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HX-HYPERGROUPS ASSOCIATED WITH THE DIRECT PRODUCTS OF SOME $\mathbf{Z}/n\mathbf{Z}$

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Abstract. One studies the HX-hypergroups, corresponding to the Chinese hypergroups associated with the direct products of some $\mathbf{Z}/n\mathbf{Z}$, calculating their fuzzy grades.

1. Introduction

One has considered the Chinese hypergroups (introduced by Corsini in [2]) associated with the HX-groups (defined by Li Hongxing [26]) of the direct product of $\mathbf{Z}/5\mathbf{Z}$, $\mathbf{Z}/6\mathbf{Z}$, $\mathbf{Z}/7\mathbf{Z}$, respectively five, six, seven times and one has found their fuzzy grade.

HX-groups and HX-hypergroups are not only a very important subject of general mathematics, but have also a remarkable role in applications to modeling, chaotic (hyperchaotic) systems and differential equations (see Baojie Zhang, Hongxing Li, [38]). Let us see what it is a HX-group and what is a Chinese hypergroup.

Let $(G, \cdot)$ be a group and $\mathcal{P}^*(G)$ be the set of all non empty subsets of G. An $HX-group$ is a nonempty subset H of $\mathcal{P}^*(G)$, which is a group, with respect to the operation (see [2], [24],

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[26], [25], [27], [36], [37]):

$$\forall (A,B) \in \mathcal{P}^*(G) \times \mathcal{P}^*(G),\ A \cdot B = \{xy \mid x \in A,\ y \in B\}.$$

Lemma 1.1. *One can see that* $\forall (x,y) \in G^* \times G^*,\ x \widehat{\circ} y = A(x) \cdot A(y)$.

It follows the next

Proposition 1.1. *If* $\mathcal{G}$ *is an* HX*-group, such that*

$$\forall (A,B) \in \mathcal{G} \times \mathcal{G},\ A \cap B \neq \emptyset \Rightarrow A = B,$$

then $(G^*, \widehat{\circ})$ *is a hypergroup.*

Let us see now what means a "fuzzy grade".

In 2003 Corsini [4] proved that with every hypergroupoid one can associate a fuzzy subset. Set

$$\forall u \in H,\ Q(u) = \{(x,y) \in H^2 \mid u \in x \circ y\},$$

$$q(u) = \mid Q(u) \mid,$$

$$A(u) = \sum_{(x,y) \in Q(u)} 1/\mid x \circ y \mid,$$

$$\mu(u) = A(u)/q(u). \tag{1}$$

Then, $\forall (x,y) \in H^2$,

$$x \circ_\mu y = \{z \in H \mid min\{\mu(x), \mu(y)\} \leq \mu(z) \leq max\{\mu(x), \mu(y)\}\}. \tag{2}$$

The hypergroupoid $(H, \circ_\mu)$ is a join space (see [3]).

So, by repeating the above correspondences, we obtain a sequence of join spaces (and fuzzy subsets) $H_m = (H, \mu_m)$. We call *fuzzy grade* of H, the minimum q, such that $(H, \mu_q) \simeq (H, \mu_{q+1}.)$

Let $\mathcal{G}$ be an HX-group with G as support and E as identity.

A Chinese hypergroupoid is a hyperstructure $(G^*, \widehat{\circ})$, where

$$G^* = \bigcup_{A \in \mathcal{G}} A \text{ and } \forall (x,y) \in G^* \times G^*,\ x \widehat{\circ} y = \bigcup_{x \in A, y \in B, \{A,B\} \subset \mathcal{G}} A \cdot B.$$

Set $\forall x \in G^*,\ \alpha(x) = \{A \mid A \in \mathcal{G},\ x \in A\}$ and $A(x) = \bigcup_{A \in \alpha(x)} A$.

In [7] one obtained a hypergroupoid from an HX-group.

2. The HX-group associated with $\mathbf{Z}/5\mathbf{Z}$

Set $G = \mathbf{Z}/5\mathbf{Z}$ and

$$H = \mathcal{G}_1^5 = \{\{1100, 2200\}, \{2200, 3300\}, \{3300, 4400\}, \{4400, 0000\}\}.$$

$\mathcal{G}_1$ is clearly a subgroup of $\mathcal{P}^*(G)$.

In $G_1^* = \bigcup_{X_i \in \mathcal{G}_1} X_i$ let us consider the following structure:

$$\forall j : \ 0 \leq j \leq 4, \ \text{set } j \otimes k = \bigcup_{j \in A, \ k \in B} AB.$$

We obtain the hypergroupoid A_0^5 represented by the following table:

$\mathcal{G}^5$	1100	2200	3300	4400	0000
1100	2200, 4400 3300	2200, 3300 4400, 0000	4400, 0000 3300, 1100	0000, 1100 4400, 2200	0000 1100, 2200
2200		H	H	H	0000, 1100 2200, 3300
3300			H	H	1100, 2200 3300, 4400
4400				H	3300, 2200 4400, 0000
0000					3300 4400, 0000

In the following, instead of 1100, 2200, 3300 we shall write 11, 22, 33 and so on.

In order to obtain the fuzzy grade, we do the following calculations:

$A_1^5(11) = (2/4 + 2/4 + 3/3) + (1/5 + 2/5 + 2/5 + 2/4)+$
$+(1/5 + 2/5 + 2/4) + 1/5 = 67/15$, whence
$q_1^5(11) = 19, \ \mu_1^5(11) = 0.23509.$

$A_1^5(33) = A_1^5(44) = (1/3 + 2/4 + 2/4) + (1/5 + 2/5 + 2/5 + 2/4)+$
$+(1/5 + 2/5 + 2/4) + 1/5 + 2/4 + 1/3 = 298/60$, whence
$q_1^5(33) = q_1^5(44) = 21, \ \mu_1^5(33) = \mu_1^5(44) = 0.236508.$

$A_1^5(00) = A_1^5(22) = (6/4 + 2/3) + (5/5 + 2/4) + 3/5+$
$+(1/5 + 2/4) + 1/3 = 106/20$, whence
$q_1^5(00) = q_1^5(22) = 22,\ \mu_1^5(00) = \mu_1^5(22) = 0.240909.$

So, we obtain:

A_1^5	00	22	33	44	11
00	00, 22	00, 22	00, 22 33, 44	00, 22 33, 44	H
22		00, 22	00, 22 33, 44	00, 22 33, 44	H
33			33, 44	33, 44	33, 44 11
44				33, 44	33, 44 11
11					11

Therefore, we obtain:

$A_2^5(00) = A_2^5(22) = 4/2 + 8/4 + 4/5 = 24/5$, whence
$q_2^5(00) = q_2^5(22) = 16,\ \mu_2^5(00) = \mu_2^5(22) = 0.3.$

$A_2^5(33) = A_2^5(44) = 4/2 + 4/3 + 8/4 + 4/5 = 92/15$, whence
$q_2^5(33) = q_2^5(44) = 20,\ \mu_2^5(33) = \mu_2^5(44) = 0.30667.$

$A_2^5(11) = 1 + 4/3 + 4/5 = 47/15$, whence
$q_2^5(11) = 9,\ \mu_2^5(11) = 0.3481.$

Therefore, $A_2^5 = A_1^5$, whence $\partial\mathcal{G}^5 = 1$.

3. The HX-group associated with $\mathbf{Z}/6\mathbf{Z}$.

Again instead of 11000 we shall write 11, instead of 22000 we shall write 22 and so on.

$\mathcal{G}^6$	11	22	33	44	55	00
11	$22, 33$ 44	$22, 33$ $44, 55$	$33, 44$ $55, 00$	$44, 55$ $00, 11$	$55, 00$ $11, 22$	$11, 22$ 00
22		$22, 33$ $44, 55$ 00	$33, 44$ $55, 00$ 11	$44, 55$ $00, 11$ 22	$55, 00$ $11, 22$ 33	$11, 22$ $33, 00$
33			$44, 55$ $00, 11$ 22	$55, 00$ $11, 22$ 33	$00, 11$ $22, 33$ 44	$11, 22$ $33, 44$
44				$00, 11$ $22, 33$ 44	$11, 22$ $33, 44$ 55	$11, 22$ $33, 44$
55					$22, 33$ $44, 55$ 00	$22, 33$ $44, 55$
66						$44, 55$ 00

One obtains:

$A_1^6(22) = (1/3 + 2/4 + 2/4 + 2/3) + (1/5 + 4/5 + 2/5 + 2/4) +$
$+ (1/5 + 4/5 + 2/4) + (1/5 + 2/5 + 2/4) + (1/5 + 2/4) = 34/5,$
whence $q_1^6(22) = 29,\ \mu_1^6(22) = 0.23448.$

$A_1^6(11) = (4/4 + 2/3) + (6/5 + 2/4) + (1/5 + 4/5 + 2/4) +$
$+ (1/5 + 2/5 + 2/4) = 358/60,$ whence
$q_1^6(11) = 26,\ \mu_1^6(11) = 0.229487.$

$A_1^6(33) = (1/3 + 4/4) + (1/5 + 4/5 + 2/4) + (4/5 + 2/4) +$
$+ (1/5 + 2/5 + 2/4) + (1/5 + 2/4) = 89/15,$ whence
$q_1^6(33) = 26,\ \mu_1^6(33) = 0.228205.$

$A_1^6(44) = (1/3 + 6/4) + (1/5 + 4/5) + (1/5 + 2/5 + 2/4) +$
$+ (1/5 + 2/5 + 2/4) + (1/5 + 2/4 + 1/3) = 91/15,$ whence
$q_1^6(44) = 26,\ \mu_1^6(44) = 0.233.$

$A_1^6(00) = 1/3 + 1/5 + 1/5 + (1/5 + 2/5 + 2/5)+$
$+(1/5 + 2/5 + 2/5 + 2/5 + 2/4)+$
$+(2/4 + 2/4 + 2/4 + 2/3) = 174/30,$
whence $q_1^6(44) = 25,\ \mu_1^6(44) = 0.232.$

$A_1^6(55) = 8/4 + (1/5 + 6/5) + (1/5 + 2/5) + 2/5+$
$+(1/5 + 2/4) + 1/3 = 326/60,$ whence
$q_1^6(55) = 24,\ \mu_1^6(55) = 0.2263888.$

Therefore,

$\mu_1^6(55) = 0.226 < \mu_1^6(33) = 0.233 < \mu_1^6(11) = 0.229 <$
$\mu_1^6(00) = 0.232 < \mu_1^6(44) = 0.233 < \mu_1^6(22) = 0.23448.$

A_1^6	22	44	00	11	33	55
22	22	22, 44	22, 44	22, 44	22, 44	H
		44	00	00, 11	00, 11, 33	H
44		44	44, 00	44, 00	44, 00	44, 00
				11	11, 33	11, 33, 55
00			00	00	00, 11	00, 11
				11	33	33, 55
11				11	11	11, 33
					33	55
33					33	33, 55
55						55

One obtains:

$A_2^6(22) = A_2^6(55) = 1 + 2/2 + 2/3 + 2/4 + 2/5 + 2/6 = 117/30,$
whence $q_2^6(22) = q_2^6(55) = 11,\ \mu_2^6(22) = \mu_2^6(55) = 0.35454.$
$A_2^6(44) = A_2^6(33) = 1 + 4/2 + 4/3 + 4/4 + 4/5 + 2/6 = 194/30,$
whence $q_2^6(44) = q_2^6(33) = 19,\ \mu_2^6(44) = \mu_2^6(33) = 0.34035.$

$A_2^6(00) = A_2^6(11) = 1 + 4/2 + 6/3 + 6/4 + 4/5 + 2/6 = 229/30,$

whence $q_2^6(00) = q_2^6(11) = 23,\ \mu_2^6(00) = \mu_2^6(11) = 0.33188.$

Hence,

A_2^6	22	55	44	33	11	00
22	22 55	22 55	22, 55 44, 33	22, 55 44, 33	H	H
55		22 55	22, 55 44, 33	22, 55 44, 33	H	H
44			44, 33	44, 33	44, 33 11, 00	44, 33 11, 00
33				44, 33	44, 33 11, 00	44, 33 11, 00
11					11, 00	11, 00
00						11, 00

We obtain:

$A_3^6(22) = A_3^6(55) = 4/2 + 8/4 + 8/6 = 32/6,$
whence $q_3^6(22) = q_3^6(55) = 20,\ \mu_3^6(22) = \mu_3^6(55) = 0.2666.$

$A_3^6(11) = A_3^6(00) = 4/2 + 8/4 + 8/6 = 32/6$
whence $q_3^6(11) = q_3^6(00) = 19,$
$\mu_3^6(11) = \mu_3^6(00) = \mu_3^6(22) = \mu_3^6(5) = 0.2666.$
$A_3^6(44) = A_3^6(33) = 4/2 + 16/4 + 8/6 = 44/6,$
whence $q_3^6(44) = q_3^6(33) = 28,\ \mu_3^6(00) = \mu_3^6(11) = 0.2619.$

Therefore,

A_3^6	22	55	00	11	44	33
22	P	P	P	P	H	H
55	P	P	P	P	H	H
00	P	P	P	P	H	H
11	P	P	P	P	H	H
44	H	H	H	H	Q	Q
33	H	H	H	H	Q	Q

where $P = \{22, 55, 00, 11\},\ Q = \{33, 44\}$.

We have

$A_4^6(22) = A_4^6(55) = A_4^6(11) = A_4^6(00) = 16/4 + 16/6 = 40/6$,
whence $q_4^6(22) = q_4^6(55) = q_4^6(11) = q_4^6(00) = 32$,
$\mu_4^6(22) = \mu_4^6(55) = \mu_4^6(11) = \mu_4^6(00) = 0.2083$.

$A_4^6(33) = A_4^6(44) = 4/2 + 16/6 = 28/6$,
whence $q_4^6(33) = q_4^6(44) = 20$,
$\mu_4^6(33) = \mu_4^6(44) = 0.233$.

Hence $A_4^6 = A_3^6$, so $\partial \mathcal{G}^6 = 3$.

4. The HX-group associated with $\mathbf{Z}/7\mathbf{Z}$.

In what follows, instead of 222000 we shall write 22. The same for 111000, 333000 etc.

$\mathcal{G}^7$	11	22	33	44	55	66	00
11	22 33 44	22, 33 44, 55	33, 44 55, 66	44, 55 66, 00	55, 66 00, 11	66, 00 11, 22	00 11, 22
22		22, 33 44, 55 66	33, 44 55, 66 00	44, 55 66, 00 11	55, 66 00, 11 22	66, 00 11, 22 33	00, 11 22, 33
33			44, 55 66, 00 11	55, 66 00, 11 22	66, 00 11, 22 33	00, 11 22, 33 44	11, 22 33, 44
44				66, 00 11, 22 33	00, 11 22, 33 44	11, 22 33, 44 55	22, 33 44, 55
55					11, 22 33, 44 555	22, 33 44, 55 666	33, 44 55, 66
66						33, 44 55, 66 00	44, 55 66, 00
00							55 66, 00

One obtains:

$A_1^7(11) = (4/4 + 2/3) + (6/5 + 2/4) + (1/5 + 6/5 + 2/4) +$
$+(1/5 + 4/5) + 1/5 = 97/15,$
whence $q_1^7(11) = 29,\ \mu_1^7(11) = 0.222988.$

$A_1^7(00) = (6/4 + 2/3) + (8/5 + 2/4) + (1/5 + 6/5) + (1/5 + 2/5) +$
$+(1/5 + 2/4) + 1/3 = 146/20,$
whence $q_1^7(00) = 32,\ \mu_1^7(00) = 0.2281.$
$A_1^7(33) = (1/3 + 4/4) + (1/5 + 4/5 + 2/4) + (4/5 + 2/4) +$
$+(1/5 + 4/5 + 2/4) + (1/5 + 2/5 + 2/4) + 1/5 = 104/15,$
whence $q_1^7(33) = 31,\ \mu_1^7(33) = 0.2236559.$

$A_1^7(55) = A_1^7(4) = 8/4 + (1/5 + 6/5) + (1/5 + 2/5)+$
$+(2/5 + 2/4) + (1/5 + 2/5 + 2/4) + (1/5 + 2/4) + 1/3 = 422/60,$
whence $q_1^7(55) = q_1^7(44) = 31,\ \mu_1^7(55) = 0.22688 = \mu_1^7(44).$

A_1^7	11	33	55	44	66	00	22
11	11	11, 33	11, 33 55, 44	11, 33 55, 44	11, 33 55, 66, 44	H	H
33		33	33, 44 55	33, 44 55	33, 55 66, 44	33, 55 66, 44 00, 22	33, 55 66, 44 00, 22
55			55, 44	55, 44	55 66, 44	55 66, 44 00, 22	55 66, 44 00, 22
44				55, 44	55 66, 44	55 66, 44 00, 22	55 66, 44 00, 22
66					6	66 00, 22	66 00, 22
00						00, 22	00, 22
22							00, 22

We have
$\mu_1^7(11) = 0.2229 < \mu_1^7(33) = 0.223 < \mu_1^7(44) = 0.22688 = \mu_1^7(55).$

$A_1^7(66) = 8/4 + (1/5 + 8/5) + (1/5 + 4/5) + 1/5+$
$+(2/5 + 2/4) + (1/5 + 2/4) + 2/3 = 109/15,$
whence $q_1^7(66) = 32,\ \mu_1^7(66) = 0.22708.$
$A_1^7(00) = (6/4 + 2/3) + (8/5 + 2/4) + 7/5 + 3/5$
$+(1/5 + 2/4) + 1/3 = 146/20,$
whence $q_1^7(00) = 32,\ \mu_1^7(00) = 0.2281 = \mu_1^7(22).$

One obtains:

$A_2^7(44) = 4/2 + 8/3 + 6/4 + 10/5 + 4/6 + 4/7 = 1660/210$,
whence $q_2^7(44) = 36$, $\mu_2^7(44) = \mu_2^7(55) = 0.2612$.

$A_2^7(66) = 1 + 8/3 + 2/4 + 10/5 + 4/6 + 4/7 = 1555/210$,
whence $q_2^7(66) = 29$, $\mu_2^7(66) = 0.2553$.

$A_2^7(11) = 1 + 2/2 + 4/4 + 2/5 + 4/7 = 139/35$,
whence $q_2^7(11) = 13$, $\mu_2^7(11) = 0.30549$.

$A_2^7(33) = 1 + 2/2 + 4/3 + 6/4 + 2/5 + 4/6 + 4/7 = 1359/210$,
whence $q_2^7(33) = 23$, $\mu_2^7(33) = 0.281366$.

$A_2^7(00) = 4/2 + 4/3 + 8/5 + 4/6 + 4/7 = 1296/210$,
whence $q_2^7(00) = 24$, $\mu_2^7(00) = 0.25714 = \mu_2^7(22)$.

A_2^7	11	33	44	55	00	22	66
11	11	11 33	11, 33 44, 55	11, 33 44, 55	11, 33 44, 55 00, 22	H 44, 55 00, 22	H
33		33	33 44, 55	33 44, 55	33 44, 55 00, 22	33 44, 55 00, 22	33, 44, 55 00, 22, 66
44			44, 55	44, 55	44, 55 00, 22	44, 55 00, 22	44, 55 00, 22 66
55				44, 55	44, 55 00, 22	44, 55 00, 22	44, 55 00, 22 66
00					00 22	00 22	00, 22 66
22						00 22	00, 22 66
66							66

Hence,

$A_3^7(11) = 1 + 2/2 + 4/4 + 4/6 + 2/7 = 146/42,$
whence $q_3^7(11) = 13,\ \mu_3^7(11) = 0.304029.$

$A_3^7(33) = 1 + 2/2 + 4/3 + 4/4 + 4/5 + 6/6 + 2/7 + 4/5 = 674/105,$
whence $q_3^7(33) = 23,\ \mu_3^7(33) = 0.279089.$

$A_3^7(44) = A_3^7(55) = 4/2 + 4/3 + 12/4 + 8/5 + 6/6 + 2/7 = 968/105,$
whence $q_3^7(44) = q_3^7(55) = 36,\ \mu_3^7(44) = \mu_3^7(55) = 0.25608.$

$A_3^7(22) = A_3^7(00) = 4/2 + 8/4 + 8/5 + 4/3 + 2/7 + 6/6 = 863/105,$
whence $q_3^7(22) = q_3^7(00) = 32,\ \mu_3^7(00) = \mu_3^7(22) = 0.256845.$

$A_3^7(66) = 1 + 4/3 + 4/4 + 2/6 + 2/7 = 788/210,$
whence $q_3^7(66) = 13,\ \mu_3^7(66) = 0.28864.$

Hence $\mu_3^7(11) = 0.304 > \mu_3^7(66) = 0.288 > \mu_3^7(33) = 0.279 >$
$> \mu_3^7(00) = \mu_3^7(22) = 0.25684 > \mu_3^7(44) = \mu_3^7(55) = 0.25608.$

So, we have

A_3^7	11	66	33	00	22	44	55
11	11	11 66	11, 66 33	11, 66 33, 00 22	11, 66 33, 00 22	H	H
66		66	66, 33	66 00, 22	66 00, 22	66 00, 22 44, 55	66 00, 22 44, 55
33			33	33 00, 22	33 00, 22	33 00, 22 44, 55	33 00, 22 44, 55
00				00, 22	00, 22	00, 22 44, 55	00, 22 44, 55
22					00, 22	00, 22 44, 55	00, 22 44, 55
44						44 55	44 55
55							44, 55

We obtain

$A_4^7(11) = 1 + 2/2 + 2/3 + 4/5 + 4/7 = 424/105$,
whence $q_4^7(11) = 13$, $\mu_4^7(11) = 0.31062$.

$A_4^7(66) = 1 + 4/2 + 2/3 + 4/5 + 4/4 + 4/7 + 4/6 = 1408/210$,
whence $q_4^7(66) = 23$, $\mu_4^7(66) = 0.29151$.

$A_4^7(33) = 1 + 2/2 + 4/3 + 4/4 + 8/5 + 4/6 + 4/2 = 1506/210$,
whence $q_4^7(33) = 27$, $\mu_4^7(33) = 0.265608$.

$A_4^7(00) = A_4^7(22) = 4/2 + 4/3 + 12/4 + 8/5 + 4/6 + /7 = 1926/210$,
whence $q_4^7(00) = q_4^7(22) = 30$, $\mu_4^7(00) = \mu_4^7(22) = 0.25476$.

$A_4^7(44) = A_4^7(55) = 4/2 + 8/4 + 4/5 + 4/6 + 4/7 = 1268/210$,
whence $q_4^7(44) = q_4^7(55) = 24$, $\mu_4^7(44) = \mu_4^7(55) = 0.25158$.

Hence $\mu_4^7(11) = 0.31062 > \mu_4^7(66) = 0.29151 > \mu_4^7(33) = 0.265608 >$
$> \mu_4^7(00) = \mu_4^7(22) = 0.25476 > \mu_4^7(44) = \mu_4^7(55) = 0.25158.$

By consequence, we have $A_4^7 = A_3^7$, whence it follows that $\partial \mathcal{G}^7 = 3$.

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