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DERIVATIONS OF UP-ALGEBRAS BY MEANS OF UP-ENDOMORPHISMS

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ABSTRACT. The notion of f-derivations of UP-algebras is introduced, some useful examples are discussed, and related properties are investigated. Moreover, we show that the fixed set and the kernel of f-derivations are UP-subalgebras of UP-algebras, and also give examples to show that the two sets are not UP-ideals of UP-algebras in general.

1. INTRODUCTION AND PRELIMINARIES

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [9], BCI-algebras [10], BCH-algebras [7], KU-algebras [25], SU-algebras [13] and others. They are strongly connected with logic. For example, BCI-algebras introduced by Iséki [10] in 1966 have connections with BCI-logic being the BCI-system in combinatorial logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki

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[9, 10] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

In the theory of rings and near rings, the properties of derivations is an important topic to study [23, 15]. In 2004, Jun and Xin [12] applied the notions of rings and near rings theory to BCI-algebras and obtained some properties. Several researches were conducted on the generalizations of the notion of derivations and application to many logical algebras such as: In 2005, Zhan and Liu [27] introduced the notion of left-right (right-left) f-derivations of BCI-algebras. In 2006, Abujabal and Al-shehri [1] investigated some fundamental properties and proved some results on derivations of BCI-algebras. In 2007, Abujabal and Al-shehri [2] introduced the notion of left derivations of BCI-algebras. In 2009, Javed and Aslam [11] investigated some fundamental properties and established some results of f-derivations of BCI-algebras. Nisar [22] introduced the notions of right F-derivations and left F-derivations of BCI-algebras. Nisar [21] characterized f-derivations of BCI-algebras. Prabpayak and Leerawat [24] studied the notions of left-right (right-left) derivations of BCC-algebras. In 2012, Al-shehri and Bawazeer [4] introduced the notion of left-right (right-left) t-derivations of BCC-algebras. Lee and Kim [16] considered the properties of f-derivations of BCC-algebras. Muhiuddin and Al-roqi [18] introduced the notion of t-derivations of BCI-algebras. Muhiuddin and Al-roqi [17] introduced the notion of (regular) (α, β) -derivations of BCI-algebras. In 2013, Bawazeer, Al-shehri and Babusal [6] introduced the notion of generalized derivations of BCC-algebras. Lee [14] introduced a new kind of derivations of BCI-algebras. Muhiuddin, Al-roqi, Jun and Ceven [20] introduced the notion of symmetric left bi-derivations of BCI-algebras. In 2014, Al-roqi [3] introduced the notion of generalized (regular) (α, β)-derivations of BCI-algebras. Muhiuddin and Al-roqi [19] introduced the notion of generalized left derivations of BCI-algebras. Ardekani and Davvaz [5] introduced the notion of (f, g)-derivations of BCI-algebras. In 2016, Sawika, Intasan, Kaewwasri and Iampan [26] introduced the notions of (l, r)-derivations, (r, l)-derivations and derivations of UP-algebras and investigated some related properties.

The notion of derivations play an important role in studying the many logical algebras. In this paper, we introduce the notion of f-derivations of UP-algebras which is the generalization of the notion of derivations [26], some useful examples are discussed, and related properties are investigated.

Before we begin our study, we will introduce to the definition of a UP-algebra.

Definition 1.1. [8] An algebra $A = (A; \cdot, 0)$ of type (2, 0) is called a *UP-algebra* if it satisfies the following axioms: for any $x, y, z \in A$,

(UP-1): $(y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0$, (UP-2): $0 \cdot x = x$, (UP-3): $x \cdot 0 = 0$, and (UP-4): $x \cdot y = y \cdot x = 0$ implies x = y.

Example 1.1. [8] Let X be a universal set. Define a binary operation \cdot on the power set of X by putting $A \cdot B = B \cap A' = A' \cap B = B - A$ for all $A, B \in \mathcal{P}(X)$. Then $(\mathcal{P}(X); \cdot, \emptyset)$ is a UP-algebra and we shall call it the power UP-algebra of type 1.

Example 1.2. [8] Let X be a universal set. Define a binary operation * on the power set of X by putting $A * B = B \cup A' = A' \cup B$ for all $A, B \in \mathcal{P}(X)$. Then $(\mathcal{P}(X); *, X)$ is a UP-algebra and we shall call it the power UP-algebra of type 2.

In what follows, let A denotes a UP-algebra unless otherwise specified. The following proposition is very important for the study of UP-algebras.

Proposition 1.1. [8] In a UP-algebra A, the following properties hold: for any $x, y, z \in A$,

(1) x ⋅ x = 0,
 (2) x ⋅ y = 0 and y ⋅ z = 0 imply x ⋅ z = 0,
 (3) x ⋅ y = 0 implies (z ⋅ x) ⋅ (z ⋅ y) = 0,
 (4) x ⋅ y = 0 implies (y ⋅ z) ⋅ (x ⋅ z) = 0,
 (5) x ⋅ (y ⋅ x) = 0,
 (6) (y ⋅ x) ⋅ x = 0 if and only if x = y ⋅ x, and
 (7) x ⋅ (y ⋅ y) = 0.

On a UP-algebra $A = (A; \cdot, 0)$, we define a binary relation \leq on A [8] as follows: for all $x, y \in A$,

 $x \leq y$ if and only if $x \cdot y = 0$.

Definition 1.2. [8] A nonempty subset B of A is called a *UP-ideal* of A if it satisfies the following properties:

- (1) the constant 0 of A is in B, and
- (2) for any $x, y, z \in A, x \cdot (y \cdot z) \in B$ and $y \in B$ imply $x \cdot z \in B$.

Clearly, A and $\{0\}$ are UP-ideals of A.

Theorem 1.3. [8] Let A be a UP-algebra and B a UP-ideal of A. Then the following statements hold: for any $x, a, b \in A$,

- (1) if $b \cdot x \in B$ and $b \in B$, then $x \in B$. Moreover, if $b \cdot X \subseteq B$ and $b \in B$, then $X \subseteq B$,
- (2) if $b \in B$, then $x \cdot b \in B$. Moreover, if $b \in B$, then $X \cdot b \subseteq B$, and
- (3) if $a, b \in B$, then $(b \cdot (a \cdot x)) \cdot x \in B$.

Definition 1.3. [8] Let $(A; \cdot, 0)$ and $(A'; \cdot', 0')$ be UP-algebras. A mapping f from A to A' is called a *UP-homomorphism* if

$$f(x \cdot y) = f(x) \cdot f(y)$$
 for all $x, y \in A$.

A UP-homomorphism $f: A \to A'$ is called a UP-endomorphism of A if A' = A.

Theorem 1.4. [8] Let $(A; \cdot, 0_A)$ and $(B; *, 0_B)$ be UP-algebras and let $f: A \to B$ be a UP-homomorphism. Then the following statements hold:

- (1) $f(0_A) = 0_B$,
- (2) for any $x, y \in A$, if $x \leq y$, then $f(x) \leq f(y)$,
- (3) if C is a UP-subalgebra of A, then the image f(C) is a UP-subalgebra of B. In particular, Im(f) is a UP-subalgebra of B,
- (4) if D is a UP-subalgebra of B, then the inverse image $f^{-1}(D)$ is a UP-subalgebra of A. In particular, Ker(f) is a UP-subalgebra of A,
- (5) if C is a UP-ideal of A, then the image f(C) is a UP-ideal of f(A),
- (6) if D is a UP-ideal of B, then the inverse image $f^{-1}(D)$ is a UP-ideal of A. In particular, Ker(f) is a UP-ideal of A, and
- (7) $\operatorname{Ker}(f) = \{0_A\}$ if and only if f is injective.

Definition 1.4. [26] For any $x, y \in A$, we define a binary operation \wedge on A by $x \wedge y = (y \cdot x) \cdot x$.

Definition 1.5. [26] A UP-algebra A is called *meet-commutative* if $x \wedge y = y \wedge x$ for all $x, y \in A$, that is, $(y \cdot x) \cdot x = (x \cdot y) \cdot y$ for all $x, y \in A$.

Proposition 1.2. [26] In a UP-algebra A, the following properties hold: for any $x \in A$,

- (1) $0 \wedge x = 0$, (2) $x \wedge 0 = 0$, and
- (3) $x \wedge x = x$.

2. Main Results

In this section, we introduce the notions of (l, r)-f-derivations, (r, l)-f-derivations, and f-derivations of UP-algebras, and study the fixed set and the kernel of (l, r)-f-derivations, (r, l)-f-derivations, and f-derivations.

Definition 2.1. Let f be a UP-endomorphism of A. A self-map $d_f \colon A \to A$ is called an (l,r)-f-derivation of A if it satisfies the identity $d_f(x \cdot y) = (d_f(x) \cdot f(y)) \land (f(x) \cdot d_f(y))$ for all $x, y \in A$. Similarly, a self-map $d_f \colon A \to A$ is called an (r, l)-f-derivation of A if it satisfies the identity $d_f(x \cdot y) = (f(x) \cdot d_f(y)) \land (d_f(x) \cdot f(y))$ for all $x, y \in A$. Moreover, if d_f is both an (l, r)-f-derivation and an (r, l)-f-derivation of A, it is called an f-derivation of A.

By using Microsoft Excel, we have all examples.

Example 2.1. Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

•	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ is a UP-algebra. We define a self-map $f: A \to A$ as follows:

$$f(0) = 0, f(1) = 0, f(2) = 1$$
 and $f(3) = 3$.

Then f is a UP-endomorphism. We define a self-map $d_f \colon A \to A$ as follows:

$$d_f(0) = 0, d_f(1) = 0, d_f(2) = 1 \text{ and } d_f(3) = 0.$$

Then d_f is an f-derivation of A.

Proposition 2.1. Each UP-endomorphism f of A is its f-derivation.

Proof. It follows from Proposition 1.2 (3). \Box

Definition 2.2. An (l, r)-f-derivation (resp. (r, l)-f-derivation, f-derivation) d_f of A is called regular if $d_f(0) = 0$.

Theorem 2.2. In a UP-algebra A, the following statements hold:

- (1) every (l,r)-f-derivation of A is regular, and
- (2) every (r, l)-f-derivation of A is regular.

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. Then

(By UP-3)
$$d_f(0) = d_f(0 \cdot 0)$$
 $= (d_f(0) \cdot f(0)) \wedge (f(0) \cdot d_f(0))$ (By Theorem 1.4 (1)) $= (d_f(0) \cdot 0) \wedge (0 \cdot d_f(0))$ (By UP-2 and UP-3) $= 0 \wedge d_f(0)$ (By Proposition 1.2 (1))

Hence, d_f is regular.

(2) Assume that d_f is an (r, l)-f-derivation of A. Then

(By UP-3) $d_f(0) = d_f(0 \cdot 0)$ $= (f(0) \cdot d_f(0)) \wedge (d_f(0) \cdot f(0))$ (By Theorem 1.4 (1)) $= (0 \cdot d_f(0)) \wedge (d_f(0) \cdot 0)$ (By UP-2 and UP-3) $= d_f(0) \wedge 0$ (By Proposition 1.2 (2))= 0.

Hence, d_f is regular. \Box

Corollary 2.3. Every *f*-derivation of *A* is regular.

Theorem 2.4. In a UP-algebra A, the following statements hold:

(1) if d_f is an (l,r)-f-derivation of A, then $d_f(x) = f(x) \wedge d_f(x)$ for all $x \in A$, and (2) if d_f is an (r,l)-f-derivation of A, then $d_f(x) = d_f(x) \wedge f(x)$ for all $x \in A$.

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. Then, for all $x \in A$,

(By UP-2)

$$d_f(x) = d_f(0 \cdot x)$$

 $= (d_f(0) \cdot f(x)) \wedge (f(0) \cdot d_f(x))$
(By Theorem 1.4 (1) and 2.2 (1))
 $= (0 \cdot f(x)) \wedge (0 \cdot d_f(x))$
 $= f(x) \wedge d_f(x).$

(2) Assume that d_f is an (r, l)-f-derivation of A. Then, for all $x \in A$,

(By UP-2)
$$d_f(x) = d_f(0 \cdot x)$$
 $= (f(0) \cdot d_f(x)) \wedge (d_f(0) \cdot f(x))$ (By Theorem 1.4 (1) and 2.2 (2)) $= (0 \cdot d_f(x)) \wedge (0 \cdot f(x))$ (By UP-2) $= d_f(x) \wedge f(x).$

Corollary 2.5. If d_f is an f-derivation of A, then $d_f(x) = d_f(x) \wedge f(x) = f(x) \wedge d_f(x)$ for all $x \in A$.

Proposition 2.2. Let d_f be an (l,r)-f-derivation of A. Then the following properties hold: for any $x, y \in A$,

(1)
$$f(x) \le d_f(x)$$
,
(2) $d_f(x) \cdot f(y) \le d_f(x \cdot y)$,
(3) if $f(d_f(x)) = d_f(x)$ or $d_f(d_f(x)) = f(x)$, then $d_f(x \cdot d_f(x)) = 0$,
(4) if $f(d_f(x)) = d_f(x)$ or $d_f(d_f(x)) = f(x)$, then $d_f(d_f(x) \cdot x) = 0$,
(5) if $d_f(f(x)) = f(x)$ or $f(f(x)) = d_f(x)$, then $d_f(x \cdot f(x)) = 0$, and
(6) if $d_f(f(x)) = f(x)$ or $f(f(x)) = d_f(x)$, then $d_f(f(x) \cdot x) = 0$.

Proof. (1) For all $x \in A$,

(By Theorem 2.4 (1))

$$f(x) \cdot d_f(x) = f(x) \cdot (f(x) \wedge d_f(x))$$

$$= f(x) \cdot ((d_f(x) \cdot f(x)) \cdot f(x))$$
(By Proposition 1.1 (5))

$$= 0.$$

Hence, $f(x) \leq d_f(x)$ for all $x \in A$. (2) For all $x, y \in A$,

$$(d_f(x) \cdot f(y)) \cdot d_f(x \cdot y) = (d_f(x) \cdot f(y)) \cdot ((d_f(x) \cdot f(y)) \wedge (f(x) \cdot d_f(y)))$$
$$= (d_f(x) \cdot f(y)) \cdot (((f(x) \cdot d_f(y)) \cdot (d_f(x) \cdot f(y))) \cdot (d_f(x) \cdot f(y)))$$

(By Proposition 1.1 (5)) = 0.

Hence, $d_f(x) \cdot f(y) \leq d_f(x \cdot y)$ for all $x, y \in A$. (3) For all $x \in A$, if $f(d_f(x)) = d_f(x)$, then

$$d_f(x \cdot d_f(x)) = (d_f(x) \cdot f(d_f(x))) \wedge (f(x) \cdot d_f(d_f(x)))$$
$$= (d_f(x) \cdot d_f(x)) \wedge (f(x) \cdot d_f(d_f(x)))$$
(By Proposition 1.1 (1))
$$= 0 \wedge (f(x) \cdot d_f(d_f(x)))$$
(By Proposition 1.2 (1))
$$= 0.$$

If $d_f(d_f(x)) = f(x)$, then

$$\begin{split} d_f(x \cdot d_f(x)) &= (d_f(x) \cdot f(d_f(x))) \wedge (f(x) \cdot d_f(d_f(x))) \\ &= (d_f(x) \cdot f(d_f(x))) \wedge (f(x) \cdot f(x)) \\ \end{split}$$
 (By Proposition 1.1 (1))
$$&= (d_f(x) \cdot f(d_f(x))) \wedge 0 \\ \end{aligned}$$
 (By Proposition 1.2 (2))
$$&= 0. \end{split}$$

(4) For all $x \in A$, if $f(d_f(x)) = d_f(x)$, then

$$\begin{aligned} d_f(d_f(x) \cdot x) &= (d_f(d_f(x)) \cdot f(x)) \wedge (f(d_f(x)) \cdot d_f(x)) \\ &= (d_f(d_f(x)) \cdot f(x)) \wedge (d_f(x) \cdot d_f(x)) \\ \end{aligned}$$
(By Proposition 1.1 (1))
$$\begin{aligned} &= (d_f(d_f(x)) \cdot f(x)) \wedge 0 \\ \end{aligned}$$
(By Proposition 1.2 (2))
$$\begin{aligned} &= 0. \end{aligned}$$

If $d_f(d_f(x)) = f(x)$, then

(By Proposition 1

$$d_f(d_f(x) \cdot x) = (d_f(d_f(x)) \cdot f(x)) \wedge (f(d_f(x)) \cdot d_f(x))$$
$$= (f(x) \cdot f(x)) \wedge (f(d_f(x)) \cdot d_f(x))$$
(By Proposition 1.1 (1))
$$= 0 \wedge (f(d_f(x)) \cdot d_f(x))$$
(By Proposition 1.2 (1))
$$= 0.$$

(5) For all $x \in A$, if $d_f(f(x)) = f(x)$, then

$$\begin{split} d_f(x \cdot f(x)) &= (d_f(x) \cdot f(f(x))) \wedge (f(x) \cdot d_f(f(x))) \\ &= (d_f(x) \cdot f(f(x))) \wedge (f(x) \cdot f(x)) \\ \end{split}$$
 (By Proposition 1.1 (1))
$$&= (d_f(x) \cdot f(f(x))) \wedge 0 \\ \end{aligned}$$
 (By Proposition 1.2 (2))
$$&= 0. \end{split}$$

If $f(f(x)) = d_f(x)$, then

$$\begin{split} d_f(x \cdot f(x)) &= (d_f(x) \cdot f(f(x))) \wedge (f(x) \cdot d_f(f(x))) \\ &= (d_f(x) \cdot d_f(x)) \wedge (f(x) \cdot d_f(f(x))) \\ \end{split} \\ \end{split}$$

$$\end{split} (By Proposition 1.1 (1)) \qquad \qquad = 0 \wedge (f(x) \cdot d_f(f(x))) \\ &= 0. \end{split}$$

(6) For all $x \in A$, if $d_f(f(x)) = f(x)$, then

$$d_f(f(x) \cdot x) = (d_f(f(x)) \cdot f(x)) \wedge (f(f(x)) \cdot d_f(x))$$
$$= (f(x) \cdot f(x)) \wedge (f(f(x)) \cdot d_f(x))$$
(By Proposition 1.1 (1))
$$= 0 \wedge (f(f(x)) \cdot d_f(x))$$
(By Proposition 1.2 (1))
$$= 0.$$

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If $f(f(x)) = d_f(x)$, then

$$\begin{aligned} d_f(f(x) \cdot x) &= (d_f(f(x)) \cdot f(x)) \wedge (f(f(x)) \cdot d_f(x)) \\ &= (d_f(f(x)) \cdot f(x)) \wedge (d_f(x) \cdot d_f(x)) \\ \end{aligned}$$
(By Proposition 1.1 (1))
$$&= (d_f(f(x)) \cdot f(x)) \wedge 0 \\ \end{aligned}$$
(By Proposition 1.2 (2))
$$&= 0. \end{aligned}$$

Proposition 2.3. Let d_f be an (r, l)-f-derivation of A. Then the following properties hold: for any $x, y \in A$,

(1)
$$f(x) \cdot d_f(y) \leq d_f(x \cdot y)$$
,
(2) if $f(d_f(x)) = d_f(x)$ or $d_f(d_f(x)) = f(x)$, then $d_f(x \cdot d_f(x)) = 0$,
(3) if $f(d_f(x)) = d_f(x)$ or $d_f(d_f(x)) = f(x)$, then $d_f(d_f(x) \cdot x) = 0$,
(4) if $d_f(f(x)) = f(x)$ or $f(f(x)) = d_f(x)$, then $d_f(x \cdot f(x)) = 0$, and
(5) if $d_f(f(x)) = f(x)$ or $f(f(x)) = d_f(x)$, then $d_f(f(x) \cdot x) = 0$.

Proof. (1) For all $x, y \in A$,

$$(f(x) \cdot d_f(y)) \cdot d_f(x \cdot y) = (f(x) \cdot d_f(y)) \cdot ((f(x) \cdot d_f(y)) \wedge (d_f(x) \cdot f(y)))$$
$$= (f(x) \cdot d_f(y)) \cdot (((d_f(x) \cdot f(y)) \cdot (f(x) \cdot d_f(y))) \cdot (f(x) \cdot d_f(y)))$$

(By Proposition 1.1 (5)) = 0.

Hence, $f(x) \cdot d_f(y) \leq d_f(x \cdot y)$ for all $x, y \in A$. (2) For all $x \in A$, if $f(d_f(x)) = d_f(x)$, then

$$\begin{split} d_f(x \cdot d_f(x)) &= (f(x) \cdot d_f(d_f(x))) \wedge (d_f(x) \cdot f(d_f(x))) \\ &= (f(x) \cdot d_f(d_f(x))) \wedge (d_f(x) \cdot d_f(x)) \\ \end{split}$$
 (By Proposition 1.1 (1))
$$&= (f(x) \cdot d_f(d_f(x))) \wedge 0 \\ \end{aligned}$$
 (By Proposition 1.2 (2))
$$&= 0. \end{split}$$

If $d_f(d_f(x)) = f(x)$, then

$$\begin{aligned} d_f(x \cdot d_f(x)) &= (f(x) \cdot d_f(d_f(x))) \wedge (d_f(x) \cdot f(d_f(x))) \\ &= (f(x) \cdot f(x)) \wedge (d_f(x) \cdot f(d_f(x))) \\ \end{aligned}$$
(By Proposition 1.1 (1))
$$\begin{aligned} &= 0 \wedge (d_f(x) \cdot f(d_f(x))) \\ &= 0. \end{aligned}$$

(3) For all $x \in A$, if $f(d_f(x)) = d_f(x)$, then

$$d_f(d_f(x) \cdot x) = (f(d_f(x)) \cdot d_f(x)) \wedge (d_f(d_f(x)) \cdot f(x))$$
$$= (d_f(x) \cdot d_f(x)) \wedge (d_f(d_f(x)) \cdot f(x))$$
Proposition 1.1 (1))
$$= 0 \wedge (d_f(d_f(x)) \cdot f(x))$$
Proposition 1.2 (1))
$$= 0.$$

If $d_f(d_f(x)) = f(x)$, then

$$d_f(d_f(x) \cdot x) = (f(d_f(x)) \cdot d_f(x)) \wedge (d_f(d_f(x)) \cdot f(x))$$
$$= (f(d_f(x)) \cdot d_f(x)) \wedge (f(x) \cdot f(x))$$
(By Proposition 1.1 (1))
$$= (f(d_f(x)) \cdot d_f(x)) \wedge 0$$
(By Proposition 1.2 (2))
$$= 0.$$

(4) For all $x \in A$, if $d_f(f(x)) = f(x)$, then

$$\begin{aligned} d_f(x \cdot f(x)) &= (f(x) \cdot d_f(f(x))) \wedge (d_f(x) \cdot f(f(x))) \\ &= (f(x) \cdot f(x)) \wedge (d_f(x) \cdot f(f(x))) \\ \end{aligned}$$
(By Proposition 1.1 (1))
$$\begin{aligned} &= 0 \wedge (d_f(x) \cdot f(f(x))) \\ &= 0. \end{aligned}$$

If $f(f(x)) = d_f(x)$, then

$$d_f(x \cdot f(x)) = (f(x) \cdot d_f(f(x))) \wedge (d_f(x) \cdot f(f(x)))$$
$$= (f(x) \cdot d_f(f(x))) \wedge (d_f(x) \cdot d_f(x))$$
(By Proposition 1.1 (1))
$$= (f(x) \cdot d_f(f(x))) \wedge 0$$
(By Proposition 1.2 (2))
$$= 0.$$

(5) For all $x \in A$, if $d_f(f(x)) = f(x)$, then

$$d_f(f(x) \cdot x) = (f(f(x)) \cdot d_f(x)) \wedge (d_f(f(x)) \cdot f(x))$$
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If $f(f(x)) = d_f(x)$, then

$$d_f(f(x) \cdot x) = (f(f(x)) \cdot d_f(x)) \wedge (d_f(f(x)) \cdot f(x))$$
$$= (d_f(x) \cdot d_f(x)) \wedge (d_f(f(x)) \cdot f(x))$$
(By Proposition 1.1 (1))
$$= 0 \wedge (d_f(f(x)) \cdot f(x))$$
(By Proposition 1.2 (1))
$$= 0.$$

Definition 2.3. A UP-ideal B of A is called f-invariant (with respect to an (l, r)-f-derivation (resp. (r, l)-f-derivation, f-derivation) d_f of A) if $d_f(B) \subseteq B$.

Example 2.6. Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

•	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ is a UP-algebra. We define a self-map $f: A \to A$ as follows:

$$f(0) = 0, f(1) = 0, f(2) = 1$$
 and $f(3) = 3$.

Then f is a UP-endomorphism. We define a self-map $d_f \colon A \to A$ as follows:

$$d_f(0) = 0, d_f(1) = 0, d_f(2) = 1 \text{ and } d_f(3) = 0.$$

Then d_f is an f-derivation of A. Let $B = \{0, 1, 2\}$ and $C = \{0, 1, 3\}$. Then B and C are UP-ideals of A and it follows that they are f-invariants with respect to an f-derivation d_f of A.

Theorem 2.7. Every ideal of A with containing the endomorphic image of f is f-invariant with respect to any (l, r)-f-derivation of A.

Proof. Assume that B is an ideal of A and d_f is an (l, r)-f-derivation of A. Let $y \in d_f(B)$. Then $y = d_f(x)$ for some $x \in B$. By Proposition 2.2 (1), we obtain $f(x) \leq d_f(x)$; that is, $f(x) \cdot d_f(x) = 0$. Thus $f(x) \cdot y = f(x) \cdot d_f(x) = 0 \in B$. Since $f(B) \subseteq B$, we have $f(x) \in B$. It follows from Theorem 1.3 (1) that $y \in B$. Hence, $d_f(B) \subseteq B$, which implies that B is f-invariant. \Box **Corollary 2.8.** Every ideal of A with containing the endomorphic image of f is f-invariant with respect to any f-derivation of A.

Definition 2.4. Let d_f be an (l, r)-f-derivation (resp. (r, l)-f-derivation, f-derivation) of A. We define a subset $\operatorname{Ker}_{d_f}(A)$ of A by

$$\operatorname{Ker}_{d_f}(A) = \{ x \in A \mid d_f(x) = 0 \}.$$

Theorem 2.9. In a UP-algebra A, the following statements hold:

- (1) if d_f is an (l,r)-f-derivation of A, then $y \wedge x \in \operatorname{Ker}_{d_f}(A)$ for all $y \in \operatorname{Ker}_{d_f}(A)$ and $x \in A$, and
- (2) if d_f is an (r,l)-f-derivation of A, then $y \wedge x \in \operatorname{Ker}_{d_f}(A)$ for all $y \in \operatorname{Ker}_{d_f}(A)$ and $x \in A$.

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. Let $y \in \text{Ker}_{d_f}(A)$ and $x \in A$. Then $d_f(y) = 0$. Thus

$$\begin{aligned} d_f(y \wedge x) &= d_f((x \cdot y) \cdot y) \\ &= (d_f(x \cdot y) \cdot f(y)) \wedge (f(x \cdot y) \cdot d_f(y)) \\ &= (d_f(x \cdot y) \cdot f(y)) \wedge (f(x \cdot y) \cdot 0) \\ \end{aligned}$$
(By UP-3)
$$\begin{aligned} &= (d_f(x \cdot y) \cdot f(y)) \wedge 0 \\ \end{aligned}$$
(By Proposition 1.2 (2))
$$\begin{aligned} &= 0. \end{aligned}$$

(By Proposition 1.2 (2))

Hence, $y \wedge x \in \operatorname{Ker}_{d_f}(A)$.

(2) Assume that d_f is an (r, l)-f-derivation of A. Let $y \in \text{Ker}_{d_f}(A)$ and $x \in A$. Then $d_f(y) = 0$. Thus

$$d_f(y \wedge x) = d_f((x \cdot y) \cdot y)$$

= $(f(x \cdot y) \cdot d_f(y)) \wedge (d_f(x \cdot y) \cdot f(y))$
= $(f(x \cdot y) \cdot 0) \wedge (d_f(x \cdot y) \cdot f(y))$
(By UP-3) = $0 \wedge (d_f(x \cdot y) \cdot f(y))$
(By Proposition 1.2 (1)) = 0 .

(By Proposition 1.2(1))

Hence, $y \wedge x \in \operatorname{Ker}_{d_f}(A)$. \Box

Corollary 2.10. If d_f is an f-derivation of A, then $y \wedge x \in \operatorname{Ker}_{d_f}(A)$ for all $y \in \operatorname{Ker}_{d_f}(A)$ and $x \in A$.

Give an example of conflict that $x \wedge y \notin \operatorname{Ker}_{d_f}(A)$ for all $y \in \operatorname{Ker}_{d_f}(A)$ and $x \in A$ in general.

Example 2.11. Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

•	0	1	2	3
0	0	1	2	3
1	0	0	0	3
2	0	0	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ is a UP-algebra. Let 1_A be an identity map on A. Then 1_A is a UPendomorphism. We define a self-map $d_{1_A}: A \to A$ as follows:

$$d_{1_A}(0) = 0, d_{1_A}(1) = 0, d_{1_A}(2) = 2 \text{ and } d_{1_A}(3) = 3.$$

Then d_{1_A} is an f-derivation of A and so $\operatorname{Ker}_{d_{1_A}}(A) = \{0,1\}$. Thus $2 \wedge 1 = 2 \notin \operatorname{Ker}_{d_{1_A}}(A)$ when $1 \in \operatorname{Ker}_{d_{1_A}}(A)$ and $2 \in A$.

Theorem 2.12. In a meet-commutative UP-algebra A, the following statements hold:

- (1) if d_f is an (l,r)-f-derivation of A and for any $x, y \in A$ is such that $y \leq x$ and $y \in \operatorname{Ker}_{d_f}(A)$, then $x \in \operatorname{Ker}_{d_f}(A)$, and
- (2) if d_f is an (r,l)-f-derivation of A and for any $x, y \in A$ is such that $y \leq x$ and $y \in \operatorname{Ker}_{d_f}(A)$, then $x \in \operatorname{Ker}_{d_f}(A)$.

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. Let $x, y \in A$ be such that $y \leq x$ and $y \in \operatorname{Ker}_{d_f}(A)$. Then $y \cdot x = 0$ and $d_f(y) = 0$. Thus

 $\begin{array}{ll} \text{(By UP-2)} & d_f(x) = d_f(0 \cdot x) \\ & = d_f((y \cdot x) \cdot x) \\ & = d_f((x \cdot y) \cdot y) \\ & = (d_f(x \cdot y) \cdot f(y)) \wedge (f(x \cdot y) \cdot d_f(y)) \\ & = (d_f(x \cdot y) \cdot f(y)) \wedge (f(x \cdot y) \cdot 0) \\ & = (d_f(x \cdot y) \cdot f(y)) \wedge (f(x \cdot y) \cdot 0) \\ & = (d_f(x \cdot y) \cdot f(y)) \wedge 0 \\ & = 0. \end{array}$

Hence, $x \in \operatorname{Ker}_{d_f}(A)$.

(2) Assume that d_f is an (r, l)-f-derivation of A. Let $x, y \in A$ be such that $y \leq x$ and $y \in \operatorname{Ker}_{d_f}(A)$. Then $y \cdot x = 0$ and $d_f(y) = 0$. Thus

$$\begin{array}{ll} \text{(By UP-2)} & d_f(x) = d_f(0 \cdot x) \\ & = d_f((y \cdot x) \cdot x) \\ & = d_f((x \cdot y) \cdot y) \\ & = (f(x \cdot y) \cdot d_f(y)) \wedge (d_f(x \cdot y) \cdot f(y)) \\ & = (f(x \cdot y) \cdot 0) \wedge (d_f(x \cdot y) \cdot f(y)) \\ & = 0 \wedge (d_f(x \cdot y) \cdot f(y)) \\ & = 0. \end{array}$$

Hence, $x \in \operatorname{Ker}_{d_f}(A)$. \Box

Corollary 2.13. If d_f is an f-derivation of a meet-commutative UP-algebra A and for any $x, y \in A$ is such that $y \leq x$ and $y \in \operatorname{Ker}_{d_f}(A)$, then $x \in \operatorname{Ker}_{d_f}(A)$.

Theorem 2.14. In a UP-algebra A, the following statements hold:

- (1) if d_f is an (l,r)-f-derivation of A, then $y \cdot x \in \operatorname{Ker}_{d_f}(A)$ for all $x \in \operatorname{Ker}_{d_f}(A)$ and $y \in A$, and
- (2) if d_f is an (r,l)-f-derivation of A, then $y \cdot x \in \operatorname{Ker}_{d_f}(A)$ for all $x \in \operatorname{Ker}_{d_f}(A)$ and $y \in A$.

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. Let $x \in \text{Ker}_{d_f}(A)$ and $y \in A$. Then $d_f(x) = 0$. Thus

$$d_f(y \cdot x) = (d_f(y) \cdot f(x)) \wedge (f(y) \cdot d_f(x))$$
$$= (d_f(y) \cdot f(x)) \wedge (f(y) \cdot 0)$$
$$= (d_f(y) \cdot f(x)) \wedge 0$$

(By Proposition 1.2 (2)) = 0.

Hence, $y \cdot x \in \operatorname{Ker}_{d_f}(A)$.

(2) Assume that d_f is an (r, l)-f-derivation of A. Let $x \in \text{Ker}_{d_f}(A)$ and $y \in A$. Then $d_f(x) = 0$. Thus

$$\begin{aligned} d_f(y \cdot x) &= (f(y) \cdot d_f(x)) \wedge (d_f(y) \cdot f(x)) \\ &= (f(y) \cdot 0) \wedge (d_f(y) \cdot f(x)) \\ \end{aligned}$$
 (By UP-3)
$$\begin{aligned} &= 0 \wedge (d_f(y) \cdot f(x)) \\ &= 0. \end{aligned}$$

Hence, $y \cdot x \in \operatorname{Ker}_{d_f}(A)$. \Box

(By UP-2 and UP-3) (

(By Proposition 1.2(1))

Corollary 2.15. If d_f is an f-derivation of A, then $y \cdot x \in \text{Ker}_{d_f}(A)$ for all $x \in \text{Ker}_{d_f}(A)$ and $y \in A$.

Example 2.16. From Example 2.1, we have $\operatorname{Ker}_{d_f}(A) = \{0, 1, 3\}$. Then $3 \cdot 2 = 2 \notin \operatorname{Ker}_{d_f}(A)$ when $3 \in \operatorname{Ker}_{d_f}(A)$ and $2 \in A$.

Theorem 2.17. In a UP-algebra A, the following statements hold:

- (1) if d_f is an (l,r)-f-derivation of A, then $\operatorname{Ker}_{d_f}(A)$ is a UP-subalgebra of A, and
- (2) if d_f is an (r, l)-f-derivation of A, then $\operatorname{Ker}_{d_f}(A)$ is a UP-subalgebra of A.

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. By Theorem 2.2 (1), we have $d_f(0) = 0$ and so $0 \in \operatorname{Ker}_{d_f}(A) \neq \emptyset$. Let $x, y \in \operatorname{Ker}_{d_f}(A)$. Then $d_f(x) = 0$ and $d_f(y) = 0$. Thus

$$\begin{split} d_f(x \cdot y) &= (d_f(x) \cdot f(y)) \wedge (f(x) \cdot d_f(y)) \\ &= (0 \cdot f(y)) \wedge (f(x) \cdot 0) \\ (\text{By UP-2 and UP-3}) &= f(y) \wedge 0 \\ (\text{By Proposition 1.2 (2)}) &= 0. \end{split}$$

Hence, $x \cdot y \in \operatorname{Ker}_{d_f}(A)$, so $\operatorname{Ker}_{d_f}(A)$ is a UP-subalgebra of A. (2) Assume that d_f is an (r, l)-f-derivation of A. By Theorem 2.2 (2), we have $d_f(0) = 0$ and so $0 \in \operatorname{Ker}_{d_f}(A) \neq \emptyset$. Let $x, y \in \operatorname{Ker}_{d_f}(A)$. Then $d_f(x) = 0$ and $d_f(y) = 0$. Thus

$$d_f(x \cdot y) = (f(x) \cdot d_f(y)) \wedge (d_f(x) \cdot f(y))$$
$$= (f(x) \cdot 0) \wedge (0 \cdot f(y))$$
$$= 0 \wedge f(y)$$
$$= 0.$$

Hence, $x \cdot y \in \operatorname{Ker}_{d_f}(A)$, so $\operatorname{Ker}_{d_f}(A)$ is a UP-subalgebra of A. \Box

Corollary 2.18. If d_f is an f-derivation of A, then $\operatorname{Ker}_{d_f}(A)$ is a UP-subalgebra of A.

Give an example of conflict that $\operatorname{Ker}_{d_f}(A)$ is not a UP-ideal of A in general.

Example 2.19. Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

•	0	1	2	3
0	0	1	2	3
1	0	0	0	0
2	0	1	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ is a UP-algebra. Let 1_A be an identity map on A. Then 1_A is a UPendomorphism. We define a self-map $d_{1_A}: A \to A$ as follows:

$$d_{1_A}(0) = 0, d_{1_A}(1) = 0, d_{1_A}(2) = 2 \text{ and } d_{1_A}(3) = 0.$$

Then d_{1_A} is an (l,r)- 1_A -derivation of A and $\operatorname{Ker}_{d_{1_A}}(A) = \{0,1,3\}$. Since $0 \cdot (1 \cdot 2) = 0 \in \operatorname{Ker}_{d_{1_A}}(A), 1 \in \operatorname{Ker}_{d_{1_A}}(A)$ but $0 \cdot 2 = 2 \notin \operatorname{Ker}_{d_{1_A}}(A)$, we conclude that $\operatorname{Ker}_{d_{1_A}}(A)$ is not a UP-ideal of A.

Definition 2.5. Let d_f be an (l, r)-f-derivation (resp. (r, l)-f-derivation, f-derivation) of A. We define a subset $\operatorname{Fix}_{d_f}(A)$ of A by

$$Fix_{d_f}(A) = \{x \in A \mid d_f(x) = f(x)\}$$

Theorem 2.20. In a UP-algebra A, the following statements hold:

- (1) if d_f is an (l,r)-f-derivation of A, then $\operatorname{Fix}_{d_f}(A)$ is a UP-subalgebra of A, and
- (2) if d_f is an (r, l)-f-derivation of A, then $\operatorname{Fix}_{d_f}(A)$ is a UP-subalgebra of A.

Proof. (1) Assume that d_f is an (l, r)-f-derivation of A. By Theorem 2.2 (1) and 1.4 (1), we have $d_f(0) = 0 = f(0)$ and so $0 \in \operatorname{Fix}_{d_f}(A) \neq \emptyset$. Let $x, y \in \operatorname{Fix}_{d_f}(A)$. Then $d_f(x) = f(x)$ and $d_f(y) = f(y)$. Thus

$$d_f(x \cdot y) = (d_f(x) \cdot f(y)) \wedge (f(x) \cdot d_f(y))$$
$$= (f(x) \cdot f(y)) \wedge (f(x) \cdot f(y))$$
$$= f(x \cdot y) \wedge f(x \cdot y)$$
$$= f(x \cdot y).$$

(By Proposition 1.2(3))

Hence, $x \cdot y \in \operatorname{Fix}_d(A)$, so $\operatorname{Fix}_d(A)$ is a UP-subalgebra of A.

(2) Assume that d_f is an (r, l)-f-derivation of A. By Theorem 2.2 (2) and 1.4 (1), we have $d_f(0) = 0 = f(0)$ and so $0 \in \operatorname{Fix}_{d_f}(A) \neq \emptyset$. Let $x, y \in \operatorname{Fix}_{d_f}(A)$. Then $d_f(x) = f(x)$ and $d_f(y) = f(y)$. Thus

$$d_f(x \cdot y) = (f(x) \cdot d_f(y)) \wedge (d_f(x) \cdot f(y))$$
$$= (f(x) \cdot f(y)) \wedge (f(x) \cdot f(y))$$
$$= f(x \cdot y) \wedge f(x \cdot y)$$
$$= f(x \cdot y).$$

(By Proposition 1.2(3))

Hence, $x \cdot y \in \operatorname{Fix}_{d_f}(A)$, so $\operatorname{Fix}_d(A)$ is a UP-subalgebra of A. \Box

Corollary 2.21. If d_f is an f-derivation of A, then $\operatorname{Fix}_{d_f}(A)$ is a UP-subalgebra of A.

Give an example of conflict that $\operatorname{Fix}_{d_f}(A)$ is not a UP-ideal of A in general.

Example 2.22. Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

•	0	1	2	3
0	0	1	2	3
1	0	0	0	0
2	0	1	0	3
3	0	1	2	0

Then $(A; \cdot, 0)$ is a UP-algebra. Let 1_A be an identity map on A. Then 1_A is a UPendomorphism. We define a self-map $d_{1_A}: A \to A$ as follows:

$$d_{1_A}(0) = 0, d_{1_A}(1) = 1, d_{1_A}(2) = 2 \text{ and } d_{1_A}(3) = 0.$$

Then d_{1_A} is an (l,r)- 1_A -derivation of A and $\operatorname{Fix}_{d_{1_A}}(A) = \{0,1,2\}$. Since $2 \cdot (1 \cdot 3) = 0 \in \operatorname{Fix}_{d_{1_A}}(A), 1 \in \operatorname{Fix}_{d_{1_A}}(A)$ but $2 \cdot 3 = 3 \notin \operatorname{Fix}_{d_{1_A}}(A)$, we conclude that $\operatorname{Fix}_{d_{1_A}}(A)$ is not a UP-ideal of A.

Theorem 2.23. In a UP-algebra A, the following statements hold:

(1) if d_f is an (l, r)-f-derivation of A, then $x \wedge y \in \operatorname{Fix}_{d_f}(A)$ for all $x, y \in \operatorname{Fix}_{d_f}(A)$, and (2) if d_f is an (r, l)-f-derivation of A, then $x \wedge y \in \operatorname{Fix}_{d_f}(A)$ for all $x, y \in \operatorname{Fix}_{d_f}(A)$. *Proof.* (1) Assume that d_f is an (l, r)-f-derivation of A. Let $x, y \in \text{Fix}_{d_f}(A)$. Then $d_f(x) = f(x)$ and $d_f(y) = f(y)$. By Theorem 2.20 (1), we get $d_f(y \cdot x) = f(y \cdot x)$. Thus

$$\begin{aligned} d_f(x \wedge y) &= d_f((y \cdot x) \cdot x) \\ &= (d_f(y \cdot x) \cdot f(x)) \wedge (f(y \cdot x) \cdot d_f(x)) \\ &= (f(y \cdot x) \cdot f(x)) \wedge (f(y \cdot x) \cdot f(x)) \\ \end{aligned}$$
(By Proposition 1.2 (3))
$$\begin{aligned} &= f(y \cdot x) \cdot f(x) \\ &= f((y \cdot x) \cdot x) \\ &= f(x \wedge y). \end{aligned}$$

Hence, $x \wedge y \in \operatorname{Fix}_{d_f}(A)$.

(2) Assume that d_f is an (r, l)-f-derivation of A. Let $x, y \in \text{Fix}_{d_f}(A)$. Then $d_f(x) = f(x)$ and $d_f(y) = f(y)$. By Theorem 2.20 (2), we get $d_f(y \cdot x) = f(y \cdot x)$. Thus

$$d_f(x \wedge y) = d_f((y \cdot x) \cdot x)$$

= $(f(y \cdot x) \cdot d_f(x)) \wedge (d_f(y \cdot x) \cdot f(x))$
= $(f(y \cdot x) \cdot f(x)) \wedge (f(y \cdot x) \cdot f(x))$
= $f(y \cdot x) \cdot f(x)$
= $f((y \cdot x) \cdot x)$
= $f((x \wedge y).$

Hence, $x \wedge y \in \operatorname{Fix}_{d_f}(A)$. \Box

(By Proposition 1.2 (3)

Corollary 2.24. If d_f is an f-derivation of A, then $x \wedge y \in \operatorname{Fix}_{d_f}(A)$ for all $x, y \in \operatorname{Fix}_{d_f}(A)$.

Competing Interests

The author declares that no competing interests exist.

References

 H. A. S. Abujabal, N. O. Al-shehri, Some results on derivations of BCI-algebras, J. Nat. Sci. Math. 46 (no. 1&2) (2006), 13–19.

[2] H. A. S. Abujabal, N. O. Al-shehri, On left derivations of BCI-algebras, Soochow J. Math. 33 (no. 3) (2007), 435–444.

[3] A. M. Al-roqi, On generalized (α, β) -derivations in BCI-algebras, J. Appl. Math. Inform. **32** (no. 1–2) (2014), 27–38.

- [4] N. O. Al-shehri, S. M. Bawazeer, On derivations of BCC-algebras, Int. J. Algebra 6 (no. 32) (2012), 1491– 1498.
- [5] L. K. Ardekani, B. Davvaz, On generalized derivations of BCI-algebras and their properties, J. Math. 2014 (2014), Article ID 207161, 10 pages.
- [6] S. M. Bawazeer, N. O. Alshehri, R. S. Babusail, Generalized derivations of BCC-algebras, Int. J. Math. Math. Sci. 2013 (2013), Article ID 451212, 4 pages.
- [7] Q. P. Hu, X. Li, On BCH-algebras, Math. Semin. Notes, Kobe Univ. 11 (1983), 313–320.
- [8] A. Iampan, A new branch of the logical algebra: UP-algebras, Manuscript submitted for publication, April 2016.
- [9] Y. Imai, K. Iséki, On axiom system of propositional calculi, XIV, Proc. Japan Acad. 42 (no. 1) (1966), 19–22.
- [10] K. Iséki, An algebra related with a propositional calculus, Proc. Japan Acad. 42 (no. 1) (1966), 26–29.
- [11] M. A. Javed, M. Aslam, A note on f-derivations of BCI-algebras, Commun. Korean Math. Soc. 24 (no. 3) (2009), 321–331.
- [12] Y. B. Jun, X. L. Xin, On derivations of BCI-algebras, Inform. Sci. 159 (2004), 167–176.
- [13] S. Keawrahun, U. Leerawat, On isomorphisms of SU-algebras, Sci. Magna 7 (no. 2) (2011), 39–44.
- [14] K. J. Lee, A new kind of derivation in BCI-algebras, Appl. Math. Sci. 7 (no. 84) (2013), 4185–4194.
- [15] P. H. Lee, T. K. Lee, On derivations of prime rings, Chinese J. Math. 9 (1981), 107–110.
- [16] S. M. Lee, K. H. Kim, A note on f-derivations of BCC-algebras, Pure Math. Sci. 1 (no. 2) (2012), 87–93.
- [17] G. Muhiuddin, A. M. Al-roqi, On (α, β) -derivations in BCI-algebras, Discrete Dyn. Nat. Soc. **2012** (2012), Article ID 403209, 11 pages.
- [18] G. Muhiuddin, A. M. Al-roqi, On t-derivations of BCI-algebras, Abstr. Appl. Anal. 2012 (2012), Article ID 872784, 12 pages.
- [19] G. Muhiuddin, A. M. Al-roqi, On generalized left derivations in BCI-algebras, Appl. Math. Inf. Sci. 8 (no. 3) (2014), 1153–1158.
- [20] G. Muhiuddin, A. M. Al-roqi, Y. B. Jun, Y. Ceven, On symmetric left bi-derivations in BCI-algebras, Int.
 J. Math. Math. Sci. 2013 (2013), Article ID 238490, 6 pages.
- [21] F. Nisar, Characterization of f-derivations of a BCI-algebra, East Asian Math. J. 25 (no. 1) (2009), 69–87.
- [22] F. Nisar, On F-derivations of BCI-algebras, J. Prime Res. Math. 5 (2009), 176-191.
- [23] E. Posner, Derivations in prime rings, Proc. Amer. Math. Soc. 8 (1957), 1093–1100.
- [24] C. Prabpayak, U. Leerawat, On derivation of BCC-algebras, Kasetsart J. (Nat. Sci.) 43 (2009), 398-401.
- [25] C. Prabpayak, U. Leerawat, On ideals and congruences in KU-algebras, Sci. Magna 5 (no. 1) (2009), 54–57.
- [26] K. Sawika, R. Intasan, A. Kaewwasri, A. Iampan, Derivations of UP-algebras, Korean J. Math. 24 (no. 3) (2016), 345–367.
- [27] J. Zhan, Y. L. Liu, On f-derivations of BCI-algebras, Int. J. Math. Math. Sci. 2005 (2005), 1675–1684.

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