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ON TRANSITIVE SOFT SETS OVER SEMIHYPERGROUPS

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Abstract. The aim of this paper is to initiate and investigate new soft sets over semihypergroups, named special soft sets and transitive soft sets and denoted by S_H and T_H, respectively. It is shown that $T_H = S_H$ if and only if $\beta = \beta^*$. We also introduce the derived semihypergroup from a special soft set and study some properties of this class of semihypergroups.

1. Introduction

In [12] Molodtsov introduced the notion of soft sets in order to deal with uncertainties which is free from the difficulties of theories such as probabilities, rough sets, interval mathematics. Some applications of soft sets have been initiated in [4, 5, 7, 10, 6]. Initiation of hyperstructure theory is nineteen thirty four (1934) at the eighth congress of Scandinavian Mathematicions, where Marty [11] introduced the hypergroup notion as a generalization of groups. For the first time, he utilized it in solving some problems of groups, algebraic functions and rational

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fractions. Some investigations of the theory of hyperstructures are accessible in the books of Corsini [1], Vougiouklis [13], Corsini and Leoreanu [2], Davvaz and Leoreanu-Fotea [3]. We recall here some basic notions of hypergroup theory.

Let H be a nonempty set and $P^*(H)$ be the set of all nonempty subsets of H. Let $\cdot$ be a *hyperoperation* (or *join operation*) on H, that is, $\cdot$ is a function from $H \times H$ into $P^*(H)$. If $(a,b) \in H \times H$, its image under $\cdot$ in $P^*(H)$ is denoted by $a \cdot b$. The join operation is extended to subsets of H in a natural way, that is, for nonempty subsets A, B of H, $A \cdot B = \cup\{ab \mid a \in A, b \in B\}$. The notation $a \cdot A$ is used for $\{a\} \cdot A$ and $A \cdot a$ for $A \cdot \{a\}$. Generally, the singleton $\{a\}$ is identified with its member a. The structure $(H, \cdot)$ is called a *semihypergroup* if $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all $a, b, c \in H$, and is called a *hypergroup* if it is a semihypergroup and $a \cdot H = H \cdot a = H$ for all $a \in H$. A nonempty subset K of a (semi)hypergroup $(H, \cdot)$ is called sub(semi)hypergroup if $(K, \cdot)$ is a (semi)hypergroup. A semihypergroup $(H, \cdot)$ is called *complete* if for all $n, m \geqslant 2$ and for all $(x_1, x_2, ..., x_n) \in H^n$ and $(y_1, y_2, ..., y_m) \in H^m$ we have the following implication:

$$\prod_{i=1}^{n} x_i \bigcap \prod_{j=1}^{m} y_j \neq \emptyset \Rightarrow \prod_{i=1}^{n} x_i = \prod_{j=1}^{m} y_j.$$

Theorem 1.1. *(Theorem 138,* [1]*) A semihypergroup $(H, \circ)$ is complete if $H = \cup_{s \in S} A_s$, where S and A_s satisfy the conditions:*

(i) *$(S, \cdot)$ is a semigroup;*
(ii) *for all $(s,t) \in S^2$, where $s \neq t$ we have $A_s \cap A_t = \emptyset$;*
(iii) *if $(a,b) \in A_s \times A_t$, then $a \circ b = A_{s \cdot t}$.*

Suppose that $(H, \cdot)$ and $(H', \circ)$ are semihypergroups. A function $f : H \to H'$ is called a *homomorphism* if $f(a \cdot b) \subseteq f(a) \circ f(b)$ for all a and b in H. We say that f is a *good homomorphism* if for all a and b in H, $f(a \cdot b) = f(a) \circ f(b)$. $(H, \cdot)$ and $(H', \circ)$ are isomorphic if and only if there exists a good homomorphism between them that is also bijective, in this case we write $H \cong H'$.

If $(H, \cdot)$ is a hypergroup and $\rho \subseteq H^2$ is an equivalence, we set

$$A \overline{\overline{\rho}} B \Leftrightarrow a \rho b, \quad \forall a \in A, \forall b \in B,$$

for all pairs (A, B) of nonempty subsets of H^2.

The relation ρ is called *strongly regular on the left* (*on the right*) if $x \rho y \Rightarrow a \cdot x \overline{\overline{\rho}} a \cdot y$ ($x \rho y \Rightarrow x \cdot a \overline{\overline{\rho}} y \cdot a$, respectively), for all $(x, y, a) \in H^3$. Moreover, ρ is called *strongly regular* if it is strongly regular on the right and on the left.

Theorem 1.2. *(Theorem 31, [1]). If $(H, \cdot)$ is a semihypergroup (hypergroup) and ρ is a strongly regular relation on H, then the quotient H/ρ is a semigroup (group) under the operation:*

$$\rho(x) \otimes \rho(y) = \rho(z), \text{ for all } z \in x \cdot y.$$

We denote $\rho(x)$ by $\bar{x}$ and instead of $\bar{x} \otimes \bar{y}$ we write $\bar{x}\bar{y}$.

For all $n > 1$, we define the relation β_n on a semihypergroup H, as follows:

$$a \, \beta_n \, b \Leftrightarrow \exists (x_1, \ldots, x_n) \in H^n \ : \ \{a, b\} \subseteq \prod_{i=1}^{n} x_i,$$

and $\beta = \bigcup_{i=1}^{n} \beta_n$, where $\beta_1 = \{(x, x) \mid x \in H\}$ is the diagonal relation on H. This relation was introduced by Koskas [9] and studied mainly by Corsini [1]. Suppose that β^* is the transitive closure of β. The relation β^* is a strongly regular relation [1]. Also, we have:

Theorem 1.3. *(Freni, [8]). If $(H, \cdot)$ is hypergroup then $\beta = \beta^*$.*

Note that, in general, for a semihypergroup may be $\beta \neq \beta^*$, see the following Example.

Example 1.4. (Remark 82, [1]), Let $(H, \cdot)$ be the following semihypergroup.

$\cdot$	e	a	b	c
e	a, b	a, c	a, c	a, c
a	a, c	a, c	a, c	a, c
b	a, c	a, c	a, c	a, c
c	a, c	a, c	a, c	a, c

Then we have $b\beta^*c$ but not $b\beta c$ hence $\beta \neq \beta^*$.

2. Special soft sets, transitive soft sets over semihypergroups and their derived semihypergroups

Definition 2.1. ([12]). A pair (f, A) is called a soft set over U, where f is a mapping given by $f : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, a may be considered as the set of a-approximate elements of the soft set (f, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [12]. Let $(H, \cdot)$ be a (semi)hypergroup, we denote the set of all subsemihypergroups of $(H, \cdot)$ by $Sub(H)$ and the set of all its subhypergroups by $Subh(H)$. Moreover, by $\mathfrak{U}_H$ or for simplicity $\mathfrak{U}$ we mean the set of all finite product of elements of H.

Definition 2.2. Let $(H, \cdot)$ be a (semi)hypergroup.

(i) A pair (f, A) is called a soft hypergroup over $(H, \cdot)$ if the following implication valid:

$$(\forall a \in A)[f(a) \neq \emptyset \Rightarrow f(a) \in Subh(H)].$$

(ii) A pair (f, A) is called a soft semihypergroup over $(H, \cdot)$ if the following implication valid:

$$(\forall a \in A)[f(a) \neq \emptyset \Rightarrow f(a) \in Sub(H)].$$

Definition 2.3. Let $(H, \cdot)$ be a semihypergroup.

(i) A pair (f, A) is called a special soft set over $(H, \cdot)$, where f is a surjection mapping given by $f : A \to \mathfrak{U}$;

(ii) A secial soft set (f, A) is called transitive if the following implication valid:

$$(\forall (a, a') \in A^2)[f(a) \cap f(a') \neq \emptyset \Rightarrow f(a) \times f(a') \subseteq \cup_{c \in A} f(c)^2].$$

Example 2.4. Let $(S, \cdot)$ be a semigroup. Then (id_S, S) is a transitive soft set over $(S, \cdot)$.

Let us denote S_H and T_H be the set of all special soft sets and the set of all transitive soft sets over $(H, \cdot)$, respectively.

Theorem 2.5. *Let $(H, \cdot)$ be a semihypergroup. Then $T_H = S_H$ if and only if $\beta = \beta^*$.*

Proof. Suppose that $(f, A) \in S_H$ and $x\beta^* x'$, then there exists $(x_0, x_1, ..., x_n) \in H^n$, such that $x_0 \beta x_1 ... x_{n-1} \beta x_n$, where $x_0 = x$ and $x_n = x'$. By induction we prove that $x\beta x'$. For $n = 1$, its obvious, now suppose that the assertion is held for $m < n$. Let $u_i \in \mathfrak{U}$ such that $\{x_{i-1}, x_i\} \subseteq u_i$, by transitivity of f, there exists $(a_1, ... a_n) \in A^n$ such that $f(a_i) = u_i$, for every $1 \leq i \leq n$. Moreover, there exists $c_j \in A$ such that $\{x_{j-2}, x_j\} \subseteq f(c_j)$, for every $2 \leq j \leq n$. Hence $x_0 \beta x_2 ... x_{n-1} \beta x_n$. By hypothesis of induction we conclude that $x\beta x'$. Now suppose that $\beta = \beta^*$. Consider the special soft set (f, A) over $(H, \cdot)$ and suppose that $f(a) \cap f(a') \neq \emptyset$, for some $(a, a') \in A^2$. Hence $x\beta^* x'$, for all $(x, x') \in f(a) \times f(a')$ and consequently $x\beta x'$, that is there exists $c \in A$ such that $\{x, x'\} \subseteq f(c)$. Thus $f(a) \times f(a') \subseteq \cup_{c \in A} f(c)^2$, consequently (f, A) is transitive over $(H, \cdot)$. $\square$

Corollary 2.6. *If $(H, \cdot)$ is a hypergroup, then $T_H = S_H$*

Proposition 2.7. *Let $(H, \circ)$ be a semihypergroup and (f, A) be a special soft set over $(H, \circ)$. Define hyperoperation $\circ_f$ on A by: $a \circ_f a' = f^{-1}(f(a)f(a'))$. Then $(A, \circ_f)$ is a complete semihypergroup and $T_A = S_A$*

Proof. Suppose $(a_1, a_2, a_3) \in A^3$ and for simplicity $\circ_f = \circ$. We have $(a_1 \circ a_2) \circ a_3 = f^{-1}(f(a_1)f(a_2)f(a_3)) = a_1 \circ (a_2 \circ a_3)$. Let $n, m \geq 2$ and $(a_1, a_2, ..., a_n) \in A^n$ and $(a'_1, a'_2, ..., a'_m) \in A^m$. If $a \in a_1 \circ a_2 ... \circ a_n \cap a'_1 \circ a'_2 ... \circ a'_m$ then $f(a) = f(a_1)f(a_2)...f(a_n) =$

$f(a'_1)f(a'_2)...f(a'_m)$. Now let $x \in a_1 \circ a_2... \circ a_n$ so $f(x) = f(a)$ and hence $x \in f^{-1}(f(a'_1)f(a'_2)...f(a'_m)) = a'_1 \circ a'_2... \circ a'_m$ therefore $a_1 \circ a_2... \circ a_n \subseteq a'_1 \circ a'_2... \circ a'_m$. Similarly we have $a'_1 \circ a'_2... \circ a'_m \subseteq a_1 \circ a_2... \circ a_n$. Thus $a_1 \circ a_2... \circ a_n = a'_1 \circ a'_2... \circ a'_m$. Because $(A, \circ)$ is a complete semihypergroup we have $\beta^* = \beta$ and so by the previous theorem $T_A = S_A$ follows. □

Remark 2.8. From now on we name $\mathbb{A} = (A, \circ_f)$ the associated semihypergroup from the special soft set (f, A) over $(H, \circ)$.

Example 2.9. Let $H = \{e, a, b\}$ and $\circ$ be a hyperoperation on H as follows:

$\circ$	e	a	b
e	e	e	H
a	a	a	H
b	b	b	H

It easy to see that (f, A) is a transitive soft set over $(H, \circ)$, where $A = \{0, 1, 2, 3, 4\}$ and $f : A \to \mathfrak{U}$, $f(0) = f(4) = e$, $f(1) = a$ $f(2) = b$, $f(3) = H$. The associated semihypergroup $\mathbb{A} = (A, \circ_f)$ has the following table

$\circ_f$	0	1	2	3	4
0	0, 4	0, 4	3	3	0, 4
1	0, 4	1	3	3	1
2	3	3	3	3	2
3	0, 4	0, 4	3	3	3
4	0, 4	1	2	3	0, 4

Proposition 2.10. *If $\mathbb{A} = (A, \circ_f)$ is the associated semihypergroup from the special soft set (f, A) over $(H, \circ)$ then $\mathbb{A}/\beta^* \cong \mathfrak{U}$.*

Proof. For each $\bar{a} \in \mathbb{A}/\beta^*$ define $\varphi : \mathbb{A}/\beta^* \to \mathfrak{U}$, $\varphi(\bar{a}) = f(a)$. If $\bar{a} = \bar{b}$ then $a\beta b$ therefore $f(a) = f(b)$. Hence φ is a well defined map. Suppose $\{\bar{a}, \bar{b}\} \subseteq \mathbb{A}/\beta^*$, $\varphi(\bar{a}\bar{b}) = \varphi(\bar{c})$, where $c \in a \circ_f b$. Hence $f(c) = f(a)f(b)$ and so $\varphi(\bar{a}\bar{b}) = \varphi(\bar{a})\varphi(\bar{b})$. Now suppose that $\varphi(\bar{a}) = \varphi(\bar{b})$, so there exist $n \in \mathbb{N}$ and $(a_1, ..., a_n) \in A^n$ that $f(a) = f(b) = f(a_1)f(a_2)...f(a_n)$. Hence $a, b \in f^{-1}(f(a_1)f(a_2)...f(a_n))$. Therefore $\bar{a} = \bar{b}$ and so φ is a injection map. Thus $\mathbb{A}/\beta^* \cong \mathfrak{U}$. □

Definition 2.11. Let $\circ$ be a hyperoperation on H such that $x \circ y \neq H$, for all $(x, y) \in H^2$. We define the hyperoperation $\circ^c$ on H as follows:

$$x \circ^c y = H \setminus (x \circ y).$$

From now on we call $\circ^c$, *the complementary hyperoperation* of $\circ$ on H.

Definition 2.12. A (semi)hypergroup $(H, \circ)$ is called *complementary feasible* if $(H, \circ^c)$ is a (semi)hypergroup.

Example 2.13. Let $H = \{0, 1, 2, 3\}$. Then the semihypergroup $(H, \circ)$, where the hyperoperation $(\circ)$ is as follows:

$\circ$	0	1	2	3
0	0	1	2, 3	3
1	1	0, 1	2, 3	2, 3
2	2	2, 3	0, 1	0, 1
3	2, 3	2, 3	0, 1	0, 1

is not a complementary feasible semihypergroup.

Proposition 2.14. *Every nontrivial group is a complementary feasible hypergroup.*

Proof. Suppose $(G, \circ)$ is a nontrivial group. It is easy to see that $(x \circ^c y) \circ^c z = G = x \circ^c (y \circ^c z)$, for every $(x, y, z) \in G^3$. $\square$

Example 2.15. Let $S = \{e, a, b\}$ and $\circ$ be an operation on S as follows:

$\circ$	e	a	b
e	e	e	e
a	e	a	a
b	e	b	b

$(S, \circ)$ is a semigroup which is not complementary feasible semigroup.

Theorem 2.16. *Let $(H, \circ)$ be a semihypergroup that $H = \cup_{s \in S} A_s$, where S and A_s satisfy the conditions:*

(i) *$(S, \cdot)$ is a semigroup;*
(ii) *for all $(s, t) \in S^2$, where $s \neq t$ we have $A_s \cap A_t = \emptyset$;*
(iii) *if $(a, b) \in A_s \times A_t$, then $a \circ b = A_{s \cdot t}$.*

Then $(H, \circ)$ is complementary feasible if and only if $(S, \cdot)$ is complementary feasible.

Proof. Suppose $(S,\cdot)$ is complementary feasible and $(x,y,z) \in A_s \times A_t \times A_u$. Because S is complementary feasible we have $x \circ y \neq H$. Moreover,

$$(x \circ^c y) \circ^c z = (\cup_{l\in s\cdot^c t} A_l) \circ^c z = \cup_{v \in (s\cdot^c t)\cdot^c u} A_v.$$

On the other hand

$$x \circ^c (y \circ^c z) = x \circ^c (\cup_{l \in t\cdot^c u} A_l) = \cup_{v \in s\cdot^c (t\cdot^c u)} A_v.$$

Hence $(H,\circ)$ is complementary feasible. Similarly we have the converse. □

Using Theorem 1.1 and Theorem 2.15 then we have

Corollary 2.17. *Every complete hypergroup is a complementary feasible hypergroup.*

Corollary 2.18. *If $\mathbb{A} = (A, \circ_f)$ is the associated semihypergroup from the special soft set (f,A) over $(H,\circ)$ then $\mathbb{A}/\beta^*$ is a complementary feasible semigroup if and only if $\mathbb{A}$ is a complementary feasible semihypergroup.*

Let (f,A), (g,A) be special soft sets over $(H,\cdot)$. We define the relation $\sim$ on S_H as $f \sim g$ if and only if $f(a) - g(a)$ or $g(a) - f(a)$ is a finite subset of H and moreover, $f(a) \cap g(a) \neq \emptyset$, for all $a \in A$.

Theorem 2.19. *Let (f,A), (g,A) be special soft set over the hypergroup $(H,\circ)$ and $f \sim g$. Then $f \in T_H$ if and only is $g \in T_H$.*

Proof. Suppose that $a \in A$ and $f(a) = \prod_{i=1}^{n} x_i$ and $g(a) = \prod_{i=1}^{m} y_i$ and $f(a) - g(a) = \{b_1, b_2, ..., b_t\}$. Since H is a hypergroup, there exists $(c_1, d_1) \in H^2$ such that $y_m \in c_1 \circ b_1$, $b_1 \in a \circ d_1$, so we have:

$$\begin{aligned}
\prod_{i=1}^{m} y_i &\subseteq \prod_{i=1}^{m-1} y_i \circ\ c_1 \circ b_1 \\
&\subseteq \prod_{i=1}^{m-1} y_i \circ\ c_1 \circ a \circ d_1 \\
&\subseteq \prod_{i=1}^{m-1} y_i \circ\ c_1 \circ \prod_{i=1}^{n} x_i \circ d_1.
\end{aligned}$$

On the other hand,

$$\begin{aligned} b_1 &\in a \circ d_1 \\ &\subseteq \prod_{i=1}^{m-1} y_i \circ \ y_m \circ d_1 \\ &\subseteq \prod_{i=1}^{m-1} y_i \circ \ c_1 \circ b_1 \circ d_1 \\ &\subseteq \prod_{i=1}^{m-1} y_i \circ \ c_1 \circ \prod_{i=1}^{n} x_i \circ d_1. \end{aligned}$$

Denote $\prod_{i=1}^{k_1} z_{1,i} = \prod_{i=1}^{m-1} y_i \circ \ c_1 \circ \prod_{i=1}^{n} x_i \circ d_1$, thus $\{b_1\} \cup \prod_{i=1}^{m} y_i \subseteq \prod_{i=1}^{k_1} z_{1,i}$. Using again that H is a hypergroup, there exists $(c_2, d_2) \in H^2$ such that $z_{1,k_1} \in c_2 \circ b_2$ and $b_2 \in b_1 \circ d_2$. Denote $\prod_{i=1}^{k_2} z_{2,i} = \prod_{i=1}^{k_1-1} z_{1,i} \circ c_2 \circ \prod_{i=1}^{n} x_i \circ d_2$. Similarly as above, $\{b_2\} \cup \prod_{i=1}^{k_1} z_{1,i} \subseteq \prod_{i=1}^{k_2} z_{2,i}$ and hence $\{b_1, b_2\} \cup \prod_{i=1}^{m} y_i \subseteq \prod_{i=1}^{k_2} z_{2,i}$. After t steps, we obtain $\prod_{i=1}^{k_t} z_{t,i}$ such that $\{b_1, b_2, ..., b_t\} \cup \prod_{i=1}^{m} y_i \subseteq \prod_{i=1}^{k_t} z_{t,i}$, thus $f(a) \cup g(a) \subseteq \prod_{i=1}^{k_t} z_{t,i} = f(c_a) = g(c'_a)$, where $(c_a, c'_a) \in A^2$. Now suppose $g(b) \cap g(b') \neq \emptyset$, for some $(b, b') \in A^2$. Therefore there exists $(c_b, c_{b'}) \in A^2$ such that $f(b) \cup g(b) \subseteq f(c_b)$ and $f(b') \cup g(b') \subseteq f(c_{b'})$. Because $f \in T_H$ we have $f(c_b) \times f(c_{b'}) \subseteq \cup_{c \in A} f(c)^2$ thus $g(b) \times g(b') \subseteq \cup_{c \in A} g(c)^2$ that is $g \in T_H$. Similarly the converse holds. □

We recall that a K_H-semihypergroup is a semihypergroup constructed from a semihypergroup $(H, \circ)$ and a family $\{A_x\}_{x \in H}$ of nonempty and mutually disjoint subsets of H. Set $K_H = \bigcup_{x \in H} A_x$ and define the hyperoperation $\otimes$ on K_H as follows,

$$\forall (a, b) \in K_H^2; \ \ a \in A_x, b \in A_y, \quad a \otimes b = \bigcup_{z \in x \circ y} A_z$$

$(H, \circ)$ is a hypergroup if and only if $(K_H, \otimes)$ is a hypergroup (see [2]).

For all $u = \prod_{i=1}^{n} x_i \in \mathfrak{U}$, set $K_u = \prod_{i=1}^{n} y_i$, where $y_i \in A_{x_i}$, for every $1 \leq i \leq n$.

Let (f, A) be special soft set over semihypergroup H we define (K_f, K_A) is a soft set over the semihypergroup K_H, where $\{A(c)\}_{c \in A}$ is a family of nonempty mutually disjoint sets and $K_f : K_A \to \mathfrak{U}_{K_H}$, $K_f(x) = K_{f(c)}$, $x \in A(c)$, for all $x \in K_A = \cup_{c \in A} A(c)$. Then we have the following results.

Proposition 2.20. *$(f, A) \in T_H$ if and only if $(K_f, K_A) \in T_{K_H}$.*

Proof. It is easy to see that $(f, A) \in S_H$ if and only if $(K_f, K_A) \in S_{K_H}$. The rest of the proof follows from the following realities.

(i) If $x \in A(c)$ and $x' \in A(c')$ then $K_f(x) \cap K_f(x') \neq \emptyset$ if and only if $f(c) \cap f(c') \neq \emptyset$;

(ii) if $x \in A(c)$ and $x' \in A(c')$ then $K_f(x) \times K_f(x') \subseteq \cup_{t \in K_A} K_f(t)^2$ if and only if $f(c) \times f(c') \subseteq \cup_{c \in A} f(c)^2$.

Therefore we have the result. □

Proposition 2.21. *(f, A) is a soft (semi)hypergroup if and only if (K_f, K_A) is a soft (semi)hypergroup.*

Theorem 2.22. *Let $(A, \circ_f)$ and $(K_A, \circ_{K_f})$ be the semihypergroups constructed from the special soft sets (f, A) and (K_f, K_A), respectively. Then $(K_A, \circ_{K_f})$ is a constructed semihypergroup from $(A, \circ_f)$.*

Proof. Suppose that $x \in A(a)$ and $y \in A(a')$, where $(a, a') \in A^2$ and $f(a) = \prod_{i=1}^{n} h_i$, $f(a') = \prod_{i=n+1}^{n+m} h_i$, where $(h_1, ..., h_{n+m}) \in H^{n+m}$. Moreover suppose that $K_f(x) = \prod_{i=1}^{n} t_i$, $K_f(y) = \prod_{i=n+1}^{n+m} t_i$, where $t_i \in B(h_i)$, for every i, $1 \leq i \leq n+m$.
Now we have $x \circ_{K_f} y = K_f^{-1}(K_f(x)K_f(y))$. On the other hand

$$x \otimes y = \cup_{w \in a \circ_f a'} A(w) = \cup_{w \in f^{-1}(f(a)f(a'))} A(w).$$

If $z \in x \otimes y$ then $z \in A(w)$, where $w \in f^{-1}(f(a)f(a'))$ therefore $f(w) = f(a)f(a') = \prod_{i=1}^{n+m} h_i$, so $K_f(z) = \prod_{i=1}^{n+m} t_i = K_f(x)K_f(y)$, hence $x \otimes y = x \circ_{K_f} y$. □

Corollary 2.23. *Using Theorem 100 in* [2] *then we have $(K_A, \circ_{K_f})$ is a hypergroup if and only if $(A, \circ_f)$ is a hypergroup.*

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