Theorem 3.19 obviously we have L/θ'^* is a lattice and θ'^* is the smallest strong regular relation on L.

Proposition 3.21. *Let $(L,\otimes,\oplus)$ be a hyperlattice and $\{A_x : x\in L\}$ be a family of non-empty disjoint sets for all elements of L. Then, $(K_L,\sqcap,\sqcup)$ is a hyperlattice, where*

$$a\sqcap b=\cup_{u\in x\otimes y}A_u \quad and \quad a\sqcup b=\cup_{u\in x\oplus y}A_u,$$

for which $a\in A_x$ and $b\in A_y$, and $K_L=\cup_{x\in L}A_x$. We call K_L the generated hyperlattice by L.

Proof. Let $(a,b,c)\in K_L^3$ and $a\in A_x$, $b\in A_y$ and $c\in A_z$. Then we have

(i) $a\sqcap a=\cup_{u\in x\otimes x}A_u$. Therefore $a\in A_x\subseteq\cup_{u\in x\otimes x}A_u$, and so $a\in a\sqcap a$.

(ii) $a\sqcap b=\cup_{u\in x\otimes y}A_u=\cup_{u\in y\otimes x}A_u=b\sqcap a$.

(iii)

$$\begin{aligned}(a \sqcap b) \sqcap c &= [\cup_{u \in x \otimes y} A_u] \sqcap c \\ &= \cup_{u \in x \otimes y} [A_u \sqcap c] \\ &= \cup_{u \in x \otimes y, v \in u \otimes z} A_v \\ &= \cup_{v \in (x \otimes y) \otimes z} A_v \\ &= \cup_{v \in x \otimes (y \otimes z)} A_v \\ &= a \sqcap [\cup_{u \in y \otimes z} A_u] \\ &= a \sqcap (b \sqcap c).\end{aligned}$$

(iv) $a \sqcap (a \sqcup b) = a \sqcap [\cup_{u \in x \oplus y} A_u] = \cup_{v \in x \otimes (x \oplus y)} A_v$, hence $a \in A_x \subseteq \cup_{v \in x \otimes (x \oplus y)} A_v$. Thus $a \in a \sqcap (a \sqcup b)$. It is easy to see that if we replace $\sqcap$ with $\sqcup$ in above facts then they hold as well.

□

Proposition 3.22. *Let $\mathcal{L} = (L, \wedge, \vee)$ be a lattice. Then, K_L is distributive if and only if L is distributive.*

Proof. If $(a, b, c) \in K_L^3$ and $a \in A_x$, $b \in A_y$ and $c \in A_z$, then we have

$$a \sqcap (b \sqcup c) = A_{x \wedge (y \vee z)} = A_{(x \wedge y) \vee (x \wedge z)} = (a \sqcap b) \sqcup (a \sqcap c).$$

Similarly

$$a \sqcup (b \sqcap c) = (a \sqcup b) \sqcap (a \sqcup c).$$

□

Example 3.23. Let $\mathcal{L} = (L, \wedge, \vee)$ be the distributive lattice in Example 3.8. i.e.

$\wedge$	e	a
e	e	e
a	e	a

$\vee$	e	a
e	e	a
a	a	a

If $A_e = \{0, 1\}$ and $A_a = \{2\}$ are the disjoint sets then, K_L is a distributive hyperlattice which has the following tables:

$\sqcap$	0	1	2
0	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
1	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
2	$\{0,1\}$	$\{0,1\}$	$\{2\}$

$\sqcup$	0	1	2
0	$\{0,1\}$	$\{0,1\}$	$\{2\}$
1	$\{0,1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{2\}$	$\{2\}$

Proposition 3.24. *Let L be a distributive hyperlattice and $\upsilon^*_{K_L}$ and υ^*_L be the smallest strongly equivalence relations on K_L and L, respectively. Then*

$$a\upsilon_{K_L} b \Leftrightarrow x\upsilon_L y,$$

where $a \in A_x$ and $b \in A_y$.

Proof. Let $a \in A_x$, $b \in A_y$ and $a\upsilon_{K_L} b$, so there exists $(c_{11}, c_{12}, ..., c_{1n}, ..., c_{mn}) \in K_L^{m\times n}$ such that $\{a, b\} \subseteq \otimes_{i=1}^n \oplus_{j=1}^m c_{ij}$. Thus there exists $(z_{11}, z_{12}, ..., z_{1n}, ..., z_{mn}) \in L^{m\times n}$ such that $a_{ij} \in A_{z_{ij}}$ for all $1 \leq i \leq n$, $1 \leq j \leq m$, and $\otimes_{i=1}^n \oplus_{j=1}^m c_{ij} = \cup_{u\in\otimes_{i=1}^n\oplus_{j=1}^m z_{ij}} A_u$. Therefore $\{x, y\} \subseteq \otimes_{i=1}^n \oplus_{j=1}^m z_{ij}$ and so $x\upsilon_L y$ follows. The converse also hold and the proof is completed. □

Corollary 3.25. *If $(a, b) \in K_L^2$ and $a \in A_x$, $b \in A_y$, then*

$$a\upsilon^*_{K_L} b \Leftrightarrow x\upsilon^*_L y.$$

4. Construction of hyperlattices from genetic codes

In this section we introduce the induced right and left genetic code hyperlattices. The genetic code is the biochemical system primarily used to establish the rules by which the nucleotide sequence of a gene is transcribed into an mRNA codon sequence and ultimately translated into an amino acid sequence of a corresponding protein. This code is an extension of the four-letter alphabet of the DNA bases: adenine, guanine, cytosine and thymine, usually denoted A, G, C, T where the T in the RNA is changed to U, uracil. It is well known that there is an association between the base in the second position and hydrophobicity where the amino acid having U at the second position of its codon is hydrophobic: $\{I, L, M, F, V\}$ (amino acids are written using one-letter symbols), whereas those having A at the second position are hydrophilic (polar amino acids): $\{D, E, H, N, K, Q, Y\}$ [6]. In [8] pointed out that to some extent 'related' amino acids have related codons and [6] considered that the amino acids in the genetic code table do not seem to be allocated totally at random. So, naturally, one can think that a kind of partial order in the codon set should reflect the physico-chemical properties of amino acids ([11], [15]). A number of mathematical models have been proposed for the understanding of the origin and evolution of the genetic code ([2], [9], [10]). In [26] R. Sanchez, et al. introduced 2 Boolean lattices of the four DNA bases as bellow:

1) Primal Boolean Algebra,

$\vee$	G	A	U	C
G	G	A	U	C
A	A	A	C	C
U	U	C	U	C
C	C	C	C	C

$\wedge$	G	A	U	C
G	G	G	G	G
A	G	A	G	A
U	G	G	U	U
C	G	A	U	C

2) Dual Boolean Algebra,

$\vee$	C	U	A	G
C	C	U	A	G
U	U	U	G	G
A	A	G	A	G
G	G	G	G	G

$\wedge$	C	U	A	G
C	C	C	C	C
U	C	U	C	U
A	C	C	A	A
G	C	U	A	G

Now consider the following table obtained from physico-chemical properties of amino acids of four DNA bases according to [17].

X	Symbols of a genetic letter from a viewpoint of a kind of the binary-opposite attributes	C	A	G	U
$N_1(X)$	0 - pyrimidines (one ring in a molecule); 1 - purines (two rings in a molecule)	0	1	1	0
$N_2(X)$	0 - a letter with amino-mutating property (amino); 1 - a letter without it (keto)	0	0	1	1

We can define the ρ_p-ordered Primal Boolean Algebra and ρ_d-ordered dual Boolean Algebra as bellow:

$$X\rho_p Y \Leftrightarrow N_2(X) = N_2(Y), N_1(X) \geq N_1(Y),$$

$$X\rho_d Y \Leftrightarrow N_2(X) = N_2(Y), N_1(X) \leq N_1(Y),$$

for all X, Y in $\{C, A, G, U\}$. We can see that

$$\rho_p = \{(A, C), (G, U), (A, A), (C, C), (G, G), (U, U)\},$$

and

$$\rho_d = \{(C, A), (U, G), (A, A), (C, C), (G, G), (U, U)\}.$$

Moreover, the right induced hyperlattice from ρ_p-ordered primal Boolean Algebra of primal Boolean Lattice and ρ_p; i.e., $\mathcal{RH}(\mathcal{L},\rho)$ is as bellow:

$\oplus_p$	G	A	U	C
G	$\{U,G\}$	$\{C,A\}$	U	C
A	$\{C,A\}$	$\{C,A\}$	C	C
U	U	C	U	C
C	C	C	C	C

$\otimes_p$	G	A	U	C
G	$\{U,G\}$	$\{U,G\}$	$\{U,G\}$	$\{U,G\}$
A	$\{U,G\}$	$\{C,A\}$	$\{U,G\}$	$\{C,A\}$
U	$\{U,G\}$	$\{U,G\}$	U	U
C	$\{U,G\}$	$\{C,A\}$	U	C

Using ρ_d and the dual Boolean lattice we have the following hyperlattice.

$\oplus_d$	C	U	A	G
C	$\{A,C\}$	$\{G,U\}$	A	G
U	$\{G,U\}$	$\{G,U\}$	G	G
A	A	G	A	G
G	G	G	G	G

$\otimes_d$	C	U	A	G
C	$\{A,C\}$	$\{A,C\}$	$\{A,C\}$	$\{A,C\}$
U	$\{A,C\}$	$\{G,U\}$	$\{A,C\}$	$\{G,U\}$
A	$\{A,C\}$	$\{A,C\}$	A	A
G	$\{A,C\}$	$\{G,U\}$	A	G

5. Conclusion

In the last few years, researchers have investigated hyperstructure theory, principally from a theoretical point of view. They have found interesting and useful applications in chemistry, algebraic geometry, cryptography, scheme theory. Here the authors continue the study on the research topic started in [23] about lattices derived from hyperlattices. First using ρ-ordered lattice $\mathcal{L}$, right induced hyperlattice $\mathcal{RH}(\mathcal{L},\rho)$, left induced hyperlattice $\mathcal{LH}(\mathcal{L},\rho)$) and the hyperlattice generated by a hyperlattice have been introduced. Some properties of these classes of hyperlattices are studied. Finally, the right induced hyperlattices of two Boolean lattices of the four DNA bases which are called Primal Boolean Algebra and Dual Boolean Algebra, and two order relations ρ_p and ρ_d which is associated from physico-chemical properties of amino acids of four DNA bases are investigated. We hope that the proposed work helps to broaden the applications of hyperstructures in future works.

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