Journal of Algebraic Structures and Their Applications
ISSN: 2382-9761
www.as.yazd.ac.ir

Algebraic Structures and Their Applications Vol. 3 No. 2 (2016), pp 1-20.

DERIVATIONS OF UP-ALGEBRAS BY MEANS OF UP-ENDOMORPHISMS

AIYARED IAMPAN

Communicated by B. Davvaz

Abstract. The notion of f-derivations of UP-algebras is introduced, some useful examples are discussed, and related properties are investigated. Moreover, we show that the fixed set and the kernel of f-derivations are UP-subalgebras of UP-algebras, and also give examples to show that the two sets are not UP-ideals of UP-algebras in general.

1. Introduction and Preliminaries

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [9], BCI-algebras [10], BCH-algebras [7], KU-algebras [25], SU-algebras [13] and others. They are strongly connected with logic. For example, BCI-algebras introduced by Iséki [10] in 1966 have connections with BCI-logic being the BCI-system in combinatorial logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki

MSC(2010): Primary: 03G25. Secondary: 13N15.

Keywords: UP-algebra, UP-subalgebra, UP-ideal, f-derivation.

Received: 01 November 2016, Accepted: 14 April 2017.

∗Corresponding author

This work was financially supported by the National Research Council of Thailand (NRCT) and the University of Phayao (UP), Project Number: RD59002.

[9, 10] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

In the theory of rings and near rings, the properties of derivations is an important topic to study [23, 15]. In 2004, Jun and Xin [12] applied the notions of rings and near rings theory to BCI-algebras and obtained some properties. Several researches were conducted on the generalizations of the notion of derivations and application to many logical algebras such as: In 2005, Zhan and Liu [27] introduced the notion of left-right (right-left) f-derivations of BCI-algebras. In 2006, Abujabal and Al-shehri [1] investigated some fundamental properties and proved some results on derivations of BCI-algebras. In 2007, Abujabal and Al-shehri [2] introduced the notion of left derivations of BCI-algebras. In 2009, Javed and Aslam [11] investigated some fundamental properties and established some results of f-derivations of BCI-algebras. Nisar [22] introduced the notions of right F-derivations and left F-derivations of BCI-algebras. Nisar [21] characterized f-derivations of BCI-algebras. Prabpayak and Leerawat [24] studied the notions of left-right (right-left) derivations of BCC-algebras. In 2012, Al-shehri and Bawazeer [4] introduced the notion of left-right (right-left) t-derivations of BCC-algebras. Lee and Kim [16] considered the properties of f-derivations of BCC-algebras. Muhiuddin and Al-roqi [18] introduced the notion of t-derivations of BCI-algebras. Muhiuddin and Al-roqi [17] introduced the notion of (regular) (α, β)-derivations of BCI-algebras. In 2013, Bawazeer, Al-shehri and Babusal [6] introduced the notion of generalized derivations of BCC-algebras. Lee [14] introduced a new kind of derivations of BCI-algebras. Muhiuddin, Al-roqi, Jun and Ceven [20] introduced the notion of symmetric left bi-derivations of BCI-algebras. In 2014, Al-roqi [3] introduced the notion of generalized (regular) (α, β)-derivations of BCI-algebras. Muhiuddin and Al-roqi [19] introduced the notion of generalized left derivations of BCI-algebras. Ardekani and Davvaz [5] introduced the notion of (f, g)-derivations of BCI-algebras. In 2016, Sawika, Intasan, Kaewwasri and Iampan [26] introduced the notions of (l, r)-derivations, (r, l)-derivations and derivations of UP-algebras and investigated some related properties.

The notion of derivations play an important role in studying the many logical algebras. In this paper, we introduce the notion of f-derivations of UP-algebras which is the generalization of the notion of derivations [26], some useful examples are discussed, and related properties are investigated.

Before we begin our study, we will introduce to the definition of a UP-algebra.

Definition 1.1. [8] An algebra $A = (A; \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra* if it satisfies the following axioms: for any $x, y, z \in A$,

(UP-1): $(y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0$,
(UP-2): $0 \cdot x = x$,

(UP-3): $x \cdot 0 = 0$, and

(UP-4): $x \cdot y = y \cdot x = 0$ implies $x = y$.

Example 1.1. [8] *Let X be a universal set. Define a binary operation $\cdot$ on the power set of X by putting $A \cdot B = B \cap A' = A' \cap B = B - A$ for all $A, B \in \mathcal{P}(X)$. Then $(\mathcal{P}(X); \cdot, \emptyset)$ is a UP-algebra and we shall call it the power UP-algebra of type 1.*

Example 1.2. [8] *Let X be a universal set. Define a binary operation $*$ on the power set of X by putting $A * B = B \cup A' = A' \cup B$ for all $A, B \in \mathcal{P}(X)$. Then $(\mathcal{P}(X); *, X)$ is a UP-algebra and we shall call it the power UP-algebra of type 2.*

In what follows, let A denotes a UP-algebra unless otherwise specified. The following proposition is very important for the study of UP-algebras.

Proposition 1.1. [8] *In a UP-algebra A, the following properties hold: for any $x, y, z \in A$,*

(1) $x \cdot x = 0$,
(2) $x \cdot y = 0$ *and* $y \cdot z = 0$ *imply* $x \cdot z = 0$,
(3) $x \cdot y = 0$ *implies* $(z \cdot x) \cdot (z \cdot y) = 0$,
(4) $x \cdot y = 0$ *implies* $(y \cdot z) \cdot (x \cdot z) = 0$,
(5) $x \cdot (y \cdot x) = 0$,
(6) $(y \cdot x) \cdot x = 0$ *if and only if* $x = y \cdot x$, *and*
(7) $x \cdot (y \cdot y) = 0$.

On a UP-algebra $A = (A; \cdot, 0)$, we define a binary relation $\leq$ on A [8] as follows: for all $x, y \in A$,

$$x \leq y \text{ if and only if } x \cdot y = 0.$$

Definition 1.2. [8] A nonempty subset B of A is called a *UP-ideal* of A if it satisfies the following properties:

(1) the constant 0 of A is in B, and
(2) for any $x, y, z \in A, x \cdot (y \cdot z) \in B$ and $y \in B$ imply $x \cdot z \in B$.

Clearly, A and $\{0\}$ are UP-ideals of A.

Theorem 1.3. [8] *Let A be a UP-algebra and B a UP-ideal of A. Then the following statements hold: for any $x, a, b \in A$,*

(1) if $b \cdot x \in B$ and $b \in B$, then $x \in B$. Moreover, if $b \cdot X \subseteq B$ and $b \in B$, then $X \subseteq B$,
(2) if $b \in B$, then $x \cdot b \in B$. Moreover, if $b \in B$, then $X \cdot b \subseteq B$, and
(3) if $a, b \in B$, then $(b \cdot (a \cdot x)) \cdot x \in B$.

Definition 1.3. [8] Let $(A;\cdot,0)$ and $(A';\cdot',0')$ be UP-algebras. A mapping f from A to A' is called a *UP-homomorphism* if

$$f(x\cdot y)=f(x)\cdot' f(y) \text{ for all } x,y\in A.$$

A UP-homomorphism $f\colon A\to A'$ is called a *UP-endomorphism* of A if $A'=A$.

Theorem 1.4. [8] *Let $(A;\cdot,0_A)$ and $(B;*,0_B)$ be UP-algebras and let $f\colon A\to B$ be a UP-homomorphism. Then the following statements hold:*

(1) $f(0_A)=0_B$*,*

(2) for any $x,y\in A$, if $x\leq y$, then $f(x)\leq f(y)$,

(3) if C is a UP-subalgebra of A, then the image $f(C)$ is a UP-subalgebra of B. In particular, $\mathrm{Im}(f)$ *is a UP-subalgebra of B,*

(4) if D is a UP-subalgebra of B, then the inverse image $f^{-1}(D)$ is a UP-subalgebra of A. In particular, $\mathrm{Ker}(f)$ *is a UP-subalgebra of A,*

(5) if C is a UP-ideal of A, then the image $f(C)$ is a UP-ideal of $f(A)$,

(6) if D is a UP-ideal of B, then the inverse image $f^{-1}(D)$ is a UP-ideal of A. In particular, $\mathrm{Ker}(f)$ *is a UP-ideal of A, and*

(7) $\mathrm{Ker}(f)=\{0_A\}$ *if and only if f is injective.*

Definition 1.4. [26] For any $x,y\in A$, we define a binary operation $\wedge$ on A by $x\wedge y=(y\cdot x)\cdot x$.

Definition 1.5. [26] A UP-algebra A is called *meet-commutative* if $x\wedge y=y\wedge x$ for all $x,y\in A$, that is, $(y\cdot x)\cdot x=(x\cdot y)\cdot y$ for all $x,y\in A$.

Proposition 1.2. [26] *In a UP-algebra A, the following properties hold: for any $x\in A$,*

(1) $0\wedge x=0$*,*

(2) $x\wedge 0=0$*, and*

(3) $x\wedge x=x$*.*

2. Main Results

In this section, we introduce the notions of (l,r)-f-derivations, (r,l)-f-derivations, and f-derivations of UP-algebras, and study the fixed set and the kernel of (l,r)-f-derivations, (r,l)-f-derivations, and f-derivations.

Definition 2.1. Let f be a UP-endomorphism of A. A self-map $d_f\colon A\to A$ is called an *(l,r)-f-derivation* of A if it satisfies the identity $d_f(x\cdot y)=(d_f(x)\cdot f(y))\wedge(f(x)\cdot d_f(y))$ for all $x,y\in A$. Similarly, a self-map $d_f\colon A\to A$ is called an *(r,l)-f-derivation* of A if it satisfies the identity $d_f(x\cdot y)=(f(x)\cdot d_f(y))\wedge(d_f(x)\cdot f(y))$ for all $x,y\in A$. Moreover, if d_f is both an (l,r)-f-derivation and an (r,l)-f-derivation of A, it is called an *f-derivation* of A.

By using Microsoft Excel, we have all examples.

Example 2.1. *Let* $A = \{0, 1, 2, 3\}$ *be a set with a binary operation* $\cdot$ *defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ *is a UP-algebra. We define a self-map* $f : A \to A$ *as follows:*

$$f(0) = 0, f(1) = 0, f(2) = 1 \text{ and } f(3) = 3.$$

Then f *is a UP-endomorphism. We define a self-map* $d_f : A \to A$ *as follows:*

$$d_f(0) = 0, d_f(1) = 0, d_f(2) = 1 \text{ and } d_f(3) = 0.$$

Then d_f *is an* f*-derivation of* A.

Proposition 2.1. *Each UP-endomorphism* f *of* A *is its* f*-derivation.*

Proof. It follows from Proposition 1.2 (3). □

Definition 2.2. An (l, r)-f-derivation (resp. (r, l)-f-derivation, f-derivation) d_f of A is called *regular* if $d_f(0) = 0$.

Theorem 2.2. *In a UP-algebra* A*, the following statements hold:*

(1) every (l, r)-f*-derivation of* A *is regular, and*

(2) every (r, l)-f*-derivation of* A *is regular.*

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. Then

(By UP-3) $d_f(0) = d_f(0 \cdot 0)$

$= (d_f(0) \cdot f(0)) \wedge (f(0) \cdot d_f(0))$

(By Theorem 1.4 (1)) $= (d_f(0) \cdot 0) \wedge (0 \cdot d_f(0))$

(By UP-2 and UP-3) $= 0 \wedge d_f(0)$

(By Proposition 1.2 (1)) $= 0.$

Hence, d_f is regular.

(2) Assume that d_f is an (r,l)-f-derivation of A. Then

$$\begin{aligned}
&\text{(By UP-3)} & d_f(0) &= d_f(0\cdot 0)\\
&& &= (f(0)\cdot d_f(0)) \wedge (d_f(0)\cdot f(0))\\
&\text{(By Theorem 1.4 (1))} & &= (0\cdot d_f(0)) \wedge (d_f(0)\cdot 0)\\
&\text{(By UP-2 and UP-3)} & &= d_f(0) \wedge 0\\
&\text{(By Proposition 1.2 (2))} & &= 0.
\end{aligned}$$

Hence, d_f is regular. □

Corollary 2.3. *Every f-derivation of A is regular.*

Theorem 2.4. *In a UP-algebra A, the following statements hold:*

(1) if d_f is an (l,r)-f-derivation of A, then $d_f(x) = f(x) \wedge d_f(x)$ for all $x \in A$, and

(2) if d_f is an (r,l)-f-derivation of A, then $d_f(x) = d_f(x) \wedge f(x)$ for all $x \in A$.

Proof. (1) Assume that d_f is an (l,r)-f-derivation of A. Then, for all $x \in A$,

$$\begin{aligned}
&\text{(By UP-2)} & d_f(x) &= d_f(0\cdot x)\\
&& &= (d_f(0)\cdot f(x)) \wedge (f(0)\cdot d_f(x))\\
&\text{(By Theorem 1.4 (1) and 2.2 (1))} & &= (0\cdot f(x)) \wedge (0\cdot d_f(x))\\
&\text{(By UP-2)} & &= f(x) \wedge d_f(x).
\end{aligned}$$

(2) Assume that d_f is an (r,l)-f-derivation of A. Then, for all $x \in A$,

$$\begin{aligned}
&\text{(By UP-2)} & d_f(x) &= d_f(0\cdot x)\\
&& &= (f(0)\cdot d_f(x)) \wedge (d_f(0)\cdot f(x))\\
&\text{(By Theorem 1.4 (1) and 2.2 (2))} & &= (0\cdot d_f(x)) \wedge (0\cdot f(x))\\
&\text{(By UP-2)} & &= d_f(x) \wedge f(x).
\end{aligned}$$

□

Corollary 2.5. *If d_f is an f-derivation of A, then $d_f(x) = d_f(x) \wedge f(x) = f(x) \wedge d_f(x)$ for all $x \in A$.*

Proposition 2.2. *Let d_f be an (l,r)-f-derivation of A. Then the following properties hold: for any $x, y \in A$,* *(1)* $f(x) \leq d_f(x)$,

(2) $d_f(x) \cdot f(y) \leq d_f(x \cdot y)$,

(3) if $f(d_f(x)) = d_f(x)$ *or* $d_f(d_f(x)) = f(x)$, *then* $d_f(x \cdot d_f(x)) = 0$,

(4) if $f(d_f(x)) = d_f(x)$ *or* $d_f(d_f(x)) = f(x)$, *then* $d_f(d_f(x) \cdot x) = 0$,

(5) if $d_f(f(x)) = f(x)$ *or* $f(f(x)) = d_f(x)$, *then* $d_f(x \cdot f(x)) = 0$, *and*

(6) if $d_f(f(x)) = f(x)$ *or* $f(f(x)) = d_f(x)$, *then* $d_f(f(x) \cdot x) = 0$.

Proof. (1) For all $x \in A$,

$$\begin{aligned} &\text{(By Theorem 2.4 (1))} & f(x) \cdot d_f(x) &= f(x) \cdot (f(x) \wedge d_f(x)) \\ & & &= f(x) \cdot ((d_f(x) \cdot f(x)) \cdot f(x)) \\ &\text{(By Proposition 1.1 (5))} & &= 0. \end{aligned}$$

Hence, $f(x) \leq d_f(x)$ for all $x \in A$.

(2) For all $x, y \in A$,

$$\begin{aligned} (d_f(x) \cdot f(y)) \cdot d_f(x \cdot y) &= (d_f(x) \cdot f(y)) \cdot ((d_f(x) \cdot f(y)) \wedge (f(x) \cdot d_f(y))) \\ &= (d_f(x) \cdot f(y)) \cdot (((f(x) \cdot d_f(y)) \cdot (d_f(x) \cdot f(y))) \cdot (d_f(x) \cdot f(y))) \\ \text{(By Proposition 1.1 (5))} \quad &= 0. \end{aligned}$$

Hence, $d_f(x) \cdot f(y) \leq d_f(x \cdot y)$ for all $x, y \in A$.

(3) For all $x \in A$, if $f(d_f(x)) = d_f(x)$, then

$$\begin{aligned} & & d_f(x \cdot d_f(x)) &= (d_f(x) \cdot f(d_f(x))) \wedge (f(x) \cdot d_f(d_f(x))) \\ & & &= (d_f(x) \cdot d_f(x)) \wedge (f(x) \cdot d_f(d_f(x))) \\ &\text{(By Proposition 1.1 (1))} & &= 0 \wedge (f(x) \cdot d_f(d_f(x))) \\ &\text{(By Proposition 1.2 (1))} & &= 0. \end{aligned}$$

If $d_f(d_f(x)) = f(x)$, then

$$\begin{aligned} & & d_f(x \cdot d_f(x)) &= (d_f(x) \cdot f(d_f(x))) \wedge (f(x) \cdot d_f(d_f(x))) \\ & & &= (d_f(x) \cdot f(d_f(x))) \wedge (f(x) \cdot f(x)) \\ &\text{(By Proposition 1.1 (1))} & &= (d_f(x) \cdot f(d_f(x))) \wedge 0 \\ &\text{(By Proposition 1.2 (2))} & &= 0. \end{aligned}$$

(4) For all $x \in A$, if $f(d_f(x)) = d_f(x)$, then

$$\begin{aligned} d_f(d_f(x)\cdot x) &= (d_f(d_f(x))\cdot f(x)) \wedge (f(d_f(x))\cdot d_f(x)) \\ &= (d_f(d_f(x))\cdot f(x)) \wedge (d_f(x)\cdot d_f(x)) \\ \text{(By Proposition 1.1 (1))} \quad &= (d_f(d_f(x))\cdot f(x)) \wedge 0 \\ \text{(By Proposition 1.2 (2))} \quad &= 0. \end{aligned}$$

If $d_f(d_f(x)) = f(x)$, then

$$\begin{aligned} d_f(d_f(x)\cdot x) &= (d_f(d_f(x))\cdot f(x)) \wedge (f(d_f(x))\cdot d_f(x)) \\ &= (f(x)\cdot f(x)) \wedge (f(d_f(x))\cdot d_f(x)) \\ \text{(By Proposition 1.1 (1))} \quad &= 0 \wedge (f(d_f(x))\cdot d_f(x)) \\ \text{(By Proposition 1.2 (1))} \quad &= 0. \end{aligned}$$

(5) For all $x \in A$, if $d_f(f(x)) = f(x)$, then

$$\begin{aligned} d_f(x\cdot f(x)) &= (d_f(x)\cdot f(f(x))) \wedge (f(x)\cdot d_f(f(x))) \\ &= (d_f(x)\cdot f(f(x))) \wedge (f(x)\cdot f(x)) \\ \text{(By Proposition 1.1 (1))} \quad &= (d_f(x)\cdot f(f(x))) \wedge 0 \\ \text{(By Proposition 1.2 (2))} \quad &= 0. \end{aligned}$$

If $f(f(x)) = d_f(x)$, then

$$\begin{aligned} d_f(x\cdot f(x)) &= (d_f(x)\cdot f(f(x))) \wedge (f(x)\cdot d_f(f(x))) \\ &= (d_f(x)\cdot d_f(x)) \wedge (f(x)\cdot d_f(f(x))) \\ \text{(By Proposition 1.1 (1))} \quad &= 0 \wedge (f(x)\cdot d_f(f(x))) \\ \text{(By Proposition 1.2 (1))} \quad &= 0. \end{aligned}$$

(6) For all $x \in A$, if $d_f(f(x)) = f(x)$, then

$$\begin{aligned} d_f(f(x)\cdot x) &= (d_f(f(x))\cdot f(x)) \wedge (f(f(x))\cdot d_f(x)) \\ &= (f(x)\cdot f(x)) \wedge (f(f(x))\cdot d_f(x)) \\ \text{(By Proposition 1.1 (1))} \quad &= 0 \wedge (f(f(x))\cdot d_f(x)) \\ \text{(By Proposition 1.2 (1))} \quad &= 0. \end{aligned}$$

If $f(f(x)) = d_f(x)$, then

$$d_f(f(x) \cdot x) = (d_f(f(x)) \cdot f(x)) \wedge (f(f(x)) \cdot d_f(x))$$
$$= (d_f(f(x)) \cdot f(x)) \wedge (d_f(x) \cdot d_f(x))$$
(By Proposition 1.1 (1)) $$= (d_f(f(x)) \cdot f(x)) \wedge 0$$
(By Proposition 1.2 (2)) $$= 0.$$

□

Proposition 2.3. *Let d_f be an (r,l)-f-derivation of A. Then the following properties hold: for any $x, y \in A$,*

(1) $f(x) \cdot d_f(y) \leq d_f(x \cdot y)$,
(2) if $f(d_f(x)) = d_f(x)$ or $d_f(d_f(x)) = f(x)$, then $d_f(x \cdot d_f(x)) = 0$,
(3) if $f(d_f(x)) = d_f(x)$ or $d_f(d_f(x)) = f(x)$, then $d_f(d_f(x) \cdot x) = 0$,
(4) if $d_f(f(x)) = f(x)$ or $f(f(x)) = d_f(x)$, then $d_f(x \cdot f(x)) = 0$, and
(5) if $d_f(f(x)) = f(x)$ or $f(f(x)) = d_f(x)$, then $d_f(f(x) \cdot x) = 0$.

Proof. (1) For all $x, y \in A$,

$$(f(x) \cdot d_f(y)) \cdot d_f(x \cdot y) = (f(x) \cdot d_f(y)) \cdot ((f(x) \cdot d_f(y)) \wedge (d_f(x) \cdot f(y)))$$
$$= (f(x) \cdot d_f(y)) \cdot (((d_f(x) \cdot f(y)) \cdot (f(x) \cdot d_f(y))) \cdot (f(x) \cdot d_f(y)))$$
(By Proposition 1.1 (5)) $$= 0.$$

Hence, $f(x) \cdot d_f(y) \leq d_f(x \cdot y)$ for all $x, y \in A$.
(2) For all $x \in A$, if $f(d_f(x)) = d_f(x)$, then

$$d_f(x \cdot d_f(x)) = (f(x) \cdot d_f(d_f(x))) \wedge (d_f(x) \cdot f(d_f(x)))$$
$$= (f(x) \cdot d_f(d_f(x))) \wedge (d_f(x) \cdot d_f(x))$$
(By Proposition 1.1 (1)) $$= (f(x) \cdot d_f(d_f(x))) \wedge 0$$
(By Proposition 1.2 (2)) $$= 0.$$

If $d_f(d_f(x)) = f(x)$, then

$$d_f(x \cdot d_f(x)) = (f(x) \cdot d_f(d_f(x))) \wedge (d_f(x) \cdot f(d_f(x)))$$
$$= (f(x) \cdot f(x)) \wedge (d_f(x) \cdot f(d_f(x)))$$
(By Proposition 1.1 (1)) $$= 0 \wedge (d_f(x) \cdot f(d_f(x)))$$
(By Proposition 1.2 (1)) $$= 0.$$

(3) For all $x \in A$, if $f(d_f(x)) = d_f(x)$, then

$$\begin{aligned}
d_f(d_f(x) \cdot x) &= (f(d_f(x)) \cdot d_f(x)) \wedge (d_f(d_f(x)) \cdot f(x)) \\
&= (d_f(x) \cdot d_f(x)) \wedge (d_f(d_f(x)) \cdot f(x)) \\
\text{(By Proposition 1.1 (1))} \quad &= 0 \wedge (d_f(d_f(x)) \cdot f(x)) \\
\text{(By Proposition 1.2 (1))} \quad &= 0.
\end{aligned}$$

If $d_f(d_f(x)) = f(x)$, then

$$\begin{aligned}
d_f(d_f(x) \cdot x) &= (f(d_f(x)) \cdot d_f(x)) \wedge (d_f(d_f(x)) \cdot f(x)) \\
&= (f(d_f(x)) \cdot d_f(x)) \wedge (f(x) \cdot f(x)) \\
\text{(By Proposition 1.1 (1))} \quad &= (f(d_f(x)) \cdot d_f(x)) \wedge 0 \\
\text{(By Proposition 1.2 (2))} \quad &= 0.
\end{aligned}$$

(4) For all $x \in A$, if $d_f(f(x)) = f(x)$, then

$$\begin{aligned}
d_f(x \cdot f(x)) &= (f(x) \cdot d_f(f(x))) \wedge (d_f(x) \cdot f(f(x))) \\
&= (f(x) \cdot f(x)) \wedge (d_f(x) \cdot f(f(x))) \\
\text{(By Proposition 1.1 (1))} \quad &= 0 \wedge (d_f(x) \cdot f(f(x))) \\
\text{(By Proposition 1.2 (1))} \quad &= 0.
\end{aligned}$$

If $f(f(x)) = d_f(x)$, then

$$\begin{aligned}
d_f(x \cdot f(x)) &= (f(x) \cdot d_f(f(x))) \wedge (d_f(x) \cdot f(f(x))) \\
&= (f(x) \cdot d_f(f(x))) \wedge (d_f(x) \cdot d_f(x)) \\
\text{(By Proposition 1.1 (1))} \quad &= (f(x) \cdot d_f(f(x))) \wedge 0 \\
\text{(By Proposition 1.2 (2))} \quad &= 0.
\end{aligned}$$

(5) For all $x \in A$, if $d_f(f(x)) = f(x)$, then

$$\begin{aligned}
d_f(f(x) \cdot x) &= (f(f(x)) \cdot d_f(x)) \wedge (d_f(f(x)) \cdot f(x)) \\
&= (f(f(x)) \cdot d_f(x)) \wedge (f(x) \cdot f(x)) \\
\text{(By Proposition 1.1 (1))} \quad &= (f(f(x)) \cdot d_f(x)) \wedge 0 \\
\text{(By Proposition 1.2 (2))} \quad &= 0.
\end{aligned}$$

If $f(f(x)) = d_f(x)$, then

$$\begin{aligned} d_f(f(x) \cdot x) &= (f(f(x)) \cdot d_f(x)) \wedge (d_f(f(x)) \cdot f(x)) \\ &= (d_f(x) \cdot d_f(x)) \wedge (d_f(f(x)) \cdot f(x)) \\ \text{(By Proposition 1.1 (1))} \quad &= 0 \wedge (d_f(f(x)) \cdot f(x)) \\ \text{(By Proposition 1.2 (1))} \quad &= 0. \end{aligned}$$

□

Definition 2.3. A UP-ideal B of A is called *f-invariant* (with respect to an (l, r)-f-derivation (resp. (r, l)-f-derivation, f-derivation) d_f of A) if $d_f(B) \subseteq B$.

Example 2.6. *Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ is a UP-algebra. We define a self-map $f \colon A \to A$ as follows:

$$f(0) = 0, f(1) = 0, f(2) = 1 \text{ and } f(3) = 3.$$

Then f is a UP-endomorphism. We define a self-map $d_f \colon A \to A$ as follows:

$$d_f(0) = 0, d_f(1) = 0, d_f(2) = 1 \text{ and } d_f(3) = 0.$$

Then d_f is an f-derivation of A. Let $B = \{0, 1, 2\}$ and $C = \{0, 1, 3\}$. Then B and C are UP-ideals of A and it follows that they are f-invariants with respect to an f-derivation d_f of A.

Theorem 2.7. *Every ideal of A with containing the endomorphic image of f is f-invariant with respect to any (l, r)-f-derivation of A.*

Proof. Assume that B is an ideal of A and d_f is an (l, r)-f-derivation of A. Let $y \in d_f(B)$. Then $y = d_f(x)$ for some $x \in B$. By Proposition 2.2 (1), we obtain $f(x) \leq d_f(x)$; that is, $f(x) \cdot d_f(x) = 0$. Thus $f(x) \cdot y = f(x) \cdot d_f(x) = 0 \in B$. Since $f(B) \subseteq B$, we have $f(x) \in B$. It follows from Theorem 1.3 (1) that $y \in B$. Hence, $d_f(B) \subseteq B$, which implies that B is f-invariant. □

Corollary 2.8. *Every ideal of A with containing the endomorphic image of f is f-invariant with respect to any f-derivation of A.*

Definition 2.4. Let d_f be an (l,r)-f-derivation (resp. (r,l)-f-derivation, f-derivation) of A. We define a subset $\mathrm{Ker}_{d_f}(A)$ of A by

$$\mathrm{Ker}_{d_f}(A) = \{x \in A \mid d_f(x) = 0\}.$$

Theorem 2.9. *In a UP-algebra A, the following statements hold:*

(1) if d_f is an (l,r)-f-derivation of A, then $y \wedge x \in \mathrm{Ker}_{d_f}(A)$ for all $y \in \mathrm{Ker}_{d_f}(A)$ and $x \in A$, and

(2) if d_f is an (r,l)-f-derivation of A, then $y \wedge x \in \mathrm{Ker}_{d_f}(A)$ for all $y \in \mathrm{Ker}_{d_f}(A)$ and $x \in A$.

Proof. (1) Assume that d_f is an (l,r)-f-derivation of A. Let $y \in \mathrm{Ker}_{d_f}(A)$ and $x \in A$. Then $d_f(y) = 0$. Thus

$$\begin{aligned}
d_f(y \wedge x) &= d_f((x \cdot y) \cdot y) \\
&= (d_f(x \cdot y) \cdot f(y)) \wedge (f(x \cdot y) \cdot d_f(y)) \\
&= (d_f(x \cdot y) \cdot f(y)) \wedge (f(x \cdot y) \cdot 0) \\
\text{(By UP-3)} \quad &= (d_f(x \cdot y) \cdot f(y)) \wedge 0 \\
\text{(By Proposition 1.2 (2))} \quad &= 0.
\end{aligned}$$

Hence, $y \wedge x \in \mathrm{Ker}_{d_f}(A)$.

(2) Assume that d_f is an (r,l)-f-derivation of A. Let $y \in \mathrm{Ker}_{d_f}(A)$ and $x \in A$. Then $d_f(y) = 0$. Thus

$$\begin{aligned}
d_f(y \wedge x) &= d_f((x \cdot y) \cdot y) \\
&= (f(x \cdot y) \cdot d_f(y)) \wedge (d_f(x \cdot y) \cdot f(y)) \\
&= (f(x \cdot y) \cdot 0) \wedge (d_f(x \cdot y) \cdot f(y)) \\
\text{(By UP-3)} \quad &= 0 \wedge (d_f(x \cdot y) \cdot f(y)) \\
\text{(By Proposition 1.2 (1))} \quad &= 0.
\end{aligned}$$

Hence, $y \wedge x \in \mathrm{Ker}_{d_f}(A)$. □

Corollary 2.10. *If d_f is an f-derivation of A, then $y \wedge x \in \mathrm{Ker}_{d_f}(A)$ for all $y \in \mathrm{Ker}_{d_f}(A)$ and $x \in A$.*

Give an example of conflict that $x \wedge y \notin \mathrm{Ker}_{d_f}(A)$ for all $y \in \mathrm{Ker}_{d_f}(A)$ and $x \in A$ in general.

Example 2.11. *Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	0	3
2	0	0	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ is a UP-algebra. Let 1_A be an identity map on A. Then 1_A is a UP-endomorphism. We define a self-map $d_{1_A} : A \to A$ as follows:

$$d_{1_A}(0) = 0, d_{1_A}(1) = 0, d_{1_A}(2) = 2 \text{ and } d_{1_A}(3) = 3.$$

Then d_{1_A} is an f-derivation of A and so $\mathrm{Ker}_{d_{1_A}}(A) = \{0, 1\}$. Thus $2 \wedge 1 = 2 \notin \mathrm{Ker}_{d_{1_A}}(A)$ when $1 \in \mathrm{Ker}_{d_{1_A}}(A)$ and $2 \in A$.

Theorem 2.12. *In a meet-commutative UP-algebra A, the following statements hold:*

(1) if d_f is an (l, r)-f-derivation of A and for any $x, y \in A$ is such that $y \leq x$ and $y \in \mathrm{Ker}_{d_f}(A)$, then $x \in \mathrm{Ker}_{d_f}(A)$, and
(2) if d_f is an (r, l)-f-derivation of A and for any $x, y \in A$ is such that $y \leq x$ and $y \in \mathrm{Ker}_{d_f}(A)$, then $x \in \mathrm{Ker}_{d_f}(A)$.

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. Let $x, y \in A$ be such that $y \leq x$ and $y \in \mathrm{Ker}_{d_f}(A)$. Then $y \cdot x = 0$ and $d_f(y) = 0$. Thus

$$\begin{aligned}
\text{(By UP-2)} \quad d_f(x) &= d_f(0 \cdot x) \\
&= d_f((y \cdot x) \cdot x) \\
&= d_f((x \cdot y) \cdot y) \\
&= (d_f(x \cdot y) \cdot f(y)) \wedge (f(x \cdot y) \cdot d_f(y)) \\
&= (d_f(x \cdot y) \cdot f(y)) \wedge (f(x \cdot y) \cdot 0) \\
\text{(By UP-3)} \quad &= (d_f(x \cdot y) \cdot f(y)) \wedge 0 \\
\text{(By Proposition 1.2 (2))} \quad &= 0.
\end{aligned}$$

Hence, $x \in \mathrm{Ker}_{d_f}(A)$.

(2) Assume that d_f is an (r,l)-f-derivation of A. Let $x, y \in A$ be such that $y \leq x$ and $y \in \mathrm{Ker}_{d_f}(A)$. Then $y \cdot x = 0$ and $d_f(y) = 0$. Thus

$$
\begin{aligned}
&\text{(By UP-2)} & d_f(x) &= d_f(0 \cdot x) \\
&& &= d_f((y \cdot x) \cdot x) \\
&& &= d_f((x \cdot y) \cdot y) \\
&& &= (f(x \cdot y) \cdot d_f(y)) \wedge (d_f(x \cdot y) \cdot f(y)) \\
&& &= (f(x \cdot y) \cdot 0) \wedge (d_f(x \cdot y) \cdot f(y)) \\
&\text{(By UP-3)} & &= 0 \wedge (d_f(x \cdot y) \cdot f(y)) \\
&\text{(By Proposition 1.2 (1))} & &= 0.
\end{aligned}
$$

Hence, $x \in \mathrm{Ker}_{d_f}(A)$. □

Corollary 2.13. *If d_f is an f-derivation of a meet-commutative UP-algebra A and for any $x, y \in A$ is such that $y \leq x$ and $y \in \mathrm{Ker}_{d_f}(A)$, then $x \in \mathrm{Ker}_{d_f}(A)$.*

Theorem 2.14. *In a UP-algebra A, the following statements hold:*

(1) if d_f is an (l,r)-f-derivation of A, then $y \cdot x \in \mathrm{Ker}_{d_f}(A)$ for all $x \in \mathrm{Ker}_{d_f}(A)$ and $y \in A$, and

(2) if d_f is an (r,l)-f-derivation of A, then $y \cdot x \in \mathrm{Ker}_{d_f}(A)$ for all $x \in \mathrm{Ker}_{d_f}(A)$ and $y \in A$.

Proof. (1) Assume that d_f is an (l,r)-f-derivation of A. Let $x \in \mathrm{Ker}_{d_f}(A)$ and $y \in A$. Then $d_f(x) = 0$. Thus

$$
\begin{aligned}
&& d_f(y \cdot x) &= (d_f(y) \cdot f(x)) \wedge (f(y) \cdot d_f(x)) \\
&& &= (d_f(y) \cdot f(x)) \wedge (f(y) \cdot 0) \\
&\text{(By UP-3)} & &= (d_f(y) \cdot f(x)) \wedge 0 \\
&\text{(By Proposition 1.2 (2))} & &= 0.
\end{aligned}
$$

Hence, $y \cdot x \in \mathrm{Ker}_{d_f}(A)$.

(2) Assume that d_f is an (r,l)-f-derivation of A. Let $x \in \mathrm{Ker}_{d_f}(A)$ and $y \in A$. Then $d_f(x) = 0$. Thus

$$\begin{aligned} d_f(y \cdot x) &= (f(y) \cdot d_f(x)) \wedge (d_f(y) \cdot f(x)) \\ &= (f(y) \cdot 0) \wedge (d_f(y) \cdot f(x)) \\ \text{(By UP-3)} \quad &= 0 \wedge (d_f(y) \cdot f(x)) \\ \text{(By Proposition 1.2 (1))} \quad &= 0. \end{aligned}$$

Hence, $y \cdot x \in \mathrm{Ker}_{d_f}(A)$. □

Corollary 2.15. *If d_f is an f-derivation of A, then $y \cdot x \in \mathrm{Ker}_{d_f}(A)$ for all $x \in \mathrm{Ker}_{d_f}(A)$ and $y \in A$.*

Example 2.16. *From Example 2.1, we have* $\mathrm{Ker}_{d_f}(A) = \{0, 1, 3\}$*. Then* $3 \cdot 2 = 2 \notin \mathrm{Ker}_{d_f}(A)$ *when* $3 \in \mathrm{Ker}_{d_f}(A)$ *and* $2 \in A$*.*

Theorem 2.17. *In a UP-algebra A, the following statements hold:*

(1) if d_f is an (l,r)-f-derivation of A, then $\mathrm{Ker}_{d_f}(A)$ *is a UP-subalgebra of A, and*

(2) if d_f is an (r,l)-f-derivation of A, then $\mathrm{Ker}_{d_f}(A)$ *is a UP-subalgebra of A.*

Proof. (1) Assume that d_f is an (l,r)-f-derivation of A. By Theorem 2.2 (1), we have $d_f(0) = 0$ and so $0 \in \mathrm{Ker}_{d_f}(A) \neq \emptyset$. Let $x, y \in \mathrm{Ker}_{d_f}(A)$. Then $d_f(x) = 0$ and $d_f(y) = 0$. Thus

$$\begin{aligned} d_f(x \cdot y) &= (d_f(x) \cdot f(y)) \wedge (f(x) \cdot d_f(y)) \\ &= (0 \cdot f(y)) \wedge (f(x) \cdot 0) \\ \text{(By UP-2 and UP-3)} \quad &= f(y) \wedge 0 \\ \text{(By Proposition 1.2 (2))} \quad &= 0. \end{aligned}$$

Hence, $x \cdot y \in \mathrm{Ker}_{d_f}(A)$, so $\mathrm{Ker}_{d_f}(A)$ is a UP-subalgebra of A.

(2) Assume that d_f is an (r,l)-f-derivation of A. By Theorem 2.2 (2), we have $d_f(0) = 0$ and so $0 \in \mathrm{Ker}_{d_f}(A) \neq \emptyset$. Let $x, y \in \mathrm{Ker}_{d_f}(A)$. Then $d_f(x) = 0$ and $d_f(y) = 0$. Thus

$$\begin{aligned} d_f(x \cdot y) &= (f(x) \cdot d_f(y)) \wedge (d_f(x) \cdot f(y)) \\ &= (f(x) \cdot 0) \wedge (0 \cdot f(y)) \\ \text{(By UP-2 and UP-3)} \quad &= 0 \wedge f(y) \\ \text{(By Proposition 1.2 (1))} \quad &= 0. \end{aligned}$$

Hence, $x \cdot y \in \mathrm{Ker}_{d_f}(A)$, so $\mathrm{Ker}_{d_f}(A)$ is a UP-subalgebra of A. □

Corollary 2.18. *If d_f is an f-derivation of A, then $\mathrm{Ker}_{d_f}(A)$ is a UP-subalgebra of A.*

Give an example of conflict that $\mathrm{Ker}_{d_f}(A)$ is not a UP-ideal of A in general.

Example 2.19. *Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	0	0
2	0	1	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ is a UP-algebra. Let 1_A be an identity map on A. Then 1_A is a UP-endomorphism. We define a self-map $d_{1_A} : A \to A$ as follows:

$$d_{1_A}(0) = 0, d_{1_A}(1) = 0, d_{1_A}(2) = 2 \text{ and } d_{1_A}(3) = 0.$$

Then d_{1_A} is an (l, r)-1_A-derivation of A and $\mathrm{Ker}_{d_{1_A}}(A) = \{0, 1, 3\}$. Since $0 \cdot (1 \cdot 2) = 0 \in \mathrm{Ker}_{d_{1_A}}(A), 1 \in \mathrm{Ker}_{d_{1_A}}(A)$ but $0 \cdot 2 = 2 \notin \mathrm{Ker}_{d_{1_A}}(A)$, we conclude that $\mathrm{Ker}_{d_{1_A}}(A)$ is not a UP-ideal of A.

Definition 2.5. Let d_f be an (l, r)-f-derivation (resp. (r, l)-f-derivation, f-derivation) of A. We define a subset $\mathrm{Fix}_{d_f}(A)$ of A by

$$\mathrm{Fix}_{d_f}(A) = \{x \in A \mid d_f(x) = f(x)\}.$$

Theorem 2.20. *In a UP-algebra A, the following statements hold:*

(1) if d_f is an (l, r)-f-derivation of A, then $\mathrm{Fix}_{d_f}(A)$ is a UP-subalgebra of A, and
(2) if d_f is an (r, l)-f-derivation of A, then $\mathrm{Fix}_{d_f}(A)$ is a UP-subalgebra of A.

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. By Theorem 2.2 (1) and 1.4 (1), we have $d_f(0) = 0 = f(0)$ and so $0 \in \mathrm{Fix}_{d_f}(A) \neq \emptyset$. Let $x, y \in \mathrm{Fix}_{d_f}(A)$. Then $d_f(x) = f(x)$ and $d_f(y) = f(y)$. Thus

$$\begin{aligned} d_f(x \cdot y) &= (d_f(x) \cdot f(y)) \wedge (f(x) \cdot d_f(y)) \\ &= (f(x) \cdot f(y)) \wedge (f(x) \cdot f(y)) \\ &= f(x \cdot y) \wedge f(x \cdot y) \\ \text{(By Proposition 1.2 (3))} \quad &= f(x \cdot y). \end{aligned}$$

Hence, $x \cdot y \in \mathrm{Fix}_d(A)$, so $\mathrm{Fix}_d(A)$ is a UP-subalgebra of A.

(2) Assume that d_f is an (r,l)-f-derivation of A. By Theorem 2.2 (2) and 1.4 (1), we have $d_f(0) = 0 = f(0)$ and so $0 \in \mathrm{Fix}_{d_f}(A) \neq \emptyset$. Let $x, y \in \mathrm{Fix}_{d_f}(A)$. Then $d_f(x) = f(x)$ and $d_f(y) = f(y)$. Thus

$$\begin{aligned} d_f(x \cdot y) &= (f(x) \cdot d_f(y)) \wedge (d_f(x) \cdot f(y)) \\ &= (f(x) \cdot f(y)) \wedge (f(x) \cdot f(y)) \\ &= f(x \cdot y) \wedge f(x \cdot y) \\ \text{(By Proposition 1.2 (3))} \qquad &= f(x \cdot y). \end{aligned}$$

Hence, $x \cdot y \in \mathrm{Fix}_{d_f}(A)$, so $\mathrm{Fix}_d(A)$ is a UP-subalgebra of A. □

Corollary 2.21. *If d_f is an f-derivation of A, then $\mathrm{Fix}_{d_f}(A)$ is a UP-subalgebra of A.*

Give an example of conflict that $\mathrm{Fix}_{d_f}(A)$ is not a UP-ideal of A in general.

Example 2.22. *Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	0	0
2	0	1	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ is a UP-algebra. Let 1_A be an identity map on A. Then 1_A is a UP-endomorphism. We define a self-map $d_{1_A} \colon A \to A$ as follows:

$$d_{1_A}(0) = 0, d_{1_A}(1) = 1, d_{1_A}(2) = 2 \text{ and } d_{1_A}(3) = 0.$$

Then d_{1_A} is an (l,r)-1_A-derivation of A and $\mathrm{Fix}_{d_{1_A}}(A) = \{0, 1, 2\}$. Since $2 \cdot (1 \cdot 3) = 0 \in \mathrm{Fix}_{d_{1_A}}(A), 1 \in \mathrm{Fix}_{d_{1_A}}(A)$ but $2 \cdot 3 = 3 \notin \mathrm{Fix}_{d_{1_A}}(A)$, we conclude that $\mathrm{Fix}_{d_{1_A}}(A)$ is not a UP-ideal of A.

Theorem 2.23. *In a UP-algebra A, the following statements hold:*

(1) if d_f is an (l,r)-f-derivation of A, then $x \wedge y \in \mathrm{Fix}_{d_f}(A)$ for all $x, y \in \mathrm{Fix}_{d_f}(A)$, and
(2) if d_f is an (r,l)-f-derivation of A, then $x \wedge y \in \mathrm{Fix}_{d_f}(A)$ for all $x, y \in \mathrm{Fix}_{d_f}(A)$.

Proof. (1) Assume that d_f is an (l,r)-f-derivation of A. Let $x, y \in \mathrm{Fix}_{d_f}(A)$. Then $d_f(x) = f(x)$ and $d_f(y) = f(y)$. By Theorem 2.20 (1), we get $d_f(y \cdot x) = f(y \cdot x)$. Thus

$$\begin{aligned} d_f(x \wedge y) &= d_f((y \cdot x) \cdot x) \\ &= (d_f(y \cdot x) \cdot f(x)) \wedge (f(y \cdot x) \cdot d_f(x)) \\ &= (f(y \cdot x) \cdot f(x)) \wedge (f(y \cdot x) \cdot f(x)) \\ \text{(By Proposition 1.2 (3))} \quad &= f(y \cdot x) \cdot f(x) \\ &= f((y \cdot x) \cdot x) \\ &= f(x \wedge y). \end{aligned}$$

Hence, $x \wedge y \in \mathrm{Fix}_{d_f}(A)$.

(2) Assume that d_f is an (r,l)-f-derivation of A. Let $x, y \in \mathrm{Fix}_{d_f}(A)$. Then $d_f(x) = f(x)$ and $d_f(y) = f(y)$. By Theorem 2.20 (2), we get $d_f(y \cdot x) = f(y \cdot x)$. Thus

$$\begin{aligned} d_f(x \wedge y) &= d_f((y \cdot x) \cdot x) \\ &= (f(y \cdot x) \cdot d_f(x)) \wedge (d_f(y \cdot x) \cdot f(x)) \\ &= (f(y \cdot x) \cdot f(x)) \wedge (f(y \cdot x) \cdot f(x)) \\ \text{(By Proposition 1.2 (3))} \quad &= f(y \cdot x) \cdot f(x) \\ &= f((y \cdot x) \cdot x) \\ &= f(x \wedge y). \end{aligned}$$

Hence, $x \wedge y \in \mathrm{Fix}_{d_f}(A)$. □

Corollary 2.24. *If d_f is an f-derivation of A, then $x \wedge y \in \mathrm{Fix}_{d_f}(A)$ for all $x, y \in \mathrm{Fix}_{d_f}(A)$.*

Competing Interests

The author declares that no competing interests exist.

References

[1] H. A. S. Abujabal, N. O. Al-shehri, *Some results on derivations of BCI-algebras*, J. Nat. Sci. Math. **46** (no. 1&2) (2006), 13–19.

[2] H. A. S. Abujabal, N. O. Al-shehri, *On left derivations of BCI-algebras*, Soochow J. Math. **33** (no. 3) (2007), 435–444.

[3] A. M. Al-roqi, *On generalized (α, β)-derivations in BCI-algebras*, J. Appl. Math. Inform. **32** (no. 1–2) (2014), 27–38.

[4] N. O. Al-shehri, S. M. Bawazeer, *On derivations of BCC-algebras*, Int. J. Algebra **6** (no. 32) (2012), 1491–1498.

[5] L. K. Ardekani, B. Davvaz, *On generalized derivations of BCI-algebras and their properties*, J. Math. **2014** (2014), Article ID 207161, 10 pages.

[6] S. M. Bawazeer, N. O. Alshehri, R. S. Babusail, *Generalized derivations of BCC-algebras*, Int. J. Math. Math. Sci. **2013** (2013), Article ID 451212, 4 pages.

[7] Q. P. Hu, X. Li, *On BCH-algebras*, Math. Semin. Notes, Kobe Univ. **11** (1983), 313–320.

[8] A. Iampan, *A new branch of the logical algebra: UP-algebras*, Manuscript submitted for publication, April 2016.

[9] Y. Imai, K. Iséki, *On axiom system of propositional calculi, XIV*, Proc. Japan Acad. **42** (no. 1) (1966), 19–22.

[10] K. Iséki, *An algebra related with a propositional calculus*, Proc. Japan Acad. **42** (no. 1) (1966), 26–29.

[11] M. A. Javed, M. Aslam, *A note on f-derivations of BCI-algebras*, Commun. Korean Math. Soc. **24** (no. 3) (2009), 321–331.

[12] Y. B. Jun, X. L. Xin, *On derivations of BCI-algebras*, Inform. Sci. **159** (2004), 167–176.

[13] S. Keawrahun, U. Leerawat, *On isomorphisms of SU-algebras*, Sci. Magna **7** (no. 2) (2011), 39–44.

[14] K. J. Lee, *A new kind of derivation in BCI-algebras*, Appl. Math. Sci. **7** (no. 84) (2013), 4185–4194.

[15] P. H. Lee, T. K. Lee, *On derivations of prime rings*, Chinese J. Math. **9** (1981), 107–110.

[16] S. M. Lee, K. H. Kim, *A note on f-derivations of BCC-algebras*, Pure Math. Sci. **1** (no. 2) (2012), 87–93.

[17] G. Muhiuddin, A. M. Al-roqi, *On (α, β)-derivations in BCI-algebras*, Discrete Dyn. Nat. Soc. **2012** (2012), Article ID 403209, 11 pages.

[18] G. Muhiuddin, A. M. Al-roqi, *On t-derivations of BCI-algebras*, Abstr. Appl. Anal. **2012** (2012), Article ID 872784, 12 pages.

[19] G. Muhiuddin, A. M. Al-roqi, *On generalized left derivations in BCI-algebras*, Appl. Math. Inf. Sci. **8** (no. 3) (2014), 1153–1158.

[20] G. Muhiuddin, A. M. Al-roqi, Y. B. Jun, Y. Ceven, *On symmetric left bi-derivations in BCI-algebras*, Int. J. Math. Math. Sci. **2013** (2013), Article ID 238490, 6 pages.

[21] F. Nisar, *Characterization of f-derivations of a BCI-algebra*, East Asian Math. J. **25** (no. 1) (2009), 69–87.

[22] F. Nisar, *On F-derivations of BCI-algebras*, J. Prime Res. Math. **5** (2009), 176–191.

[23] E. Posner, *Derivations in prime rings*, Proc. Amer. Math. Soc. **8** (1957), 1093–1100.

[24] C. Prabpayak, U. Leerawat, *On derivation of BCC-algebras*, Kasetsart J. (Nat. Sci.) **43** (2009), 398–401.

[25] C. Prabpayak, U. Leerawat, *On ideals and congruences in KU-algebras*, Sci. Magna **5** (no. 1) (2009), 54–57.

[26] K. Sawika, R. Intasan, A. Kaewwasri, A. Iampan, *Derivations of UP-algebras*, Korean J. Math. **24** (no. 3) (2016), 345–367.

[27] J. Zhan, Y. L. Liu, *On f-derivations of BCI-algebras*, Int. J. Math. Math. Sci. **2005** (2005), 1675–1684.

Aiyared Iampan

Department of Mathematics, School of Science

University of Phayao, Phayao 56000, Thailand
aiyared.ia@up.ac.th