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Research Paper

HOPFCITY AND JACOBSON SMALL SUBMODULES

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Abstract. The study of modules by properties of their endomorphisms has long been of interest. In this paper, we introduce the notion of jacobson weakly Hopfian modules. It is shown that over a ring R, every projective (free) R-module is jacobson weakly Hopfian if and only if R has no nonzero semisimple projective R-module. Let L be a module such that L satisfies ascending chain conditions on jacobson-small submodules. Then it is shown that L is jacobson weakly Hopfian. Some basic characterizations of projective jacobson weakly Hopfian modules are proved.

1. Introduction

Throughout this paper all rings have identity and all modules are unitary right modules. Let L be an R-module, for submodules X and Y of L, $X \leq Y$ denotes that X is a submodule of Y, $X \leq^{\oplus} L$ denotes that X is a direct summand of L, $\mathrm{Rad}(L)$ denote the radical of L and $\mathrm{End}_R(L)$ denote the ring of endomorphisms of L.

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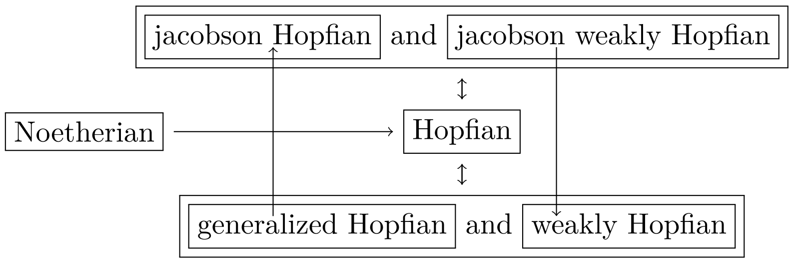
Recall that a submodule K of an R-module L is said to be small in L, if for every submodule $H \leq L$ with $K + H = L$ implies $H = L$, and in this case we write $K \ll L$. The socle of L is defined as the sum of all its simple submodules and can be shown to coincide with the intersection of all the essential submodules of L. It is a fully invariant submodule of L. Note that L is semisimple precisely when $L = \mathrm{Soc}(L)$ (see [1]). The radical of an R-module L defined as a dual of the socle of L, is the intersection of all maximal submodules of L, taking $\mathrm{Rad}(L) = L$ when L has no maximal submodules. A submodule K of L is said to be jacobson-small in L ($K \ll_J L$), in case $L = K + H$ with $\mathrm{Rad}(L/H) = L/H$, implies $L = H$ (see [7]). It is clear that if A is a small submodule of L, then A is a jacobson-small submodule of L, but the converse is not true in general. By [7], if $\mathrm{Rad}(L) = L$ and $K \leq L$, then K is small in L if and only if K is jacobson-small in L. For a right R-module L, Talebi and Vanaja [13], defined the submodule $\overline{Z}(L) = \cap\{Kerf : f \in Hom(L, N), N \in S\} = \cap\{K \subset L, L/K \in S\}$ as a dual of singular submodule, where S denotes the class of all small right R-modules. A module L is called cosingular (resp. noncosingular) if $\overline{Z}(L) = 0$ (resp. $\overline{Z}(L) = L$). Recall that a submodule N of L is said to be γ-small in L (denoted by $N \ll_\gamma L$), if $L = N + X$ with L/X noncosingular implies $L = X$. In other words, $L \neq N + X$ for every proper submodule X of L with L/X noncosingular (see [8]). A submodule K of an R-module L is said to be δ-small in L, written $K \ll_\delta L$, if for every submodule N of L such that $K + N = L$ with L/N singular implies $N = L$ (see [16])

The concept of Hopfian modules has been extensively studied in the literature. Recall that a module L is said to be Hopfian if every surjective endomorphism of L is an automorphism (see [6]), and a module L is said to be co-Hopfian if every injective endomorphism of L is an automorphism (see [14]). In [5], a proper generalization of Hopfian modules, called generalized Hopfian modules, was given. A right R-module L is called generalized Hopfian, if any surjective endomorphism of L has a small kernel. In [15], an other proper generalization of Hopfian modules, called weakly Hopfian modules, was given. A right R-module L is called weakly Hopfian, if any small surjective endomorphism of L is an automorphism. It is clear that a right R-module L is Hopfian if and only if L is both generalized Hopfian and weakly Hopfian. In [4], the concept of γ-Hopfian modules was investigated. A right R-module L is called γ-Hopfian if any surjective endomorphism of L has a γ-small kernel. In [3], the concept of δ-weakly Hopfian modules was introduced. A right R-module L is called δ-weakly Hopfian if any δ-small surjective endomorphism of L is an automorphism. In [2], the concept of jacobson Hopfian modules was studied. A right R-module L is called jacobson Hopfian if any surjective endomorphism of L has a jacobson-small kernel. Such modules and others generalizations have been considered by many authors ([2, 3, 4, 5, 6, 14, 15]).

Motivated by the above-mentioned works, we are interested in introducing a new generalization of Hopfian modules namely jacobson weakly Hopfian modules. We call a module jacobson weakly Hopfian if any its jacobson-small surjective endomorphism is an automorphism, which implies that a right R-module L is Hopfian if and only if L is both jacobson Hopfian and jacobson weakly Hopfian.

Therefore, we obtain the following diagram:



The paper is organized as follows:

In Section 2, some basic characterizations of projective jacobson weakly Hopfian modules are proved in (Theorem 2.6). It is proved that a projective module L is jacobson weakly Hopfian if and only if whenever $f \in \mathrm{End}_R(L)$ has a right inverse and $Ker(f)$ is semisimple, then f has a left inverse in $\mathrm{End}_R(L)$. We show also that if every projective (free) R-module is jacobson weakly Hopfian if and only if R has no nonzero semisimple projective R-module (Theorem 2.7).

In [15], Yongduo Wang proved that if the ACC holds on small submodules of L, then L is weakly Hopfian. In [3], El Moussaouy, Moniri Hamzekolaee, Ziane and Khoramdel showed that if the ACC holds on δ-small submodules of L, then L is δ-weakly Hopfian. Also we know that Noetherian modules are Hopfian modules. Thus it is natural to prove that if the ACC holds on jacobson-small submodules of L, then L is jacobson weakly Hopfian (Theorem 2.20).

At the end of the paper, some open problems are given.

We list some properties of jacobson-small submodules that will be used in the paper.

Lemma 1.1. [2] *Let L be an R-module and $K \leq L$. The following are equivalent.*

(1) $K \ll_J L$.

(2) *If $X + K = L$, then $L = X \oplus H$ for a semisimple submodule H of L.*

Lemma 1.2. [7].

Let $L = L_1 \oplus L_2$ be an R-module and let $A_1 \leq L_1$ and $A_2 \leq L_2$. Then $A_1 \oplus A_2 \ll_J L_1 \oplus L_2$ if and only if $A_1 \ll_J L_1$ and $A_2 \ll_J L_2$.

2. Modules whose jacobson-small surjective endomorphisms are isomorphism

Motivated by the definition of Hopfian modules and the definition of δ-weakly Hopfian modules, we introduce the key definition of this paper.

Definition 2.1. Let L be an R-module. We say that L is jacobson weakly Hopfian if any jacobson-small surjective endomorphism of L is an automorphism.

The following example introduces a module which is not jacobson weakly Hopfian.

Example 2.2. There exists a jacobson-small epimorphism which is not an isomorphism. Let $G = Z_{p^\infty}$, since in G every proper subgroup is jacobson-small, so every surjective endomorphism of G has a jacobson-small kernel, but the multiplication by p induces an epimorphism of G which is not an isomorphism.

Lemma 2.3. *For a nonzero module L, the following statements are equivalent.*

(i) L is jacobson weakly Hopfian;

(ii) $L/K \cong L$ for any jacobson-small submodule K of L if and only if $K = 0$.

Proof. (i) $\Rightarrow$ (ii) Suppose $L \cong L/K$ for some $K \ll_J L$. Let $\varphi : L/K \to L$ be an isomorphism and $\pi : L \to L/K$ the canonical epimorphism. Then the map $\varphi\pi$ is an epimorphism with $Ker(\varphi\pi) = K$. Then $\varphi\pi$ is a jacobson-small epimorphism. So $\varphi\pi$ is an isomorphism by (i), and so $K = 0$.

(ii) $\Rightarrow$ (i) Let $f : L \to L$ be a jacobson-small epimorphism. Then $L \cong L/Ker(f)$ by first isomorphism theorem. From (ii), we get $Ker(f) = 0$. This shows f is an isomorphism. Hence L is jacobson weakly Hopfian. □

Proposition 2.4. *Let L be a jacobson weakly Hopfian module. If $L \cong L \oplus N$ for some semisimple module N, then $N = 0$. Moreover, if L is projective, then the converse holds.*

Proof. Let L be a jacobson weakly Hopfian module and $L \cong L \oplus N$ for some semisimple module N. It is easy to see that $L \cong K \oplus H$ where $K \cong N$ and $H \cong L$. Note that K is a jacobson-small submodule of L as N is semisimple by Lemma 1.1. Since $L/K \cong H \cong L$, $K = 0$ by Lemma 2.3.

For the moreover statement, assume that L is projective and f is a surjective endomorphisme of L, where $Ker(f) \ll_J L$. Then $L = Kerf \oplus T$, where $T \leq L$ and $T \cong L$. Since $Ker(f) \ll_J L$, we have $L = P \oplus T$ where P is a semisimple submodule of $Ker(f)$, by Lemma 1.1. Now, modular law implies that $Ker(f) = P$. Therefore $L \cong Ker(f) \oplus L$ and $Kerf$ is semisimple. Hence $Ker(f) = 0$ and L will be a jacobson weakly Hopfian module. □

Proposition 2.5. *Let R be a semisimple artinian ring. Then a free R-module F is jacobson weakly Hopfian if and only if it has finite rank.*

Proof. Let F be a free R-module jacobson weakly Hopfian. If F has infinite rank, then $R^{\mathbb{N}}$ is jacobson weakly Hopfian (because $R^{\mathbb{N}}$ is a direct summand of F). Since $R^{\mathbb{N}} \cong R^{\mathbb{N}} \oplus R^{\mathbb{N}}$ and $R^{\mathbb{N}} \neq 0$, it is impossible, by Proposition 2.4. Hence F has finite rank. Conversely, If F has finite rank, then it is Hopfian and so it is jacobson weakly Hopfian. □

In the following, we explore some basic characterizations of projective jacobson weakly Hopfian modules.

Theorem 2.6. *Let L be a projective R-module and $f \in \mathrm{End}_R(L)$. Then the following statements are equivalent:*

(1) *L is jacobson weakly Hopfian.*
(2) *If f has a right inverse and $Ker(f)$ is semisimple, then f has a left inverse in $\mathrm{End}_R(L)$.*
(3) *If f has a right inverse and $Ker(f) \ll_J L$, then f has a left inverse in $\mathrm{End}_R(L)$.*
(4) *If f has a right inverse g and $(1 - gf)L \ll_J L$, then f has a left inverse in $\mathrm{End}_R(L)$.*
(5) *If f is a surjective endomorphism and $Ker(f)$ is semisimple, then f has a left inverse in $\mathrm{End}_R(L)$.*

Proof. If L is projective and $f \in \mathrm{End}_R(L)$, then f is a surjective endomorphism if and only if f has a right inverse g. Therefore $Ker(f) = (1 - gf)L$ and $L = Ker(f) \oplus (gf)L$.

(1) $\Rightarrow$ (2) If f has a right inverse g, then $fg = 1$. Since $Ker(f) \leq^{\oplus} L$, it is projective. Hence $L \cong L \oplus Kerf$ where $Kerf$ is semisimple. Now by Proposition 2.4, $Ker(f) = 0$.

(2) $\Rightarrow$ (3) Suppose that f has a right inverse and $Ker(f) \ll_J L$. Since $Ker(f) \leq^{\oplus} L$, $Ker(f)$ is semisimple. Therefore f has a left inverse in $\mathrm{End}_R(L)$.

(3) $\Rightarrow$ (4) It is clear, because $Ker(f) = (1 - gf)L \ll_J L$

(4) $\Rightarrow$ (5) It is clear, because $Ker(f) = (1-gf)L \ll_J L$ if and only if $Ker(f)$ is semisimple.

(5) $\Rightarrow$ (1) Let f be a surjective endomorphism of L and $Ker(f) \ll_J L$. Since L is projective, f has a right inverse g and $Ker(f) = (1-gf)L \leq^{\oplus} L$. Hence $Ker(f)$ is semisimple. Therefore by (5), f has a left inverse and it is an automorphism. □

Theorem 2.7. *Let R be a ring. Then the following statements are equivalent:*

(1) *Every projective R-module is jacobson weakly Hopfian.*
(2) *Every free R-module is jacobson weakly Hopfian.*
(3) *Every maximal right ideal of R is essential in R_R.*
(4) *R has no nonzero semisimple projective R-module.*

Proof. (1) $\Rightarrow$ (2) Is clear.

(2) $\Rightarrow$ (1) Is clear by Proposition 2.14

(1) $\Rightarrow$ (3) Assume that m is a maximal right ideal of R. Then either m is a direct summand of R_R or it is essential in R_R. If m is a direct summand of R_R, then $L = (R/m)^{(\mathbb{N})}$ is semisimple and projective. Therefore L is jacobson weakly Hopfian by (1). Since $L \cong L \oplus L$, $L = 0$, by Proposition 2.4, which is impossible, and so m is essential in R_R.

(3) $\Rightarrow$ (4) Is clear.

(4) $\Rightarrow$ (1) Assume that L is a projective module and $f : L \to L$ is an epimorphism where $Ker(f) \ll_J L$. Since L is projective, there existe an endomorphism g of L which makes the following diagram commutative.

$$\begin{tikzcd}
 & L \arrow[dl, dotted, "g"'] \arrow[d, "id"] & \\
L \arrow[r, "f"] & L \arrow[r] & 0
\end{tikzcd}$$

Therefore, $fg = id$ and $L = Kerf \oplus Img$. Since $Ker(f) \ll_J L$, then by Lemma 1.1, $L = N \oplus Img$, for some semisimple submodule N of $Ker(f)$. And N is projective, as $N \leq^{\oplus} L$. Hence by modular law $Ker(f) = N \oplus (Im(g) \cap Ker(f)) = N$. Since R has no nonzero semisimple projective R-module, $N = 0$, hence $Kerf = 0$. Therefore f is an automorphism and L is jacobson weakly Hopfian. □

Recall that a ring R is a right GV-ring provided every simple R-module is either projective or injective. It is known that R is a right GV-ring if and only if every simple singular R-module is injective (see [10]). A ring R is called CP in case every cosingular right R-module is projective. A ring R is right GV if and only if every small right R-module is projective (see [11]).

Corollary 2.8. *Let L be an R-module. Then the following statements hold.*

(1) *If R is right GV, then every indecomposable small right R-module is jacobson-weakly Hopfian.*

(2) *If R is right CP, then every indecomposable cosingular right R-module is jacobson-weakly Hopfian.*

Recall that a ring R is right CD if and only if every cosingular right R-module is discrete (see [12]).

Proposition 2.9. [12, Proposition 2.26] *Let R be a commutative domain. Then the following are equivalent:*

(1) *R is CD;*

(2) *Every cosingular R-module is projective.*

Corollary 2.10. *Let R be a commutative domain and L an R-module. If R is right CD, then every indecomposable cosingular right R-module is jacobson-weakly Hopfian.*

Proposition 2.11. *Let L be an R-module. If L/N is jacobson weakly Hopfian for any nonzero jacobson-small submodule N of L, then L itself is jacobson-weakly Hopfian.*

Proof. If L is not jacobson weakly Hopfian. Then there exists a jacobson-small surjection f of L which is not an isomorphism, and f induces an isomorphism $g : L/Kerf \to L$. If $\pi : L \to L/Kerf$ denotes the canonical quotient morphism, then $\pi g : L/Kerf \to L/Kerf$ is a jacobson-small surjection which is not an isomorphism. This is a contradiction. □

Example 2.12. Let P be a set of all primes and $\mathbb{Q}/\mathbb{Z} = \bigoplus_{p\in P} \mathbb{Z}_{p^\infty}$. If $\bigoplus_{p\in P} \mathbb{Z}_{p^\infty}$ is jacobson weakly Hopfian $\mathbb{Z}$-module, hence $\mathbb{Z}_{p^\infty}$ is jacobson weakly Hopfian by Proposition 2.14, contradiction with example 2.2. Then $\mathbb{Q}/\mathbb{Z}$ is not jacobson weakly Hopfian, but $\mathbb{Q}$ is jacobson weakly Hopfian $\mathbb{Z}$-module.

Proposition 2.13. *Let L be a quasi-projective module, if L is co-Hopfian, then it is jacobson weakly Hopfian.*

Proof. Let $f : L \to L$ be a jacobson-small surjective endomorphism, since L is quasi-projective, there exists $g : L \to L$, such that $fg = id_L$, then g is a injective endomorphism, since L is co-Hopfian, so g is automorphism, which shows that f is an automorphism, then L is jacobson weakly Hopfian. □

Proposition 2.14. *Any direct summand of a jacobson weakly Hopfian module L is jacobson weakly Hopfian.*

Proof. Let $K \leq^{\oplus} L$. Then there exists N a submodule of L such that $L = K \oplus N$. Let $f : K \to K$ be a jacobson-small surjective endomorphism of K, then f induces a surjective endomorphism of L, $f \oplus 1_N : L \to L$ with $(f \oplus 1_N)(k + n) = f(k) + n$, where $k \in K$ and $n \in N$. Thus by lemma 1.2, $Ker(f \oplus 1_N) = Ker(f) \oplus 0 \ll_J K \oplus N$. Since L is jacobson weakly Hopfian, $f \oplus 1_N$ is automorphism of L and hence f is an automorphism of K, then K is jacobson weakly Hopfian. □

The next result gives a condition that a direct sum of two jacobson weakly Hopfian modules is jacobson weakly Hopfian.

Proposition 2.15. *Let $L = L_1 \oplus L_2$ and let L_1, L_2 be fully invariant submodules under any surjection of L. Then L is jacobson weakly Hopfian if and only if L_1, L_2 are jacobson weakly Hopfian.*

Proof. $\Rightarrow$) Clear by Proposition 2.14.

$\Leftarrow$) Let $f : L \to L$ be a jacobson-small epimorphism, then $f|_{L_i} : L_i \to L_i$ where $i \in \{1; 2\}$, is a jacobson-small surjection. By assumption, $f|_{L_i}$ is automorphism. Let $f(x_1 + x_2) = 0$, then $f(x_1) + f(x_2) = 0$ and so $x_1 = x_2 = 0$. Thus f is injective. Then L is jacobson weakly Hopfian. □

Definition 2.16. [9] A module L is called duo, provided that every submodule of L is fully invariant.

Corollary 2.17. *Let $L = L_1 \oplus L_2$ be a duo module. Then L is jacobson weakly Hopfian if and only if L_1 and L_2 are jacobson weakly Hopfian.*

It is clear that every jacobson weakly Hopfian module is weakly Hopfian. The following examples shows that the converse is not true, in general.

Example 2.18. Note that $L = \mathbb{Z}_6$ is a semisimple $\mathbb{Z}$-module. Since for any semisimple module L we have $\mathrm{Rad}(L) = 0$, so any proper submodule is jacobson-small in L. Hence L is not jacobson weakly Hopfian. But L has non nonzero small submodule, then L is weakly Hopfian.

Lemma 2.19. *Let M, N and L be modules. If $f : M \to N$ and $g : N \to L$ are two jacobson-small epimorphisms. Then gf is jacobson-small epimorphism.*

Proof. Suppose that $Kergf + K = M$, with $\mathrm{Rad}(M/K) = M/K$, then $Kerg + f(K) = f(M)$. As $\mathrm{Rad}(M/K) = M/K$ and $f(\mathrm{Rad}(M/K)) \subseteq \mathrm{Rad}(f(M/K))$. Hence $f(M/K) = f(M)/f(K) \subseteq \mathrm{Rad}(f(M)/f(K))$. Then $\mathrm{Rad}(f(M)/f(K)) = f(M)/f(K)$. And since $Kerg \ll_J f(M) = N$, $f(M) = f(K)$, then $M = Kerf + K$. Since $Kerf \ll_J M$ and $\mathrm{Rad}(M/K) = M/K$, $M = K$. Thus gf is jacobson-small epimorphism. □

Theorem 2.20. *Let L be an R-module with ACC on jacobson-small submodules. Then L is jacobson weakly Hopfian.*

Proof. Let L be an R-module and $f : L \to L$ be a jacobson-small epimorphism of L. Then $Kerf \subseteq Kerf^2 \subseteq ... \subseteq Kerf^n \subseteq ...$ is an ascending chain of jacobson-small submodules of L by Lemma 2.19. Since L satisfies the ACC on jacobson-small submodules, then there exists a positive number n such that $Kerf^n = Kerf^{n+1}$. Let $x \in Kerf$, then $f(x) = 0$. Since f is an epimorphism, there exists $x_1 \in L$ such that $f(x_1) = x$. Since f is an epimorphism, there exists $x_2 \in L$ such that $f(x_2) = x_1$. Repeating the process, we obtain $x_{n-1} \in L$ with $f(x_n) = x_{n-1}$. Thus

$$x = f(x_1) = f^2(x_2) = ... = f^n(x_n).$$

Since $x \in Kerf$, $0 = f(x) = f(f^n(x_n))$, that is, $f^{n+1}(x_n) = 0$. So $x_n \in Kerf^{n+1} = Kerf^n$. Consequently, $f^n(x_n) = 0$ and hence $x = 0$, so $Kerf = 0$ and f is an isomorphism. Then L is jacobson weakly Hopfian. □

Open Problems

(1) What is the structure of rings whose finitely generated right modules are jacobson weakly Hopfian?

(2) Let R be a ring with identity, and M a jacobson weakly Hopfian module. Is $M[X, X^{-1}]$ jacobson weakly Hopfian in $R[X, X^{-1}]$-module?

(3) Let R be a jacobson weakly Hopfian ring and $n \geq 1$ an integer. Is the matrix ring $M_n(R)$ jacobson weakly Hopfian?

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