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Research Paper

CHARACTERIZATIONS OF J-PRIME IDEALS AND M_J-IDEALS IN POSETS

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Abstract. In this paper, we introduce the concepts of J-prime ideals and M_J-ideals in posets, and obtain some of their interesting characterizations in posets. Furthermore, we discuss the properties of J-ideals that are analogous to J-prime ideals and M_J-ideals in posets. Finally, we establish a set of equivalent conditions for an ideal in a poset $\mathcal{P}$ containing an ideal J is an J-ideal, and for a semi-prime ideal J to be an M_J-ideal of $\mathcal{P}$.

1. Introduction

A relation is a mathematical concept that specifies how items are related to one another. Database and scheduling applications typically use relationships. Science and technology are intended to help people make better decisions. Human expectancies, multiple choice results, and confidence levels must all be identified before making decisions. This information will be used to create partial orders. Many fields of mathematics use ordered structures. At its most

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fundamental level, ordering is a key aspect of natural numbers, and the greater than/less than relationship is often used in everyday reasoning.

The notion of the partially ordered set (poset) is derived from this relationship, and structures based on posets have a variety of applications in computer science. Today, partial order and lattice theory are widely used in electrical and computer engineering. They are used in cloud computing (vector clocks, global predicate detection), concurrency theory (pomsets, occurrence nets), and data mining (concept analysis). Combinatorics, number theory, and group theory all benefit from them. Partially ordered sets occur naturally when working with multidimensional systems of qualitative ordinal variables in social science. Instead of standard techniques, partial order theory and partially ordered sets can be used to generate composite indicators for evaluating well-being, quality of life, and multidimensional poverty. They can be used to make decisions or analyse data in the same way that multi-criteria analysis and social choice theory are. They are also useful in social network analysis, where math is used to investigate network topologies and dynamics.

For nearly four decades, many authors have been captivated by the partially ordered set of prime ideals. The basis of representation theory is prime elements, irreducible elements, and coatoms, which play an important role in order structure.

In 1973, Cornish [1] investigated the properties of α-ideals in distributive lattices. The most interesting result obtained by him was that the distributive lattice L is a generalized Stone lattice if and only if each prime ideal contains a unique prime α-ideal.

In 1986, Jayaram [3] generalized the concept of α-ideal in 0-distributive lattices. In fact, he established the separation theorem for prime α-ideals on 0-distributive lattices. In 2010, Y. S. Pawar and S. S. Khopade [11] proved that the image and inverse image of α-ideal are α-ideals under annihilator preserving homomorphism of a 0-distributive lattice.

In 2015, Khalid A. Mokbel [10] introduced the concept of α-ideal in posets and obtained its characterization. He also proved that the minimality of the prime ideal is sufficient for its being an α-ideal in a 0-distributive poset. An ideal K of $\mathcal{P}$ is said to be an α-ideal if $\langle\langle x, \{0\}\rangle, \{0\}\rangle \subseteq K$ for all $x \in K$ [10].

In 2016, Vinayak Joshi and Nilesh Mundlik [8] extended these concepts to posets. In fact, they defined Baer ideals in posets and obtained many characterizations of Baer ideals in 0-distributive posets. Furthermore, they extended the concept of quasi-complementedness to posets and obtained characterizations of quasi-complemented poset. Moreover, they proved that in distributive poset, every ideal is Baer if and only if every prime ideal is Baer.

In this work, we define J-prime and M_J-ideals in posets and examine their many features We obtain an equivalent condition for M_J-ideals to be J-ideals. We also define the J^*-property and provide an equivalent condition for an ideal of $\mathcal{P}$ to be a J-ideal. We also establish some features and characterizations of the set $\widetilde{S(J)}$ in posets.

2. Preliminaries

Throughout the paper, $(\mathcal{P}, \leq)$ denotes a poset with the smallest element 0. For basic terminology and notations for posets, we refer to [2] and [9].

For $A \subseteq \mathcal{P}$, let $L(A) = \{x \in \mathcal{P} : x \leq a \text{ for all } a \in A\}$ denotes the lower cone of A in $\mathcal{P}$ and $U(A) = \{x \in \mathcal{P} : a \leq x \text{ for all } a \in A\}$ denote the upper cone of A in $\mathcal{P}$. For $X, Y \subseteq \mathcal{P}$, we write $L(X, Y)$ instead of $L(X \cup Y)$. If $A = \{a_1, a_2, ..., a_n\}$ is a finite subset of $\mathcal{P}$, then we use the notation $L(a_1, a_2, ..., a_n)$ instead of $L(\{a_1, a_2, ..., a_n\})$.

It is clear that for any subset X of $\mathcal{P}$, we have $X \subseteq L(U(X))$ and $X \subseteq U(L(X))$. If $X \subseteq Y$, then $L(Y) \subseteq L(X)$ and $U(Y) \subseteq U(X)$. Furthermore, $LUL(X) = L(X)$ and $ULU(X) = U(X)$.

Following [12], a non-empty subset K of $\mathcal{P}$ is called semi-ideal if $s \in K$ and $r \leq s$ imply $r \in K$. A subset K of $\mathcal{P}$ is called an ideal of $\mathcal{P}$ if $r, s \in K$ implies $L(U(r, s)) \subseteq K$. For any subset X of $\mathcal{P}$, $[X]$ denotes the smallest ideal of $\mathcal{P}$ containing X.

An ideal K of $\mathcal{P}$ is called semi-prime if $L(r, s) \subseteq K$ and $L(r, t) \subseteq K$ together imply $L(r, U(s, t)) \subseteq K$ for $r, s, t \in \mathcal{P}$ [9]. A proper semi-ideal (ideal) K of $\mathcal{P}$ is called prime if $L(r, s) \subseteq K$ implies that either $r \in K$ or $s \in K$ [2]. The set of all prime ideals of $\mathcal{P}$ is denoted by $Spec(\mathcal{P})$.

For $r \in \mathcal{P}$, $(r] = L(r) = \{x \in \mathcal{P} : x \leq r\}$ is the principal ideal of $\mathcal{P}$ generated by r. A non-empty subset A of $\mathcal{P}$ is called an up-directed set if $A \cap U(r, s) \neq$ for all $r, s \in A$. If an ideal K of $\mathcal{P}$ is an up-directed set of $\mathcal{P}$, then it is called a u-ideal of $\mathcal{P}$. A subset F ($\neq$) of $\mathcal{P}$ is referred to as an m-system if for $w_1, w_2 \in F, \exists w \in L(w_1, w_2)$ such that $w \in F$.

Following [5], for any semi-ideal J of $\mathcal{P}$ and a subset X of $\mathcal{P}$, we define

$$\langle X, J \rangle = \{z \in \mathcal{P} : L(r, z) \subseteq J \ for\ all\ r \in X\} = \bigcap_{r \in X} \langle r, J \rangle .$$

If $X = \{s\}$, then we shall write $\langle s, J \rangle$ instead of $\langle \{s\}, J \rangle$. It is clear that for any subset B of $\mathcal{P}$, $J \subseteq \langle B, J \rangle ; B \subseteq \langle \langle B, J \rangle, J \rangle$ and $x \in \langle \langle x, J \rangle, J \rangle$ for any semi-ideal J of $\mathcal{P}$. Moreover, if $X \subseteq Y$, then $\langle Y, J \rangle \subseteq \langle X, J \rangle$. For any semi-ideal J of $\mathcal{P}$, it is clear that $\langle \langle \langle X, J \rangle, J \rangle, J \rangle = \langle X, J \rangle$ for any subset X of $\mathcal{P}$. Note that for an ideal J and a subset B of $\mathcal{P}$, $\langle B, J \rangle$ is not necessarily to be an ideal of $\mathcal{P}$. For any ideal I of $\mathcal{P}$, a prime ideal K of $\mathcal{P}$ is said to be a minimal prime ideal of I if $I \subseteq K$ and there exists no prime ideal R of $\mathcal{P}$ such that $I \subset R \subset K$[5].

Theorem 2.1. *([9], Theorem 20) Let I be a proper ideal of $\mathcal{P}$. Then I is prime if and only if $\langle x, I \rangle = I$ for all $x \in \mathcal{P} \backslash I$.*

An ideal J of $\mathcal{P}$ is said to have the $*$-property if for any $r, s \in \mathcal{P} \backslash J$, we have either $r = s$ or $L(r, s) = \{0\}$ [4]. Following [10], we have defined the notion of J-ideal in poset $\mathcal{P}$ as follows:

for an ideal J of $\mathcal{P}$, an ideal K of $\mathcal{P}$ is said to be J-ideal if $\langle\langle r, J\rangle, J\rangle \subseteq K$ for all $r \in K$ [7]. For an ideal I of $\mathcal{P}$, we have I is an α-ideal if and only if I is a $(0]$-ideal.

The following example shows that the notions of J-ideals and α-ideals are different in posets.

Example 2.2. Consider $\mathcal{P} = \{0, a, b, c, d\}$. Define a relation $\leq$ on $\mathcal{P}$ as follows. Then $(\mathcal{P}, \leq)$

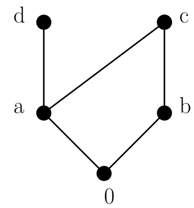

FIGURE 1. Hasse diagram of $(\mathcal{P}, \leq)$.

is a poset and $J = \{0, a\}$ is an ideal of $\mathcal{P}$. Here $I_1 = \{0, b\}$ is an α-ideal of $\mathcal{P}$, but it is not a J-ideal as $\langle\langle b, J\rangle, J\rangle = \{0, a, b, c\} \nsubseteq I_1$. Also $I_2 = \{0, a, b, c\}$ is a J-ideal of $\mathcal{P}$, but it is not an α-ideal as $\langle\langle a, \{0\}\rangle, \{0\}\rangle = \{0, a, d\} \nsubseteq I_2$. Moreover $I_3 = \{0, a, d\}$ is a both J-ideal and α-ideal of $\mathcal{P}$. □

3. J-PRIME IDEALS OF POSETS

Lemma 3.1. *Let J be a semi-prime ideal of $\mathcal{P}$. Then $\langle B, J\rangle$ is a J-ideal for any non-empty subset B of $\mathcal{P}$.*

Proof. Let J be a semi-prime ideal of $\mathcal{P}$ and $s, t \in \langle B, J\rangle$ for any non-empty subset B of $\mathcal{P}$. Then $s, t \in \langle b, J\rangle$ for all $b \in B$ and $L(s, b) \subseteq J$ and $L(t, b) \subseteq J$, which imply $L(b, U(s, t)) \subseteq J$, so $L(U(s, t)) \subseteq \langle b, J\rangle$ for all $b \in B$. Thus $L(U(s, t)) \subseteq \langle B, J\rangle$ and hence $\langle B, J\rangle$ is an ideal of $\mathcal{P}$ for any subset B of $\mathcal{P}$. For $r \in \langle B, J\rangle$, we have $\langle\langle r, J\rangle, J\rangle \subseteq \langle\langle\langle B, J\rangle, J\rangle, J\rangle = \langle B, J\rangle$. So $\langle B, J\rangle$ is J-ideal. □

Remark 3.2. Let K and J be ideals of $\mathcal{P}$. Then the following assertions hold.

(i) If $\langle\langle K, J\rangle, J\rangle = K$, then K is a J-ideal of $\mathcal{P}$.

(ii) If K is a proper J-ideal of $\mathcal{P}$ with $\langle K, J\rangle = \{0\}$, then $\mathcal{P} = \langle\langle K, J\rangle, J\rangle \neq K$. This shows that the converse of (i) is not true in general, but we have the following.

(iii) If $\langle\langle K, J\rangle, J\rangle = \bigcap\limits_{x \in K} \langle\langle x, J\rangle, J\rangle$ and $\langle K, J\rangle \neq \{0\}$, then $\langle\langle K, J\rangle, J\rangle = K$. Note that the condition $\langle\langle K, J\rangle, J\rangle = \bigcap\limits_{x \in K} \langle\langle x, J\rangle, J\rangle$ is not true in general. In Example 2.2, $J = \{0, a\}$ and $K = \{0, b\}$ are the ideals of $\mathcal{P}$, but $\langle\langle K, J\rangle, J\rangle \neq \bigcap\limits_{x \in K} \langle\langle x, J\rangle, J\rangle$.

Definition 3.3. Let J be an ideal of $\mathcal{P}$. An ideal K of $\mathcal{P}$ is said to be J-prime if $\langle K, J\rangle = J$. An element $r \in \mathcal{P}$ is called prime to J if $\langle r, J\rangle = J$. The set $S(I) = \{x \in \mathcal{P} : x$ is not prime to $I\}$ is called adjoint set of I and $\widetilde{S(J)} = \{r \in \mathcal{P} : r$ is prime to $J\}$.

It is clear that for any ideal J of $\mathcal{P}$, $\mathcal{P}$ is a J-prime ideal of $\mathcal{P}$. Also, for any ideals A, B and J of $\mathcal{P}$ with $A \subseteq B$, if A is J-prime ideal of $\mathcal{P}$, then B is also a J-prime ideal of $\mathcal{P}$. The following example shows that the converse of the later statement is not true in general.

Example 3.4. In Example 2.2, $J = \{0, a, d\}$ and $K = (c] = \{0, a, b, c\}$ are ideals of $\mathcal{P}$. Here K is a J-prime ideal of $\mathcal{P}$ and the ideal $K_1 = \{0, a\}$ is a subset of K, but it is not a J-prime ideal of $\mathcal{P}$.

The following example distinguishes prime ideals from J-prime ideals for an ideal J of $\mathcal{P}$.

Example 3.5. Consider $\mathcal{P} = \{0, a, b, c, d, e\}$ and define a relation $\leq$ on $\mathcal{P}$ as follows.

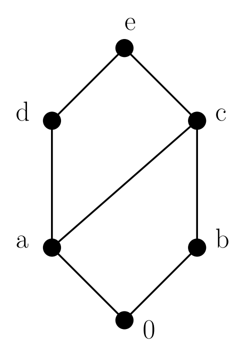

FIGURE 2. Hasse diagram of $(\mathcal{P}, \leq)$.

Then $(\mathcal{P}, \leq)$ is a poset with ideals $J = \{0, a\}$ and $K = \{0, b\}$ of $\mathcal{P}$. Here K is a prime ideal of $\mathcal{P}$, but it is not a J-prime ideal of $\mathcal{P}$ as $\langle K, J\rangle = \{0, a, d\} \neq J$. Moreover, for $J = \{0, b\}$ and $K = \{0, a\}$, we have K is a J-prime ideal of $\mathcal{P}$ as $\langle K, J\rangle = J$. □

Theorem 3.6. *Let K and J be semi-prime ideals of $\mathcal{P}$ with $J \subseteq K$. If K is a prime but not a J-prime ideal of $\mathcal{P}$, then K is a J-ideal of $\mathcal{P}$.*

Proof. Assume that K is a prime, but not a J-prime ideal of $\mathcal{P}$. Then there exists $x \in \langle K, J\rangle \setminus J$ such that $\langle\langle K, J\rangle, J\rangle \subseteq \langle x, J\rangle$, which gives $K \subseteq \langle x, J\rangle$.

Let $t \in \langle x, J\rangle$. Then $L(x, t) \subseteq J \subseteq K$. Since K is prime ideal, we have either $x \in K$ or $t \in K$. If $x \in K \subseteq \langle x, J\rangle$, then $x \in J$, a contradiction. Thus $t \in K$ and hence $K = \langle x, J\rangle$. By Lemma 3.1, K is a J-ideal of $\mathcal{P}$. □

For an ideal K and a semi-ideal J of $\mathcal{P}$, we define a subset $K_J = \{x \in \mathcal{P} : \langle a, J\rangle \subseteq \langle x, J\rangle$ for some $a \in K\}$. The condition for an ideal to be a J-ideal of a poset is given by the following theorem.

Theorem 3.7. *Let K be a u-ideal of $\mathcal{P}$. If J is a semi-prime ideal of $\mathcal{P}$, then K_J is the smallest J-ideal of $\mathcal{P}$ containing K.*

Proof. Let $x, y \in K_J$ and $z \in L(U(x,y))$. Then there exist $a, b \in K$ such that $\langle a, J\rangle \subseteq \langle x, J\rangle$ and $\langle b, J\rangle \subseteq \langle y, J\rangle$. Since J is semi-prime, we have $\langle a, J\rangle \cap \langle b, J\rangle \subseteq \langle x, J\rangle \cap \langle y, J\rangle = \langle L(U(x,y)), J\rangle$. As K is a u-ideal of P, we can find $c \in U(a,b)$ such that $c \in K$ and $\langle c, J\rangle \subseteq \langle a, J\rangle \cap \langle b, J\rangle = \langle L(U(x,y)), J\rangle \subseteq \langle z, J\rangle$. Thus $z \in K_J$ and hence K_J is an ideal of $\mathcal{P}$.

Let $x \in K_J$. Then $\langle a, J\rangle \subseteq \langle x, J\rangle$ for some $a \in K$. If $\langle \langle x, J\rangle, J\rangle \nsubseteq K_J$, then there exists $y \in \langle \langle x, J\rangle, J\rangle$ such that $\langle a, J\rangle \nsubseteq \langle y, J\rangle$. Now for $b \in \langle a, J\rangle \setminus \langle y, J\rangle$, we have $b \in \langle x, J\rangle$ as $\langle a, J\rangle \subseteq \langle x, J\rangle$, and also $y \in \langle \langle x, J\rangle, J\rangle$ gives $y \in \langle b, J\rangle$, a contradiction. So $\langle \langle x, J\rangle, J\rangle \subseteq K_J$. Hence K_J is a J-ideal of $\mathcal{P}$.

Suppose that R is a J-ideal of $\mathcal{P}$ with $K \subseteq R$ and let $x \in K_J$. Then $\langle a, J\rangle \subseteq \langle x, J\rangle$ for some $a \in K \subseteq R$. Since R is J-ideal, $x \in \langle \langle x, J\rangle, J\rangle \subseteq \langle \langle a, J\rangle, J\rangle \subseteq R$. So K_J is the smallest J-ideal of $\mathcal{P}$ containing K. $\square$

Theorem 3.8. *Let $\mathcal{P}$ be a poset with the greatest element 1, K be a u-ideal of $\mathcal{P}$ and J be a semi-prime ideal of $\mathcal{P}$. Then $K \cap \widetilde{S(J)} =$ if and only if K_J is a proper ideal of $\mathcal{P}$.*

Proof. Assume that $K \cap \widetilde{S(J)} =$. If K_J is not a proper ideal of $\mathcal{P}$, then $1 \in K_J = \mathcal{P}$ gives $\langle a, J\rangle \subseteq \langle 1, J\rangle = J$ for some $a \in K$. So $a \in \widetilde{S(J)}$, a contradiction.

Conversely, let K_J be a proper ideal of $\mathcal{P}$. Then by Theorem 3.7, K_J is a J-ideal of $\mathcal{P}$ containing K. If $K \cap \widetilde{S(J)} \neq$, then there exists $t \in K \subseteq K_J$ and $\langle t, J\rangle = J$. Since K_J is a J-ideal of $\mathcal{P}$, we have $\langle \langle x, J\rangle, J\rangle \subseteq K_J$ for all $x \in K_J$. In particular, $\langle \langle t, J\rangle, J\rangle \subseteq K_J$, which implies $\mathcal{P} = \langle J, J\rangle \subseteq K_J$, a contradiction. $\square$

Definition 3.9. Let J be an ideal of $\mathcal{P}$. Then $\mathcal{P}$ has the J^*-property if the following condition holds: If $a, b \in \mathcal{P} \setminus J$ are such that $b \nleq a$, then there exists $c \in \mathcal{P} \setminus J$ such that $c \leq b$ and $L(a,c) \subseteq J$.

Example 3.10. Consider $\mathcal{P} = \{0, 1, 2, 3, 4\}$. Define a relation $\leq$ on $\mathcal{P}$ as follows.

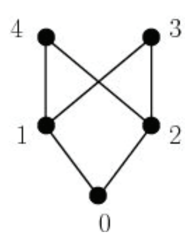

Figure 3. Hasse diagram of $(\mathcal{P}, \leq)$.

Then $(\mathcal{P}, \leq)$ is a poset. For an ideal $J = \{0, 2\}$ of $\mathcal{P}$, the poset $\mathcal{P}$ has not J^*-property, and for an ideal $I = \{0, 1, 2\}$ of $\mathcal{P}$, the poset $\mathcal{P}$ has I^*-property. □

The below theorem gives equivalent conditions for a poset $\mathcal{P}$ to have J^*-property.

Theorem 3.11. *Let J be a ideal of $\mathcal{P}$. Then the following conditions are equivalent:*

(i) $\mathcal{P}$ has J^*-property,

(ii) Every ideal of $\mathcal{P}$ is a J-ideal of $\mathcal{P}$,

(iii) For any $r, s \in \mathcal{P} \backslash J$, $\langle r, J \rangle = \langle s, J \rangle$ implies $r = s$.

Proof. (**i**) ⇒ (**ii**) Let K of $\mathcal{P}$ be an ideal of $\mathcal{P}$. If K is not a J-ideal of $\mathcal{P}$, then there exists $s \in \langle \langle r, J \rangle, J \rangle$ and $s \notin K$ for some $r \in K$, which imply $s \not\leq r$. Since $\mathcal{P}$ has J^*-property, there exists $c \in \mathcal{P} \backslash J$ with $c \leq s$ and $L(r, c) \subseteq J$, which imply $c \notin \langle s, J \rangle$. Indeed, if $c \in \langle s, J \rangle$, then $c \in L(c) = L(c, s) \subseteq J$, a contradiction.

Also, we have $\langle r, J \rangle = \langle \langle \langle r, J \rangle, J \rangle, J \rangle \subseteq \langle s, J \rangle$ with $c \in \langle r, J \rangle$ and $c \notin \langle s, J \rangle$, a contradiction. Thus K is a J-ideal of $\mathcal{P}$.

(**ii**) ⇒ (**iii**) Let $r, s \in \mathcal{P} \backslash J$ and $\langle r, J \rangle = \langle s, J \rangle$ with $r \neq s$. Then $L(r) \neq L(s)$.

Without loss of generality, let $t \in L(r)$ and $t \notin L(s)$. As $t \leq r$, we have $\langle s, J \rangle = \langle r, J \rangle \subseteq \langle t, J \rangle$, which gives $t \in \langle \langle t, J \rangle, J \rangle \subseteq \langle \langle s, J \rangle, J \rangle \subseteq L(s)$ as $L(s)$ is J-ideal, a contradiction.

(**iii**) ⇒ (**i**) Let $r, s \in \mathcal{P} \backslash J$ with $s \not\leq r$. Then $\langle s, J \rangle \subset \langle r, J \rangle$ and there exists $x \in \langle r, J \rangle$ with $x \notin \langle s, J \rangle$, which implies $L(x, r) \subseteq J$ and there exists $y \notin J$ such that $y \in L(x, s)$. Thus $y \leq s$ and $L(r, y) \subseteq J$. So $\mathcal{P}$ has J^*-property. □

Theorem 3.12. *Let J be an ideal of $\mathcal{P}$. Then the following conditions are equivalent:*

(i) For any ideals K_1 and K_2 of $\mathcal{P}$ containing J. If $K_2 \nsubseteq K_1$, then ∃ an ideal K_3 of $\mathcal{P}$ with $K_3 \nsubseteq J$ such that $K_3 \subseteq K_2$ and $K_3 \cap K_1 \subseteq J$,

(ii) Every ideal of $\mathcal{P}$ containing J is a J-ideal of $\mathcal{P}$,

(iii) For any subsets K_1 and K_2 of $\mathcal{P}$ containing J, $\langle K_1, J \rangle = \langle K_2, J \rangle$ implies $[K_1] = [K_2]$.

Proof. (**i**) ⇒ (**ii**) Let K_1 be an ideal of $\mathcal{P}$ containing J. If K_1 is not a J-ideal, then there exists an element $a \in \mathcal{P}$ with $a \in \langle \langle K_1, J \rangle, J \rangle \backslash K_1$ and $\langle K_1, J \rangle = \langle \langle \langle K_1, J \rangle, J \rangle, J \rangle \subseteq \langle a, J \rangle$.

Take $K_2 := \langle \langle K_1, J \rangle, J \rangle$. Then by hypothesis, there exists an ideal $K_3 = L(a)$ of $\mathcal{P}$ with $K_3 \nsubseteq J$.

By hypothesis, we have $K_3 \subseteq K_2$ and $K_3 \cap K_1 \subseteq J$, which imply $K_3 \subseteq \langle K_1, J \rangle$ and $K_3 \nsubseteq \langle a, J \rangle$, so $\langle K_1, J \rangle \nsubseteq \langle a, J \rangle$, a contradiction. So K_1 is a J-ideal of $\mathcal{P}$.

(**ii**) ⇒ (**iii**) Assume that K_1 and K_2 are subsets of $\mathcal{P}$ containing J such that $\langle K_1, J \rangle = \langle K_2, J \rangle$. Let $a \in K_1$. Then $\langle K_2, J \rangle = \langle K_1, J \rangle \subseteq \langle a, J \rangle$, which implies $a \in \langle \langle a, J \rangle, J \rangle \subseteq \langle \langle K_2, J \rangle, J \rangle \subseteq \langle \langle [K_2], J \rangle, J \rangle \subseteq [K_2]$ as $[K_2]$ is a J-ideal of $\mathcal{P}$. So $K_1 \subseteq [K_2]$ implies $[K_1] \subseteq [K_2]$. . Similarly, we can prove that $[K_2] \subseteq [K_1]$.

(iii) $\Rightarrow$ **(i)** Let K_1 and K_2 be the ideals of $\mathcal{P}$ containing J with $K_1 \nsubseteq K_2$. If $\langle K_1, J\rangle = \langle K_2, J\rangle$, then by hypothesis, we get $K_1 = K_2$, a contradiction. Also if $\langle K_2, J\rangle \subset \langle K_1, J\rangle$, then $\langle K_1 \cup K_2, J\rangle = \langle K_1, J\rangle \cap \langle K_2, J\rangle = \langle K_2, J\rangle$.

By hypothesis, we get $K_1 \cup K_2 = K_2$, which gives $K_1 \subset K_2$. So $\langle K_1, J\rangle \subset \langle K_2, J\rangle$. Then there exists $t \in \langle K_2, J\rangle \setminus \langle K_1, J\rangle$ with $L(t, s) \nsubseteq J$ for some $s \in K_1$. Set $K_3 = L(t, s)$. Then $K_3 \nsubseteq J$ with $K_3 = L(t, s) \subseteq (t] \subseteq \langle K_2, J\rangle$. Therefore $K_3 \cap K_2 \subseteq J$. □

Lemma 3.13. *Let J be a semi-prime ideal of $\mathcal{P}$. Then the following conditions hold.*

(i) $\langle r, J\rangle \cap \langle \langle r, J\rangle, J\rangle = J$.

(ii) $\langle \langle r, J\rangle, J\rangle \cap \langle \langle s, J\rangle, J\rangle = \langle \langle L(r, s), J\rangle, J\rangle$.

(iii) $\langle r, J\rangle \cap \langle s, J\rangle \subseteq J$ *if and only if* $\langle r, J\rangle \subseteq \langle \langle s, J\rangle, J\rangle$.

Proof. **(i)** Obviously $J \subseteq \langle r, J\rangle \cap \langle \langle r, J\rangle, J\rangle$. Let $a \in \langle r, J\rangle \cap \langle \langle r, J\rangle, J\rangle$. Then $L(a, r) \subseteq J$ and $L(a, t) \subseteq J$ for all $t \in \langle r, J\rangle$. Since $a \in \langle r, J\rangle$, we have $a \in J$. So $J \subseteq \langle r, J\rangle \cap \langle \langle r, J\rangle, J\rangle$.

(ii) As $L(r, s) \subseteq L(s)$, we have $\langle \langle L(r, s), J\rangle, J\rangle \subseteq \langle \langle L(s), J\rangle, J\rangle \subseteq \langle \langle s, J\rangle, J\rangle$. Similarly, $\langle \langle L(r, s), J\rangle, J\rangle \subseteq \langle \langle r, J\rangle, J\rangle$. So $\langle \langle L(r, s), J\rangle, J\rangle \subseteq \langle \langle r, J\rangle, J\rangle \cap \langle \langle s, J\rangle, J\rangle$.

Conversely, let $t \in \langle \langle r, J\rangle, J\rangle \cap \langle \langle s, J\rangle, J\rangle$ and $t_1 \in \langle L(r, s), J\rangle$. Then $\langle r, J\rangle \subseteq \langle t, J\rangle$ and $L(r, s) \subseteq \langle t_1, J\rangle$, which imply $L(s, t_1) \subseteq \langle t, J\rangle$, so $L(t, t_1) \subseteq \langle s, J\rangle$. Since $t \in \langle \langle s, J\rangle, J\rangle$, we have $L(t, t_1) \subseteq \langle \langle s, J\rangle, J\rangle$. Hence $L(t, t_1) \subseteq \langle s, J\rangle \cap \langle \langle s, J\rangle, J\rangle$. By (i), we have $L(t, t_1) \subseteq J$ and $t \in \langle \langle L(r, s), J\rangle, J\rangle$.

(iii) Let $\langle r, J\rangle \cap \langle s, J\rangle \subseteq J$. If $\langle r, J\rangle \nsubseteq \langle \langle s, J\rangle, J\rangle$, then there exists $t \in \langle r, J\rangle$ with $t \notin \langle \langle s, J\rangle, J\rangle$, which implies $L(t, c) \nsubseteq J$ for some $c \in \langle s, J\rangle$. As $t \in \langle r, J\rangle$ and $c \in \langle s, J\rangle$, we have $L(t, c) \subseteq J$, a contradiction.

Conversely, let $\langle r, J\rangle \subseteq \langle \langle s, J\rangle, J\rangle$ for $r, s \in \mathcal{P}$. Then by (i), $\langle r, J\rangle \cap \langle s, J\rangle \subseteq \langle \langle s, J\rangle, J\rangle \cap \langle s, J\rangle = J$. □

Theorem 3.14. *Let J be a semi-prime ideal of $\mathcal{P}$. Then the set $\widetilde{S(J)}$ is a semi-prime filter of $\mathcal{P}$.*

Proof. We first show that $\widetilde{S(J)}$ is a filter of $\mathcal{P}$. Let $x, y \in \widetilde{S(J)}$ and $z \in U(L(x, y))$. To prove $z \in \widetilde{S(J)}$, it is enough to prove that $\langle z, J\rangle \subseteq J$. Let $r \in \langle z, J\rangle$. Then $L(z, r) \subseteq J$. Since $L(x, y) \subseteq L(z)$, we have $L(x, y, r) \subseteq L(z, r) \subseteq J$. As $\langle x, J\rangle = J = \langle y, J\rangle$, we get $r \in J$.

Let $U(a, b) \subseteq \widetilde{S(J)}$ and $U(a, c) \subseteq \widetilde{S(J)}$ for $a, b, c \in \mathcal{P}$. Then $\langle U(a, b), J\rangle = J$ and $\langle U(a, c), J\rangle = J$, which imply $\langle a, J\rangle \cap \langle b, J\rangle = J$ and $\langle a, J\rangle \cap \langle c, J\rangle = J$. By Lemma 3.13(iii), we have $\langle a, J\rangle \subseteq \langle \langle b, J\rangle, J\rangle$ and $\langle a, J\rangle \subseteq \langle \langle c, J\rangle, J\rangle$.

Again by Lemma 3.13(ii), $\langle a, J\rangle \subseteq \langle \langle b, J\rangle, J\rangle \cap \langle \langle c, J\rangle, J\rangle = \langle \langle L(b, c), J\rangle, J\rangle$, so $\langle a, J\rangle \cap \langle L(b, c), J\rangle \subseteq \langle \langle L(b, c), J\rangle, J\rangle \cap \langle L(b, c), J\rangle = J$, by Lemma 3.13(i).

Let $t \in \langle U(a, L(b,c)), J\rangle$. Then $L(t,s) \subseteq J$ for all $s \in U(a, L(b,c))$, which gives $t \in \langle s, J\rangle \subseteq \langle a, J\rangle \cap \langle L(b,c), J\rangle \subseteq J$. Thus $J \subseteq \langle U(a, L(b,c)), J\rangle \subseteq J$ and hence $U(a, L(b,c)) \subseteq \widetilde{S(J)}$. So $\widetilde{S(J)}$ is semi-prime. $\square$

The following example shows that the converse of the above theorem is not true in general.

Example 3.15. Consider $\mathcal{P} = \{0, a, b, c, d, e, f\}$ and define a relation $\leq$ on $\mathcal{P}$ as follows.

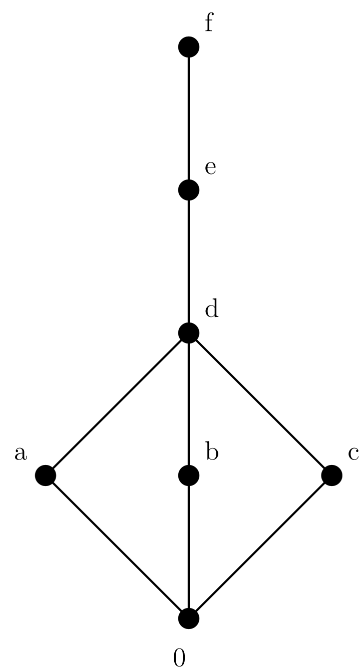

FIGURE 4. Hasse diagram of $(\mathcal{P}, \leq)$.

Then $(\mathcal{P}, \leq)$ is a poset. For the ideal $J = \{0, b\}$, we have $\widetilde{S(J)} = \{d, e, f\}$. Here $\widetilde{S(J)}$ is semi-prime filter of $\mathcal{P}$, but J is not a semi-prime ideal of $\mathcal{P}$ as $L(a,b) \subseteq J$ and $L(a,c) \subseteq J$, but $L(a, U(b,c)) \nsubseteq J$. $\square$

Theorem 3.16. *Let J be a semi-prime ideal of $\mathcal{P}$ and every semi-ideal of $\mathcal{P}$ is a u-ideal of $\mathcal{P}$. If no proper J-ideal of $\mathcal{P}$ is J-prime, then for each $r \in \mathcal{P}$, there exists $s \in \mathcal{P}$ such that $\langle r, J\rangle = \langle \langle s, J\rangle, J\rangle$.*

Proof. Let $r \in \mathcal{P}$ and $K = [\langle r, J\rangle \cup \langle \langle r, J\rangle, J\rangle]$. By hypothesis and Theorem 3.7, we have K is a u-ideal of $\mathcal{P}$ and $K_J = \{r \in \mathcal{P} : \langle a, J\rangle \subseteq \langle r, J\rangle \text{ for some } a \in K\}$ is a J-ideal of $\mathcal{P}$ containing K. As $\langle r, J\rangle \subseteq K_J$ and $\langle \langle r, J\rangle, J\rangle \subseteq K_J$, we get $\langle K_J, J\rangle \subseteq \langle \langle \langle r, J\rangle, J\rangle, J\rangle \cap \langle \langle r, J\rangle, J\rangle$. Then by Lemma 3.13(i), $\langle K_J, J\rangle \subseteq J$, which gives K_J is a J-prime ideal of $\mathcal{P}$. By hypothesis and Theorem 3.8, we get $K_J = \mathcal{P}$ and $K \cap \widetilde{S(J)} \neq$.

Let $u \in K_J \cap \widetilde{S(J)}$. Then $\langle a, J\rangle \subseteq \langle u, J\rangle$ for some $a \in K$ and $\langle u, J\rangle = J$ imply $\langle a, J\rangle \subseteq J$. Since $a \in K = [\langle r, J\rangle \cup \langle\langle r, J\rangle, J\rangle]$, we have

$$\begin{aligned}
&\langle(\langle r, J\rangle \cup \langle\langle r, J\rangle, J\rangle), J\rangle \subseteq \langle a, J\rangle \subseteq J \\
\Rightarrow\, &\langle L(U(\langle r, J\rangle \cup \langle\langle r, J\rangle, J\rangle)), J\rangle \subseteq J \\
\Rightarrow\, &\langle\langle r, J\rangle, J\rangle \cap \langle\langle\langle r, J\rangle, J\rangle, J\rangle \subseteq J \\
\Rightarrow\, &\langle s, J\rangle \subseteq \langle\langle c, J\rangle, J\rangle \,(\text{for some } s \in \langle r, J\rangle \text{ and } c \in \langle\langle r, J\rangle, J\rangle) \\
\Rightarrow\, &\langle s, J\rangle \subseteq \langle\langle c, J\rangle, J\rangle \subseteq \langle\langle r, J\rangle, J\rangle \subseteq \langle s, J\rangle \,(\text{by Lemma 3.13(iii)}).
\end{aligned}$$

So $\langle r, J\rangle = \langle\langle s, J\rangle, J\rangle$. □

4. M_J-ideal of posets

For a non-empty subset B of $\mathcal{P}$, we define the set $0(B) = \{r \in \mathcal{P} : L(b, r) = \{0\}$ for some $b \in B\}$[6]. A proper ideal K of $\mathcal{P}$ is said to be M-ideal if there exists a m-system M, where M is a proper subset and $0 \notin M$, such that $K = 0(M)$. Every M-ideal of $\mathcal{P}$ is a α-ideal of $\mathcal{P}$ and the converse need not be true in general. For any minimal prime ideal I of $\mathcal{P}$, we have $0(\mathcal{P}\backslash I) = I$.

Definition 4.1. A proper ideal K of $\mathcal{P}$ is said to be a M_J-ideal if there exists an ideal J and a proper m-system M in $\mathcal{P}$ such that $K = J(M)$, where $J(M) = \{r \in \mathcal{P} : L(a, r) \subseteq J$ for some $a \in M\}$.

Example 4.2. Consider $\mathcal{P} = \{0, a, b, c, d, e\}$. Define a relation $\leq$ on $\mathcal{P}$ as follows.

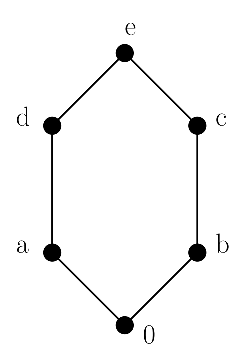

Figure 5. Hasse diagram of $(\mathcal{P}, \leq)$.

Then $(\mathcal{P}, \leq)$ is a poset. Let $J = \{0, b\}$, $K = \{0, b, c\}$ and $M = \{a, d\}$. Then K is a M_J-ideal of $\mathcal{P}$. □

Example 4.3. In Example 3.10, $J = \{0, 1, 2\}$ and $K = \{0, 2\}$ are ideals of $\mathcal{P}$ and $M = \{1, 4\}$ is a m-system of $\mathcal{P}$. Here K is a M-ideal of $\mathcal{P}$, but not a M_J-ideal of $\mathcal{P}$.

The above examples shows that the concept of M_J-ideals are different from M-ideals in poset.

Theorem 4.4. *Let K and J be ideals of $\mathcal{P}$. If K is a M_J-ideal of $\mathcal{P}$ for some m-system M of $\mathcal{P}$, then K is a J-ideal of $\mathcal{P}$.*

Proof. Let K be a M_J-ideals of $\mathcal{P}$ and $x \in K = J(M)$. Then $L(x, y) \subseteq J$ for some $y \in M$. Let $t \in \langle\langle x, J\rangle, J\rangle$. Then $\langle x, J\rangle \subseteq \langle\langle\langle x, J\rangle, J\rangle, J\rangle \subseteq \langle t, J\rangle$. Since $y \in \langle x, J\rangle$, we have $L(t, y) \subseteq J$ and $y \in M$, which imply $t \in J(M) = K$, so K is a J-ideal of $\mathcal{P}$. □

Remark 4.5. The converse of the above theorem is not true in general. In Example 4.2, for the ideals $J = \{0\}$ and $K = \{0, b, c\}$, we have K is a J-ideal of $\mathcal{P}$ and K is not a M_J-ideal of $\mathcal{P}$ for all m-system M of $\mathcal{P}$.

Theorem 4.6. *Let $J \subseteq I$ be ideals of $\mathcal{P}$, where I is prime. If J has the $*$-property and union of any two ideals is an ideal of $\mathcal{P}$, then the following conditions are equivalent:*

(i) I is a minimal prime ideal of J,

(ii) For each $x \in I$, there exists $y \in \mathcal{P}\backslash I$ and $t \in U(x)$ such that $L(t, y) \subseteq J$.

Proof. This theorem proof is similarly to that of Theorem 2.7 of [4]. □

Lemma 4.7. *Let K be a prime ideal of $\mathcal{P}$ and J be an ideal of $\mathcal{P}$ with the $*$-property. If K is minimal prime ideal of J and union of any two ideals is an ideal of $\mathcal{P}$, then K contains precisely one of r or $\langle r, J\rangle$ for any $r \in \mathcal{P}$.*

Proof. Assume the contrary that $\langle r, J\rangle \subseteq K$ for every $r \in K$. Since K is minimal prime ideal and by Theorem 4.6, for each $r \notin \mathcal{P}\backslash K$, there exists $t \in U(r)$ and $s \in \mathcal{P}\backslash K$ such that $L(t, s) \subseteq J$, it follows that $s \in K$, a contradiction.

Let $r \notin K$ and $t \in \langle r, J\rangle$. Then $L(r, t) \subseteq J \subseteq K$. Since K is prime ideal and $r \notin K$, we have $t \in K$. □

Theorem 4.8. *Let J be a semi-prime ideal of $\mathcal{P}$ with the $*$-property. If union of any two ideals is an ideal of $\mathcal{P}$, then the following conditions are equivalent:*

(i) For each $a \in \mathcal{P}$, there exists $b \in \mathcal{P}$ such that $L(a, b) \subseteq J$ and $U(a, b) \subseteq \widetilde{S(J)}$,

(ii) If K is a prime ideal of $\mathcal{P}$ with $J \subseteq K$ and $K\cap\ \widetilde{S(J)} =$, then K is minimal prime ideal of J,

(iii) If R is an ideal of $\mathcal{P}$ with $J \subseteq R$ and $R\cap\widetilde{S(J)} =$, then R is contained in some minimal

prime ideal of J,

(iv) For each $a \in \mathcal{P}$, there exists $b \in \mathcal{P}$ such that $\langle a, J\rangle = \langle \langle b, J\rangle, J\rangle$.

Proof. (**i**) $\Rightarrow$ (**ii**) Let K be a prime ideal of $\mathcal{P}$ with $J \subseteq K$, $K\cap \widetilde{S(J)} =$. By (i) for $a \in K$, there exists $b \in \mathcal{P}$ such that $L(a,b) \subseteq J$ and $U(a,b) \subseteq \widetilde{S(J)}$. Then $b \notin K$. Indeed, if $b \in K$, then $L(U(a,b)) \subseteq K$. For some $t \in U(a,b) \subseteq \widetilde{S(J)}$, we have $L(t) \in K$, which implies $t \in K \cap \widetilde{S(J)}$, a contradiction. So for $a \in \mathcal{P}$, there exists $b \in \mathcal{P}\backslash K$ such that $L(a,b) \subseteq J$ and $U(a,b) \subseteq \widetilde{S(J)}$.

Suppose K_1 is a prime ideal of $\mathcal{P}$ with $J \subseteq K_1 \subset K$. Let $a \in K$. Then there exists $b \in \mathcal{P}\backslash K$ such that $L(a,b) \subseteq J \subseteq K_1$. As K_1 is prime and $b \notin K_1$, we have $a \in K_1$. Hence K is the minimal prime ideal of J.

(**ii**) $\Rightarrow$ (**iii**) Let R be an ideal of $\mathcal{P}$ and $J \subseteq R$ with $R \cap \widetilde{S(J)} =$. We first prove that $\widetilde{S(J)}$ is an m-system of $\mathcal{P}$. Let $a, b \in \widetilde{S(J)}$. Then $\langle a, J\rangle = J$ and $\langle b, J\rangle = J$. Suppose that $L(a,b) \cap \widetilde{S(J)} =$. For $t \in L(a,b)$, there exists $r \in \mathcal{P}\backslash J$ such that $L(t,r) \subseteq J$, which implies that $L(a,b) \subseteq \langle r, J\rangle$. So $r \in J$, a contradiction. Thus $L(a,b) \cap \widetilde{S(J)} \neq$ and hence $\widetilde{S(J)}$ is an m-system of $\mathcal{P}$. By Theorem 2.6 of [4], R contained in a prime ideal K of $\mathcal{P}$ with $K \cap \widetilde{S(J)} =$. By (ii) K is a minimal prime ideal of J.

(**iii**) $\Rightarrow$ (**iv**) Let $a \in \mathcal{P}$ and $R = (a] \cup \langle a, J\rangle$. Then $R\cap \widetilde{S(J)} \neq$. Indeed, if $R\cap \widetilde{S(J)} =$, then by the hypothesis, there exists a minimal prime ideal K of J such that $(a] \cup \langle a, J\rangle \subseteq K$. By Lemma 4.7, for $a \in \mathcal{P}$, we have K contains precisely one of a or $\langle a, J\rangle$, a contradiction. So there exists $d \in \widetilde{S(J)}$ and $d \in (a] \cup \langle a, J\rangle$.

Case 1: Let $d \in \langle a, J\rangle$. Then $\langle\langle a, J\rangle, J\rangle \subseteq \langle d, J\rangle = J$. So, $\langle\langle a, J\rangle, J\rangle = \langle d, J\rangle$ for some $d \in \mathcal{P}$

Case 2: Let $d \in (a]$ and $b \in \langle a, J\rangle$. Then $\langle\langle a, J\rangle, J\rangle \subseteq \langle b, J\rangle$.

Now, consider $\langle a, J\rangle \cap \langle b, J\rangle = J \cap \langle b, J\rangle = J$. By Lemma 3.13(iii), we have $\langle b, J\rangle \subseteq \langle\langle a, J\rangle, J\rangle$.

(**iv**) $\Rightarrow$ (**i**) Let $a \in \mathcal{P}$. By the hypothesis, there exists $b \in \mathcal{P}$ such that $\langle\langle a, J\rangle, J\rangle$. Then $L(a,b) \subseteq J$. Now, $\forall t \in U(a,b)$, we have $\langle U(a,b), J\rangle \subseteq \langle t, J\rangle$, which implies $\langle U(a,b), J\rangle \subseteq \langle a, J\rangle \cap \langle b, J\rangle = \langle a, J\rangle \cap \langle\langle a, J\rangle, J\rangle = J$ by Lemma 3.13 (i). So, $t \in \widetilde{S(J)}$ and $U(a,b) \subseteq \widetilde{S(J)}$. □

Theorem 4.9. *Let J be a semi-prime ideal of $\mathcal{P}$ and every J-ideal of $\mathcal{P}$ is a M_J-ideal of $\mathcal{P}$. Then for each $a \in \mathcal{P}$, there exists $b \in \mathcal{P}$ such that $\langle a, J\rangle = \langle\langle b, J\rangle, J\rangle$.*

Proof. Assume that every J-ideal of $\mathcal{P}$ is a M_J-ideal of $\mathcal{P}$. Then for $a \in \mathcal{P}$, $\langle\langle a, J\rangle, J\rangle$ is a J-ideal of $\mathcal{P}$, so $a \in \langle\langle a, J\rangle, J\rangle = J(M)$ for some m-system M of $\mathcal{P}$, which implies $L(a,b) \subseteq J$ for some $b \in M$. Since $b \in \langle a, J\rangle$, we have $\langle\langle a, J\rangle, J\rangle \subseteq \langle b, J\rangle \subseteq J(M) = \langle\langle a, J\rangle, J\rangle$ gives $\langle b, J\rangle = \langle\langle a, J\rangle, J\rangle$. □

Theorem 4.10. *Let J be a semi-prime ideal of $\mathcal{P}$ and for each $a \in \mathcal{P}$, there exists $b \in \mathcal{P}$ such that $\langle a, J\rangle = \langle\langle b, J\rangle, J\rangle$. Then for any J-ideal K of $\mathcal{P}$, we have $K \subseteq J(M)$ for some m-system M of $\mathcal{P}$.*

Proof. Let K be an J-ideal of $\mathcal{P}$ and $M = \{a \in \mathcal{P} : \langle\langle x, J\rangle, J\rangle \subseteq \langle a, J\rangle \ \ for\ some\ x \in K\}$. By hypothesis $M \neq$. We now prove M is a m-system of $\mathcal{P}$. Let $a, b \in M$. If $M \cap L(a, b) =$, then for any $t \in L(a, b)\backslash\{0\}$, we have $\langle\langle x_1, J\rangle, J\rangle \subseteq \langle a, J\rangle$ for some $x_1 \in K\backslash\{0\}$, which implies $t \in M$, a contradiction. So M is an m-system of $\mathcal{P}$.

Let K be a J-ideal of $\mathcal{P}$ and $a \in K$. Then from the hypothesis, there exists $b \in \mathcal{P}$ such that $\langle\langle a, J\rangle, J\rangle = \langle b, J\rangle$, which gives $L(a, b) \subseteq J$ and $b \in M$. Therefore $K \subseteq J(M)$. □

The below example gives an illustration of the above theorem.

Example 4.11. In Example 4.2, for the ideals $J = \{0, b\}$, $K = \{0, b, c\}$ and an m-system $M = \{a, d\}$, we have $\langle 0, J\rangle = \langle b, J\rangle = \langle\langle e, J\rangle, J\rangle$ and $\langle a, J\rangle = \langle d, J\rangle = \langle\langle c, J\rangle, J\rangle$. Then for J-ideal K of $\mathcal{P}$, we have $K \subseteq J(M)$ for the m-system M of $\mathcal{P}$.

5. Conclusion

The notion and development of the prime ideal have a specific position in algebraic geometry and commutative algebra. These are ways for discovering algebraic structural characteristics that are realistic. The features of J-prime ideals in posets were examined in this work, as well as a wider generalization of prime ideals in posets. Examples of J-prime ideals, J-ideals and M_J-ideals were given. We also obtained the condition for every ideal of $\mathcal{P}$ to be a J-ideal. We also provided the conditions for a M_J-ideal to be a J-ideal. These results can be applied to 0-distributive posets, lattices, near lattices, semi-lattices, and 0-distributive near lattices using the notion described in this paper.

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