Algebraic Structures and Their Applications Vol. 9 No. 2 (2022) pp 1-10.

Research Paper

Λ-EXTENSION OF BINARY MATROIDS

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Abstract. In this paper, we combine two binary operations Γ-Extension and element splitting under special conditions, to extend binary matroids. For a given binary matroid M, we call a matroid obtained in this way a Λ-Extension of M. We note some attractive properties of this matroid operation, particularly constructing a chordal matroid from a chordal binary matroid.

1. Introduction

The matroid notations and terminology used here will follow Oxley [6]. Slater [8, 9] defined several operations on graphs that have important roles in graph connectivity. Azadi [1] and Azanchiler [2] define matroid generalizations of two of these operations (n-point splitting and point addition operations, respectively) and they note some attractive properties of these matroid operations. They called them, element splitting and Γ-Extension operations on binary matroids, respectively. These operations are defined as follows:

DOI: 10.22034/as.2022.2608

MSC(2010): Primary: 05B35

Keywords: Γ-extension, element splitting, binary matroid, connectivity, chordal matroid.

Received: 23 November 2020, Accepted: 13 February 2022.

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Definition 1.1. Let M be a binary matroid on a set E and A be a matrix that represents M over $GF(2)$. Consider a subset T of $E(M)$. Let A_T be the matrix that is obtained by adjoining an extra row to A whose entries are zero everywhere except in the columns corresponding to all elements of T. Let A'_T be the matrix that is obtained by adjoining an extra column to A_T with this column being zero everywhere except in the last row. Let M_T and M'_T be the matroids that are represented by the matrices A_T, A'_T, respectively. Then the transition from M to M_T and M'_T is called *the generalized splitting operation* and *element splitting operation*, respectively.

Let M be a matroid and $T \subseteq E(M)$, a circuit C of M is called an *OT-circuit* if C contains an odd number of elements of T, and C is an *ET-circuit* if C contains an even number of elements of T. The following proposition characterizes the circuits of the element splitting matroid M'_T in terms of the circuits of the original binary matroid M.

Proposition 1.2. [1] *Let $M = (E, \mathcal{C})$ be a binary matroid together with the collection of circuits $\mathcal{C}$. Suppose $T \subseteq E$ and $\alpha \notin E$. Then $M'_T = (E \cup \alpha, \mathcal{C}')$ where $\mathcal{C}' = \mathcal{C}_0 \cup \mathcal{C}_1 \cup \mathcal{C}_2$ and*
$\mathcal{C}_0 = \{C \in \mathcal{C} : C$ *is an ET-circuit*$\}$;
$\mathcal{C}_1 = \{C \cup \{\alpha\} : C \in \mathcal{C}$ *and C is an OT-circuit*$\}$;
$\mathcal{C}_2 =$ *The set of minimal members of* $\{C_1 \cup C_2 : C_1, C_2 \in \mathcal{C}, C_1 \cap C_2 = \emptyset$
and each of C_1 and C_2 is an OT-circuit$\}$.

Now, let M be a binary matroid. By using the next definition, we obtain a matroid by adding some new elements in parallel to selected members of $E(M)$ and then applying the generalized splitting operation on these new elements.

Definition 1.3. Let A be a matrix that represents a binary matroid M. Let $X = \{x_1, x_2, ..., x_k\}$ be an independent subset of $E(M)$ and let $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_k\}$ be a set such that $E(M) \cap \Gamma = \emptyset$. Let A^X be the matrix obtained from A by the following way.

(1) Obtain a matrix A' from A by adjoining k columns labeled by $\gamma_1, \gamma_2, ..., \gamma_k$ such that the column labeled by γ_i is same as the column labeled by x_i for $i \in \{1, 2, ...k\}$.
(2) Adjoin (generalized splitting) one extra row to A' which has entry 1 in the column labeled by γ_i and zero elsewhere, $i \in \{1, 2, ...k\}$.

The vector matroid of the matrix A^X, denoted by M^X, is called the Γ-*Extension* of M and the transition from M to M^X is called Γ-*Extension operation* on M.

Now we change part (2) of definition 1.3. We apply element splitting operation instead of the generalized splitting operation on part (2) such that one can select some elements even if they don't belong to part (1) and we denote the resulting matrix and its vector matroid by A^X_α

and M_α^X, respectively. We say that the transition from M to M_α^X is Γ'-*Extension operation* on M.

In the next section, we shall use the Γ'-Extension operation to characterize one special class of graphic matroids, which we use the cycle matroids of all members of this class to define a new extension of binary matroids.

2. Characterizing one special class of graphic matroid

An undirected graph is *chordal* (triangulated, rigid circuit) if every cycle of length greater than three has a chord [3]. Suppose that Λ_n is a graph that is obtained from the cycle C_n (cycle with n vertices and n edges) by adding $n-3$ chords on exactly one of its vertices where $n \geq 4$. This graph has exactly one vertex of degree $n-1$, we denote such vertex by V_λ. Let λ_1 and λ_{n-1} be adjacent edges of Λ_n such that their common endpoint is V_λ. Let $x_1, x_2, ..., x_{n-2}$ be the labels of the other edges of C_n and $\lambda_2, \lambda_3, ..., \lambda_{n-2}$ be the labels of all $n-3$ chords of C_n such that the set $\{\lambda_i, \lambda_{i+1}, x_i\}$ is a circuit of Λ_n, for $i \in \{1, 2, ..., n-2\}$. For $n \geq 4$, we denote by Λ_n^+, the graph that is obtained from Λ_n by adding a new edge to Λ_n with label x_{n-1} in parallel to the edge λ_1 or λ_{n-1} (see Figure 1). Clearly, these two graphs are chordal.

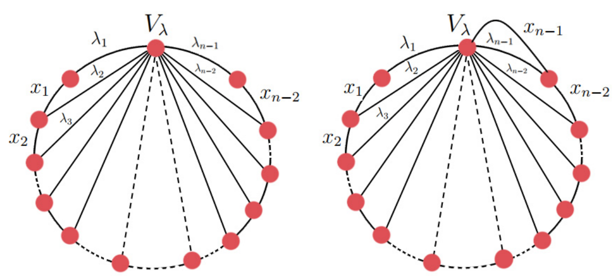

Figure 1. Two graphs Λ_n and Λ_n^+ .

Next, consider matrix L_n a special type of lower triangular matrix as follows:

$$L_{ij} = \begin{cases} 1, & i \geq j; \\ 0, & i < j. \end{cases} \tag{1}$$

Written explicitly,

$$L_n = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix} \tag{2}$$

Theorem 2.1. *For $n \geq 4$, Let $M(G)$ be the cycle matroid of the graph $G = \Lambda_n^+$. Let $A = [I_{n-1}|L_{n-1}]$ be a matrix over $GF(2)$ whose columns are labeled, in order*

$x_1, x_2, ..., x_{n-2}, x_{n-1}, \lambda_1, \lambda_2, \lambda_3, ..., \lambda_{n-1}$ *such that* L_{n-1} *is an* $(n-1) \times (n-1)$ *lower triangular matrix as (2). Then* A *represents* $M(G)$*. Moreover, a set* C *is a circuit of* $M(G)$ *if and only if* C *has one of the following forms.*

(i) $\{x_{n-1}, \lambda_{n-1}\}$*;*

(ii) $\{\lambda_i, \lambda_j, x_i, x_{i+1}, ..., x_{j-1}\}$ *for* i *and* j *in* $\{1, 2, 3, ..., n-1\}$ *such that* $i < j$*;*

(iii) $C = C'\Delta\{x_{n-1}, \lambda_{n-1}\}$ *where* C' *has the form in the part (ii) such that* $C' \cap \{x_{n-1}, \lambda_{n-1}\} \neq \emptyset$*.*

Proof. For $n \geq 4$, let G be a graph Λ_n^+ for which graphic representations are shown in Figure 1. Suppose that $X = \{x_1, x_2, ..., x_{n-2}, x_{n-1}\}$. Then X is a spanning tree of G and so X is a basis of $M(G)$. Let $A = [I_{n-1}|D]$ be a matrix over $GF(2)$ whose columns are labeled, in order

$$x_1, x_2, ..., x_{n-1}, \lambda_1, \lambda_2, \lambda_3, ..., \lambda_{n-1}$$

and A represents $M(G)$. Since $X \cup \lambda_1$ is a cycle of $M(G)$, all entries in the corresponding column of λ_1 in A are 1. Moreover, the set $(X - \{x_1\}) \cup \lambda_2$ is a cycle of $M(G)$. Thus, all entries in the corresponding column of λ_2 in A are 1 except in the first entry. For all j in $\{1, 2, ..., n-2\}$, one can easily check that the set $(X - \{x_1, x_2, ..., x_j\}) \cup \lambda_{j+1}$ is a cycle of $M(G)$ and so all entries in the corresponding column of λ_{j+1} in A are 1 except in the corresponding entries of the first row, the second row,...., the jth row. We conclude that D is an $(n-1) \times (n-1)$ lower triangular matrix L_{n-1}. Clearly, the corresponding entries in columns x_{n-1} and λ_{n-1} are the same. Therefore the set $\{x_{n-1}, \lambda_{n-1}\}$ is a circuit of $M(G)$. Moreover, for $j \neq n-1$, the set $X' = (X - \{x_1, x_2, ..., x_j\}) \cup \lambda_{j+1}$ contains x_{n-1}. Hence $X'\Delta\{x_{n-1}, \lambda_{n-1}\}$ is a circuit of $M(G)$ and has the form as part (ii). Now consider the set $\{\lambda_i, \lambda_j, x_i, x_{i+1}, ..., x_{j-1}\}$, for i and j in $\{1, 2, 3, ..., n-l\}$ such that $i < j$. The corresponding column $(n-1)$-vectors of λ_i and λ_j are

$$\lambda_i = (\underbrace{0, 0, ..., 0}_{i-1}, 1, 1, ..., 1)^T \text{ and } \lambda_j = (\underbrace{\underbrace{0, 0, ..., 0}_{i-1}, 0, 0, ..., 0}_{j-1}, 1, 1, ..., 1)^T.$$

Hence the corresponding vector of $\lambda_i + \lambda_j$ over $GF(2)$ is

$$(\underbrace{\underbrace{0, 0, ..., 0}_{i-1}, 1, 1, ..., 1}_{j-1}, 0, 0, ..., 0)^T.$$

We conclude that the elements $x_i, x_{i+1}, ..., x_{j-1}$ must be added to $\{\lambda_i, \lambda_j\}$ to have a circuit of $M(G)$. By the fact that in all binary matroids, the symmetric difference of any set of circuits is either empty or contains a circuit, all circuits of $M(G)$ have the forms in part (i)-(iii). □

Corollary 2.2. *Let M be a matroid defined on Theorem 2.1. Then*

$$M(\Lambda_n^+) \setminus x_{n-1} \cong M(\Lambda_n^+) \setminus \lambda_{n-1} \cong M(\Lambda_n).$$

Moreover, the collection of all circuits of $M(G')$ is

$$\Big\{\{\lambda_i, \lambda_j, x_i, x_{i+1}, ..., x_{j-1}\} : \textit{for } i \textit{ and } j \textit{ in } \{1, 2, 3, ..., n-1\} \textit{ such that } i < j \ \Big\}.$$

Let $\mathcal{G} = \{M(\Lambda_n) : n > 4\}$. The next theorem shows that we can construct all members of $\mathcal{G}$ from Λ_4 by a sequence of Γ'-Extension operations.

Theorem 2.3. *Let the ground set of $M(\Lambda_n)$ be $\{x_1, x_2, ..., x_{n-2}, x_{n-1}, \lambda_1, \lambda_2\ , ..., \lambda_{n-2}\}$ such that the two sets $\{x_i : 1 \leq i \leq n-1\}$ and $\{\lambda_j : 1 \leq j \leq n-2\} \cup x_{n-1}$ are bases of $M(\Lambda_n)$. Then $M(\Lambda_{n+1})$ can be obtained from $M(\Lambda_n)$ by adding the new element λ_{n-1} in parallel to x_{n-1} and applying the element splitting operation on the set $\{\lambda_1, \lambda_2, ..., \lambda_{n-2}, \lambda_{n-1}\}$*

Proof. Note that by adding the new element λ_{n-1} in parallel to x_{n-1}, we obtain $M(\Lambda_n^+)$. Suppose that $N = M(\Lambda_n^+)$. We must show that after applying the element splitting operation on N with respect to $T = \{\lambda_1, \lambda_2, ..., \lambda_{n-2}, \lambda_{n-1}\}$ we obtain $M(\Lambda_{n+1})$. First, let n be an even number greater than three and let α be a new element that it has been added to N after applying the element splitting operation. By Theorem 2.1, the set $\{x_{n-1}, \lambda_{n-1}\}$ is an OT-circuit. Therefore, by Proposition 1.2, $\{x_{n-1}, \lambda_{n-1}, \alpha\}$ is a circuit of N'_T. Moreover, the set $W = \{\lambda_i, \lambda_j, x_i, x_{i+1}, ..., x_{j-1}\}$ is an ET-circuit of N, for i and j in $\{1, 2, 3, ..., n-1\}$ such that $i < j$. By Proposition 1.2 again, W is a circuit of N'_T. Now if W contains the element λ_{n-1}, then, by Theorem 2.1, the set $W' = (W - \lambda_{n-1}) \cup x_{n-1}$ is an OT-circuit of N and so by Proposition 1.2 again, $W' \cup \alpha$ is a circuit of N'_T. Clearly, there is no two OT-circuits of N such that their intersection is empty. Hence, the collection $\mathcal{C}_2$ in Proposition 1.2 is an empty set. Now Let α plays the role of λ_n in N'_T. Then $N'_T \cong M(\Lambda_{n+1})$. Similarly, one can easily check that if n is an odd number, then $N'_T \cong M(\Lambda_{n+1})$. □

3. Λ-extension operation on binary matroids

In this section, we define a new operation on binary matroid by the matrix representation of $M(\Lambda_n)$, for $n \geq 4$. indeed, we use Γ'-extension operation to define this operation.

Definition 3.1. Let $A = [I_r|D]$ be a matrix that represents a binary rank r matroid M. Let A'' be the matrix obtained from A by the following way.

(1) Obtain a matrix A' from A by adjoining r columns to A labeled $\lambda_1, \lambda_2, ..., \lambda_r$ such that these columns form the matrix L_r.

(2) Adjoin (element splitting) one extra row to A' which has entry 1 in the column labeled by λ_i and zero elsewhere, $i \in \{1, 2, ...r\}$ and then Adjoin one extra column with this column being zero everywhere except in the last new row.

We say that the vector matroid of the matrix A'' and we denote it by $\Lambda(M)$, is Λ-extension of M and we call the transition from M to $\Lambda(M)$ is Λ-extension operation on M.

The following two theorems characterize the collection of all circuits of a matroid that is obtained by Λ-extension operation in terms of the circuits of a given binary matroid.

Theorem 3.2. *Let M and N be binary matroids with matrix representations $[I_r|D]$ and $[I_r|D|L_r]$, respectively. Then the collection of all circuits of N is $\mathcal{C}(M) \cup \mathcal{C}_0 \cup \mathcal{C}_1$ where $\mathcal{C}_0$ is a collection of all circuits of $M(\Lambda^+_{r+1})$ and a set C is a member of $\mathcal{C}_1$, if it is a member of the following collection.*

Minimal members of $\{C_1 \Delta C_2 : C_1 \in \mathcal{C}(M)$ and $C_2 \in \mathcal{C}_0$ such that $C_1 \cap C_2 \neq \emptyset\}$;

Proof. By the proof of Theorem 2.1 and by the fact that all circuits of a binary matroid can be characterized by symmetric difference operation, the proof is straightforward. □

Theorem 3.3. *Under the hypotheses of Theorem 3.2, Let $T = \{\lambda_1, \lambda_2, ..., \lambda_r\}$ be the set of all labels of the corresponding columns of submatrix L_r. Then the collection of all circuits of $\Lambda(M)$ is $\mathcal{C}(M) \cup \mathcal{C}_2 \cup \mathcal{C}_3$ where $\mathcal{C}_2$ is a collection of all circuits of $M(\Lambda_{r+2})$ and a set C is a member of $\mathcal{C}_3$, if it is a member of the following collections.*

(i) *$\{C \cup \{\lambda_{r+1}\} : C \in \mathcal{C}_1$ and C is an OT-circuit$\}$;*

(ii) *$\{C_1 \cup C_2 : C_1, C_2 \in \mathcal{C}_1, C_1 \cap C_2 = \emptyset$ and each of C_1 and C_2 is an OT-circuit$\}$.*

Proof. Let M and N be binary matroids with matrix representations $[I_r|D]$ and $[I_r|D|L_r]$, respectively. Let $T = \{\lambda_1, \lambda_2, ..., \lambda_r\}$ be the set of all labels of the corresponding columns of submatrix L_r. Then, by Definition 3.1, $\Lambda(M) = N'_T$ (The element splitting of N). The intersection of any circuit of M with T is empty, so all circuits of M are ET-circuits and by Proposition 1.2, these circuits are also circuits of $\Lambda(M)$. By Theorem 2.3, all circuits of $M(\Lambda_{r+2})$ can be obtained by applying element splitting operation on $M(\Lambda^+_{r+1})$. Now let λ_{r+1} be the new element that is added to N after applying the element splitting operation. Then, by Proposition 1.2 again, one can easily find all members of collection $\mathcal{C}_3$. □

Theorem 3.4. *Let M be a binary matroid. Let r and r' be the rank functions of M and $\Lambda(M)$, respectively. Then $r' = r + 1$.*

Proof. Let M be a binary matroid of rank r and let $A = [I_r|D]$ be the matrix that represents M over $GF(2)$. Let $M' = M[A']$ be a vector matroid obtained from A by adding r new columns to it such that $M' \setminus D \cong M(\Lambda_{r+1})$. In fact, the corresponding submatrix of these r columns is an $r \times r$ lower triangle matrix L_r. Therefore the matrix A' has the form $[I_r|D|L_r]$ and

$$r(M' \setminus D) = r(M' \setminus L_r) = r(M')$$

So $r(M') = r(M)$. Now let M'' be the matroid obtained from M' by element splitting operation with respect to all corresponding elements of L_r. By the fact that the element splitting operation increases the rank of given binary matroid always one unit, we conclude that $r(M'') = r(M) + 1$. □

Theorem 3.5. *Let M be a binary n-connected matroid, for $n \geq 3$. Then $\Lambda(M)$ is a 3-connected matroid.*

Proof. By Definition 3.1, Let A and A'' be the matrices that represent a given binary matroid M and $\Lambda(M)$, respectively. Let r and r' be the rank functions of M and $\Lambda(M)$, respectively. For some k, let $E(M) = \{x_1, x_2, ..., x_r, d_1, d_2, d_3, ..., d_k\}$ and

$$E(\Lambda(M)) = \{x_1, x_2, ..., x_r, d_1, d_2, d_3, ..., d_k, \lambda_1, \lambda_2, \lambda_3, ..., \lambda_r, \lambda_{r+1}\},$$

where the set $\{x_1, x_2, ..., x_r\}$ is a basis of M and $\{x_1, x_2, ..., x_r, \lambda_{r+1}\}$ is a basis of $\Lambda(M)$ such that the matrices A and A'' are constructed by these bases. By Theorem 3.2, the set $X = \{\lambda_i, \lambda_{i+1}, x_i\}$ is a circuit of $\Lambda(M)$, for i in $\{1, 2, ..., r\}$. Then $r'(X) = 2$. Let $Y = E(\Lambda(M)) - X$. Then, by the fact that $r \geq 3$, $\min\{|X|, |Y|\} = 3$. Now Let $z = r'(X) + r'(Y) - r'(\Lambda(M))$ and $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_{i-1}, \lambda_{i+2}, ..., \lambda_{r+1}\}$. Then

$$z = r'\big((E(M) - x_i) \cup \Lambda\big) - r(M) + 1.$$

But

$$r'\big((E(M) - \{x_i\}) \cup \Lambda\big) \leq r(M) + 1.$$

We conclude that $z \leq 2$ and this means (X, Y) is a 3-separation of $\Lambda(M)$. One can easily see that if $\Lambda(M)$ has a 2 or 1-separation, then M is not an n-connected matroid. Hence $\Lambda(M)$is a 3-connected matroid. □

Chordal matroids are the natural generalization of chordal graphs. We are interested in the following the effect of the Λ-Extension operation on the notion of a chordal matroid.

Definition 3.6. In [4, 5], a binary matroid M is said to be *chordal* if each circuit C of M that has four or more elements has a chord, that is, a circuit C of a matroid M has a chord e if there are two circuits C_1 and C_2 such that $C_1 \cap C_2 = \{e\}$ and $C = C_1 \Delta C_2$. In this case, we say that C is the sum of C_1 and C_2 and also that $C \cup \{e\}$ is split into C_1 and C_2.
In [7], the author introduces an equivalent definition of chordal binary matroids as follows: M is chordal if for every circuit C, $|C| > 3$, there is a pair of elements $c, c; \in C$ and an element $e \in E(M) - C$ such that $\{c, c', e\}$ is a triangle (A circuit of M is a triangle if it has cardinality of exactly 3).

Theorem 3.7. *Let M be a binary rank-r matroid that is not chordal. Then $\Lambda(M)$ is not a chordal matroid.*

Proof. Let M be a binary rank-r matroid that is not chordal. Let $[I_r|D]$ be a matrix representation of M with respect to basis $\{x_1, x_2, ..., x_r\}$. Let C be a circuit of M with $|C| \geq 4$ such that C has no chord. Suppose that $C = \{x_1, x_2, ..., x_i, d_1, d_2, ..., d_j\}$, for some i and j. Then, By Theorem 3.3, C is a circuit of $\Lambda(M)$. Assume the contrary, that is, suppose that C has a chord in $\Lambda(M)$. Then exactly one of the following holds.

(i) For some l,k and s and for $x_l, d_k \in C$ there is a circuit $\{x_l, d_k, \lambda_s\}$ in $\Lambda(M)$; a contradiction (by Theorem 3.3, all circuits of $\Lambda(M)$ have even cardinality with the set $\{\lambda_1, \lambda_2, ..., \lambda_{r+1}\}$).
(ii) For some l',k' and s' and for $d_{l'}, d_{k'} \in C$ there is a circuit $\{d_{l'}, d_{k'}, \lambda_{s'}\}$ in $\Lambda(M)$; a contradiction.
(iii) For some l'',k'' and s'' and for $x_{l''}, x_{k''} \in C$ there is a circuit $\{x_{l'_,}, x_{k''}, \lambda_{s''}\}$ in $\Lambda(M)$; a contradiction.

We conclude that $\Lambda(M)$ is not a chordal matroid. □

Theorem 3.8. *Let M be a binary rank-r chordal matroid. Then $\Lambda(M)$ is a chordal matroid.*

Proof. For $r \geq 3$, Suppose that $\mathcal{C}$ and $\mathcal{C}'$ are collections of all circuits of $M(\Lambda^+_{r+1})$ and $M(\Lambda_{r+2})$, respectively. Let M be a chordal binary matroid M with rank r and matrix representation $[I_r|D]$. Then $E(\Lambda(M)) = E(M) \cup E(\Lambda_{r+2})$ and every circuit of M that has four or more elements has a chord. Let N be a binary matroid with matrix representation $[I_r|D|L_r]$ whose columns are labeled, for some k, in order

$$x_1, x_2, ..., x_r, d_1, d_2, d_3, ..., d_k, \lambda_1, \lambda_2, \lambda_3, ..., \lambda_r, \lambda_{r+1}.$$

Then, By Corollary 2.2, every member of $M(\Lambda_{r+2})$ belongs to a triangle. This means for the circuit $\{\lambda_i, \lambda_j, x_i, x_{i+1}, ..., x_{j-1}\}$ from $M(\Lambda_{r+2})$, with i and j in $\{1, 2, 3, ..., r+1\}$ such that $i < j$ and $j \neq i+1$, there is a triangle $\{\lambda_i, \lambda_{i+1}, x_i\}$ and so every circuit of $M(\Lambda_{r+2})$ that has four or more elements has a chord. For $T = \{\lambda_1, \lambda_2, ..., \lambda_r\}$, we have $\Lambda(M) = N'_T$. Let λ_{r+1} be a new element that is added to N after applying the element splitting operation. Then, all circuits of N have a chord and it suffices to show that all circuits belong to the collection $\mathcal{C}_3$ of Theorem 3.3 have a chord. Let $C \in \mathcal{C}_3$ and $C = C' \cup \lambda_{r+1}$. Then, by Theorem 3.2, $C = (C_1 \Delta C_2) \cup \lambda_{r+1}$ where $C_1 \in \mathcal{C}(M)$ and $C_2 \in \mathcal{C}$ such that $C_1 \cap C_2 \neq \emptyset$ and $|C_2 \cap T|$ is odd. Therefore, by Theorem 2.1, if $\lambda_i \in C_2$, then $\lambda_{i+1} \notin C_2$, for i in $\{1, 2, ..., r-1\}$ (clearly, if $i = r$, then x_r is a chord of C). Now, if $x_i \notin C_1$, then λ_{i+1} is a chord of C; and if $x_i \in C_1$ and $C_1 \cap C_2 = x_i$, then $C = C_1 \Delta C'_2$ where $C'_2 \in \mathcal{C}'$ and so C has a chord. Finally, let $C \in \mathcal{C}_3$ and $C = C_3 \cup C_4$ where $C_3 = C'_3 \Delta C''_3$ and $C_4 = C'_4 \Delta C''_4$ such that $C'_3, C''_3 \in \mathcal{C}(M)$ and $C'_4, C''_4 \in \mathcal{C}$. Then $C = (C'_3 \Delta C'_4) \Delta (C''_3 \Delta C''_4)$. Now, there are two elements x_l and λ_l in $(C'_3 \Delta C'_4) \cap (C''_3 \Delta C''_4)$, for l in $\{1, 2, ..., r\}$ and therefore λ_{l+1} is a chord of C. We conclude that $\Lambda(M)$ is a chordal matroid. □

The next corollary is an immediate consequence of the last two theorems.

Corollary 3.9. *Let M be a binary matroid. Then M is a chordal matroid if and only if $\Lambda(M)$ is a chordal matroid.*

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