A NOTE ON VAGUE GRAPHS

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ABSTRACT. In this paper, we introduce the notions of product vague graph, balanced product vague graph, irregularity and total irregularity of any irregular vague graphs and some results are presented. Also, density and balanced irregular vague graphs are discussed and some of their properties are established. Finally we give an application of vague digraphs.

1. INTRODUCTION

The introduction of fuzzy sets by Zadeh [21] in 1965 changed the face of science and technology to a great extent. Fuzzy sets paved the way for a new philosophical thinking of 'Fuzzy Logic' which now, is an essential concept in artificial intelligence. This logic is also used in the production of a large number of electronic and other household items with 'partial' thinking ability. Fuzzy logic and the theory of fuzzy sets have been applied widely in areas like information theory, pattern recognition, clustering, expert systems, database theory, control theory, robotics, networks and nano-technology. The initial definition given by Kaufmann [7] of a fuzzy graphs was based on the fuzzy relation proposed by Zadeh [21, 22, 23]. Later Rosenfeld

[16] introduced the fuzzy analogue of several basic graph-theoretic concepts. Mordeson and Nair [8] defined the concept of complement of fuzzy graph and studied some operations on fuzzy graphs. Gau and Buehrer [6] proposed the concept of vague set in 1993, by replacing the value of an element in a set with a subinterval of $[0, 1]$. Namely, a true-membership function $t_v(x)$ and a false membership function $f_v(x)$ are used to describe the boundaries of the membership degree. Akram et al. [1, 2] introduced vague hypergraphs and certain types of vague graphs. Ramakrishna [10] introduced the concept of vague graphs and studied some of their properties. Borzooei and Rashmanlou introduced ring sum in product intuitionistic fuzzy graphs [3], domination in vague graphs and its applications [4] and degree of vertices in vague graphs [5]. Pal and Rashmanlou [9] studied irregular interval-valued fuzzy graphs. Also, they defined antipodal interval-valued fuzzy graphs [11], balanced interval-valued fuzzy graphs [12], some properties of highly irregular interval-valued fuzzy graphs [13], a study on bipolar fuzzy graphs [14]. Rashmanlou and Jun investigated complete interval-valued fuzzy graphs [15]. Talebi et al. [19, 20] studied isomorphism on vague graphs. Samanta and Pal introduced fuzzy tolerance graphs [17] and fuzzy threshold graphs [18]. In this paper, we introduce the notion of product vague graph, balanced product vague graph, irregularity and total irregularity of any irregular vague graphs and some results are presented. Also, density, balanced irregular vague graphs are discussed and some of their properties are established. Finally, we give an application of vague digraphs.

2. Preliminaries

A graph is an ordered pair $G = (V, E)$, where V is the set of vertices of G and E is the set of edges of G. A subgraph of a graph $G = (V, E)$ is a graph $H = (W, F)$, where $W \subseteq V$ and $F \subseteq E$. A simple graph is an undirected graph that has no loops and not more than one edge between any two different vertices. A simple graph with a single vertex is called trivial graph and one with no edges is called an empty graph. The set of all vertices adjacent to a vertex x in G is called the neighbour set of x, denoted by $N(x)$. A $v_0 - v_n$ path P in G is an alternating sequence of vertices and edges $v_0, e_1, v_1, e_2, \cdots, e_n, v_n$ such that $v_i v_{i+1}$ is an edge for $i = 0, 1, 2, \cdots, n - 1$. The number of edges in P is called the length of P and P is called a closed path or a cycle if $v_0 = v_n$. A fuzzy subset μ on a set X is a map $\mu : X \to [0, 1]$. A map $\nu : X \times X \to [0, 1]$ is called a fuzzy relation on μ if $\nu(x, y) \leq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$. A fuzzy graph is a pair $G = (\sigma, \mu)$, where σ is a fuzzy subset on a set V and μ is a fuzzy relation on σ. It is assumed that V is a finite and non-empty, μ is reflexive and symmetric. Thus, if G is a fuzzy graph then, $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ is such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, for all $u, v \in V$, where $\wedge$ denotes the minimum.

Definition 2.1. [6] A vague set A in an ordinary finite non-empty set X is a pair (t_A, f_A), where $t_A : X \to [0,1]$, $f_A : X \to [0,1]$ are true and false membership functions, respectively such that $0 \le t_A(x) + f_A(x) \le 1$, for all $x \in X$. Note that $t_A(x)$ is considered as the lower bound for degree of membership of x in A and $f_A(x)$ is the lower bound for negative of membership of x in A. So, the degree of membership of x in the vague graph set A is characterized by interval $[t_A(x), 1 - f_A(x)]$. Hence, a vague set is a special case of interval-valued sets studied by many mathematicians and applied in many branches of mathematics.

It is worth to mention here that interval-valued fuzzy sets are not vague sets. In interval-valued fuzzy sets, an interval valued membership value is assigned to each element of the universe considering the evidence for x only, without considering evidence against x. In vague sets both are independently proposed by the decision maker. This makes a major difference in the judgment about the grade of membership. A vague relation is a generalization of a fuzzy relation. Let X and Y be ordinary finite non-empty sets. We call a vague relation to be a vague subset of $X \times Y$, that is an expression R defined by

$$R = \{\langle (x,y),\ t_R(x,y),\ f_R(x,y) \rangle \mid x \in X, y \in Y \}$$

where $t_R : X \times Y \to [0,1]$, $f_R : X \times Y \to [0,1]$, which satisfies the condition $0 \le t_R(x,y) + f_R(x,y) \le 1$, for all $(x,y) \in X \times Y$.

Let $G^* = (V, E)$ be a crisp graph. A pair $G = (A, B)$ is called a vague graph on a crisp graph $G^* = (V, E)$, where $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that $t_B(xy) \le \min(t_A(x), t_A(y))$ and $f_B(xy) \ge \max(f_A(x), f_A(y))$, for each edge $xy \in E$. The underlying crisp graph of a vague graph $G = (A, B)$, is the graph $G = (V', E')$, where $V' = \{v \in V :\ t_A(v) > 0,\ f_A(v) > 0\}$ and $E' = \{\{u, v\} :\ t_B(\{u, v\}) > 0\ and\ f_B(\{u, v\}) > 0\}$. V' is called the vertex set and E' is called the edge set. A vague graph may be also denoted as $G = (V', E')$.

A vague graph G is said to be strong if $t_B(v_i v_j) = \min\{t_A(v_i), t_A(v_j)\}$ and $f_B(v_i v_j) = \max\{f_A(v_i), f_A(v_j)\}$, for every edge $v_i v_j \in E$.

A vague graph G is said to be complete if $t_B(v_i v_j) = \min\{t_A(v_i), t_A(v_j)\}$ and $f_B(v_I v_j) = \max\{f_A(v_i), f_A(v_j)\}$, for all $v_i, v_j \in V$.

Definition 2.2. [2] Let $G = (A, B)$ be a vague graph. Then
(i) the t-degree of a vertex u is $d_t(u) = \sum_{v \in N(u)} t_B(uv)$,
(ii) the f-degree of a vertex u is $d_f(u) = \sum_{v \in N(u)} f_B(uv)$,
(iii) the degree of a vertex u is $d(u) = \left[\sum_{v \in N(u)} t_B(uv), \sum_{v \in N(u)} f_B(uv) \right]$,
(iv) the order of G is defined to be $O(G) = (O_t(G), O_f(G))$, where $O_t(G) = \sum_{u \in V} t_A(u)$ and $O_f(G) = \sum_{u \in V} f_A(u)$,

(v) the size of G is defined to be $S(G) = (S_t(G), S_f(G))$, where $S_t(G) = \sum_{u \neq v} t_B(uv)$ and $S_f(G) = \sum_{u \neq v} f_B(uv)$.

(vi) the total degree of a vertex $u \in V$ is defined as

$$td(u) = [d_t(u) + t_A(u), d_f(u) + f_A(u)].$$

Note that if $d_t(v_i) = k_i$ and $d_f(v_i) = k_j$, for all $v_i, v_j \in V$ then, the graph is called as (k_i, k_j)-vague graph (or) regular vague graph of degree (k_i, k_j). Also, if each vertex of G has the same total degree (r_1, r_2) then, G is said to be a vague graph of total degree (r_1, r_2) or (r_1, r_2)-totally regular vague graph.

The density of a vague graph $G = (A, B)$ is $D(G) = (D_t(G), D_f(G))$, where $D_t(G)$ is defined by

$$D_t(G) = \frac{2 \sum_{u,v \in V} (t_B(uv))}{\sum_{uv \in E} (t_A(u) \wedge t_A(v))}, \text{ for } u, v \in V$$

and $D_f(G)$ is defined by

$$D_f(G) = \frac{2 \sum_{u,v \in V} (f_B(uv))}{\sum_{uv \in E} (f_A(u) \vee f_A(v))}, \text{ for } u, v \in V.$$

A vague graph $G = (A, B)$ is balanced if $D(H) \leq D(G)$, that is, $D_t(H) \leq D_t(G)$ and $D_f(H) \leq D_f(G)$, for all subgraphs H of G.

3. New concepts of balanced vague graphs

Definition 3.1. Let $G = (A, B)$ be a vague graph of a graph $G^* = (V, E)$. If $t_B(xy) \leq t_A(x) \times t_A(y)$ and $f_B(xy) \geq f_A(x) \times f_A(y)$, for all $x, y \in V$ then, the vague graph G is called product vague graph of G^*, where $\times$ represent ordinary multiplication.

Example 3.2. Consider a vague graph G such that $V = \{x, y, z\}$, $E = \{xy, yz, xz\}$. By
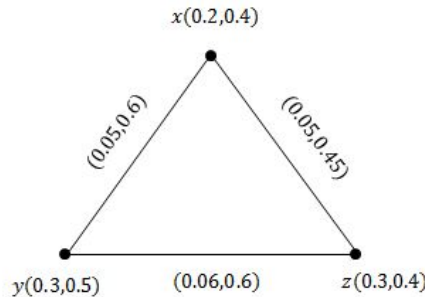

FIGURE 1. Product vague graph G

routine computation, it is easy to show that G is a product vague graph.

Definition 3.3. The density of a product vague graph $G = (A, B)$ is $D(G) = (D_t(G), D_f(G))$, where $D_t(G)$ is defined by

$$D_t(G) = \frac{2 \sum_{u,v \in V} (t_B(uv))}{\sum_{uv \in E} (t_A(u) \cdot t_A(v))}, \text{ for } u, v \in V$$

and $D_f(G)$ is defined by

$$D_f(G) = \frac{2 \sum_{u,v \in V} (f_B(uv))}{\sum_{uv \in E} (f_A(u) \cdot f_A(v))}, \text{ for } u, v \in V.$$

Definition 3.4. A product vague graph $G = (A, B)$ is balanced if $D(H) \leq D(G)$, that is, $D_t(H) \leq D_t(G)$ and $D_f(H) \leq D_f(G)$, for all subgraph H of G.

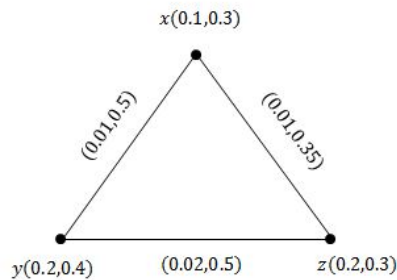**Example 3.5.** Consider a product vague graph G as in Figure 2.

FIGURE 2. Product vague graph G

$$D_t(G) = 2 \left(\frac{0.01 + 0.01 + 0.02}{0.02 + 0.04 + 0.02} \right) = 1,$$

$$D_f(G) = 2 \left(\frac{0.5 + 0.35 + 0.5}{0.12 + 0.09 + 0.12} \right) = 8.18.$$

$$D(G) = (D_t(G), D_f(G)) = (1, 8.18).$$

Definition 3.6. A product vague graph $G = (A, B)$ is said to be complete if $t_B(xy) = t_A(x) \times t_A(y)$ and $f_B(xy) = f_A(x) \times f_A(y)$, for all $x, y \in V$.

Theorem 3.7. *Every complete product vague graph is balanced.*

Proof. Let $G = (A, B)$ be a complete product vague graph then, by the definition we have, $t_B(xy) = t_A(x) \times t_A(y)$ and $f_B(xy) = f_A(x) \times f_A(y)$, for all $x, y \in V$.

$\sum_{x,y \in V}(t_B(xy)) = \sum_{x,y \in V}(t_A(x) \times t_A(y))$ and $\sum_{x,y \in V}(f_B(xy)) = \sum_{x,y \in V}(f_A(x) \times f_A(y))$.

Now

$$
\begin{aligned}
D(G) &= \left(\frac{2\sum_{x,y \in V}(t_B(xy))}{\sum(t_A(x) \times t_A(y))}, \frac{2\sum_{x,y \in V}(f_B(xy))}{\sum(f_A(x) \times f_A(y))} \right) \\
&= 2 \left(\frac{\sum_{x,y \in V}(t_A(x) \times t_A(y))}{\sum_{x,y \in V}(f_A(x) \times f_A(y))}, \frac{\sum_{x,y \in V}(f_A(x) \times f_A(y))}{\sum_{x,y \in V}(f_A(x) \times f_A(y))} \right) = (2, 2).
\end{aligned}
$$

Hence, $D(G) = (2, 2)$. Let H be a non-empty subgraph of G then, $D(H) = (2, 2)$, for every $H \subseteq G$. Thus, G is balanced. ∎

Definition 3.8. The complement of product vague graph $G = (A, B)$ is a vague graph $\overline{G} = (\overline{A}, \overline{B})$, where $\overline{A} = A = (\mu_A, \nu_A)$ and $\overline{B} = (\overline{\mu_B}, \overline{\nu_B})$ is defined by:

$$\overline{\mu_B}(xy) = \mu_A(x) \times \mu_A(y) - \mu_B(xy), \quad \overline{\nu_B}(xy) = \nu_A(x) \times \nu_A(y) - \nu_B(xy).$$

Remark 3.9. It follows that $\overline{G}$ is a product vague graph and $\overline{(\overline{G})} = G$.

Definition 3.10. A product vague graph $G = (A, B)$ is said to be strong product vague graph if $t_B(xy) = t_A(x) \times t_A(y)$ and $f_B(xy) = f_A(x) \times f_A(y)$, for every $(x, y) \in E$.

Theorem 3.11. *The complement of a strong product vague graph is balanced.*

Proof. Let $G = (A, B)$ be a strong product vague graph and $\overline{G} = (\overline{A}, \overline{B})$ be its complement. Since G is strong, $t_B(xy) = t_A(x) \times t_A(y)$ and $f_B(xy) = f_A(x) \times f_A(y)$, for every $(x, y) \in E$. In $\overline{G}$ we have, $\overline{t_B}(xy) = t_A(x) \times t_A(y) - t_B(xy)$ and $\overline{f_B}(xy) = f_A(x) \times f_A(y) - f_B(xy)$.

Since G is strong we have, $\overline{t_B}(xy) = 0$ and $\overline{f_B}(xy) = 0$, for every $xy \in E$ and $\overline{t_B}(xy) = t_A(x) \times t_A(y)$ and $\overline{f_B}(xy) = f_A(x) \times f_A(y)$, for every $xy \in \overline{E}$. So, $\overline{G}$ is a strong product vague graph and it follows that is balanced. ∎

Definition 3.12. [2] A vague graph $G = (A, B)$ is said to be

(i) irregular, if there is a vertex which is adjacent to vertices with distinct degrees,

(ii) totally irregular, if there is a vertex which is adjacent to vertices with distinct total degrees,

(iii) highly irregular, if every vertex of G is adjacent to vertices with distinct degrees.

Definition 3.13. Let $G = (A, B)$ be an irregular vague graph then, the irregularity of G is defined as $Irreg(G) = (Irreg_t(G), Irreg_f(G))$ where $Irreg_t(G) = \sum_{xy \in E} |d_t(x) - d_t(y)|$ and $Irreg_f(G) = \sum_{xy \in E} |d_f(x) - d_f(y)|$, for all $x, y \in V$.

The total irregularity of the irregular vague graph is defined as

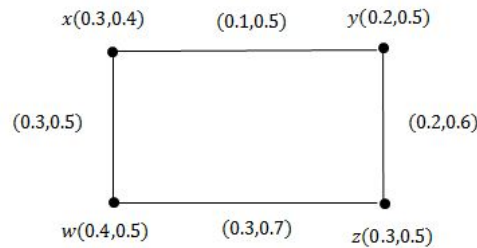$$Irreg_{total}(G) = \frac{1}{2} \sum_{x,y \in V} |d(x) - d(y)|.$$

FIGURE 3.   Irregular vague graph G

Example 3.14. Consider an irregular vague graph G as in Figure 3.   Here, $d(x) =$

$(0.4, 1)$, $d(y) = (0.3, 1.1)$, $d(z) = (0.5, 1.3)$, $d(w) = (0.6, 1.2)$. It is easy to show that $Irreg(G) = (0.1, 0.1) + (0.2, 0.2) + (0.1, 0.1) + (0.2, 0.2) = (0.6, 0.6)$. But the total irregularity is $Irreg_{total}(G) = \frac{1}{2}[(0.1, 0.1) + (0.1, 0.3) + (0.2, 0.2) + (0.3, 0.2) + (0.1, 0.1)] = (0.5, 0.55)$.

Remark 3.15. (i) For any irregular vague graph G, $Irreg_{total}(G) \leq Irreg(G)$.
(ii) Let $G = (A, B)$ be the irregular vague graph which is complete then, $Irreg(G) = 2[Irreg_{total}(G)]$.

Definition 3.16. Let $G = (A, B)$ be an irregular vague graph. The density of G is defined as $D(G) = (D_t(G), D_f(G))$ where $D_t(G) = \dfrac{2\sum_{x,y \in V} t_B(xy)}{\sum_{xy \in E} t_A(x) \wedge t_A(y)}$, for all $x, y \in V$ and $D_f(G) = \dfrac{2\sum_{x,y \in V} f_B(xy)}{\sum_{xy \in E} f_A(x) \vee f_A(y)}$, for all $x, y \in V$.

Example 3.17. In Example 3.14 we have

$$D(G) = (D_t(G), D_f(G)) = \left(\frac{2[0.1 + 0.2 + 0.3 + 0.3]}{0.2 + 0.2 + 0.3 + 0.3}, \frac{2[0.5 + 0.6 + 0.7 + 0.5]}{0.5 + 0.5 + 0.5 + 0.5} \right) = (1.8, 2.3).$$

Theorem 3.18. *Let $G = (A, B)$ be the irregular vague graph with $t_B(xy) = \dfrac{1}{2}(t_A(x) \wedge t_A(y))$ and $f_B(xy) = \dfrac{1}{2}(f_A(x) \vee f_A(y))$ then, $D(G) = (1, 1)$.*

Proof.

$$
\begin{aligned}
D(G) &= \left(\frac{2 \sum_{x,y \in V} t_B(xy)}{\sum_{xy \in E}(t_A(x) \wedge t_A(y))}, \frac{2 \sum_{x,y \in V} f_B(xy)}{\sum_{xy \in E}(f_A(x) \vee f_A(y))} \right), \ \text{for all } x, y \in V \\
&= \left(\frac{2 \sum_{x,y \in V} \frac{1}{2}(t_A(x) \wedge t_A(y))}{\sum_{xy \in E}(t_A(x) \wedge t_A(y))}, \frac{2 \sum_{x,y \in V} \frac{1}{2}(f_A(x) \vee f_A(y))}{\sum_{xy \in E}(f_A(x) \vee f_A(y))} \right) \\
&= \left(\frac{2(\frac{1}{2}) \sum_{x,y \in V}(t_A(x) \wedge t_A(y))}{\sum_{xy \in E}(t_A(x) \wedge t_A(y))}, \frac{2(\frac{1}{2}) \sum_{x,y \in V}(f_A(x) \vee f_A(y))}{\sum_{xy \in E}(f_A(x) \vee f_A(y))} \right) = (1, 1).
\end{aligned}
$$

∎

Theorem 3.19. *Let G be an irregular and complete vague graph then, $D(G) = (2,2)$.*

Proof. Since G is complete and irregular we have: $t_B(xy) = t_A(x) \wedge t_A(y)$ and $f_B(xy) = f_A(x) \vee f_A(y)$, for all $x, y \in V$. Now, $\sum_{x,y \in V} t_B(xy) = \sum_{xy \in E}(t_A(x) \wedge t_A(y))$, also $\sum_{x,y \in V} f_B(xy) = \sum_{xy \in E}(f_A(x) \vee f_A(y))$. Therefore,

$$
\begin{aligned}
D(G) &= \left(\frac{2 \sum_{x,y \in V} t_B(xy)}{\sum_{xy \in E}(t_A(x) \wedge t_A(y))}, \frac{2 \sum_{x,y \in V} f_B(xy)}{\sum_{xy \in E}(f_A(x) \vee f_A(y))} \right), \ \text{for all } x, y \in V \\
&= \left(\frac{2 \sum_{xy \in E}(t_A(x) \wedge t_A(y))}{\sum_{xy \in E}(t_A(x) \wedge t_A(y))}, \frac{2 \sum_{xy \in E}(f_A(x) \vee f_A(y))}{\sum_{xy \in E}(f_A(x) \vee f_A(y))} \right) = (2, 2).
\end{aligned}
$$

∎

Definition 3.20. An irregular vague graph G is said to be balanced if $D_t(H) \leq D_t(G)$ and $D_f(H) \leq D_f(G)$, for all subgraph H of G.

Example 3.21. Consider an irregular vague graph G as in Figure 4.   Here, $D(G) =$

FIGURE 4. Balanced irregular vague graph G

$(D_t(G), D_f(G)) = (1.5, 2)$, and subgraph of G are $H_1 = \{x, y\}$, $H_2 = \{x, z\}$, $H_3 = \{x, w\}$, $H_4 = \{y, z\}$, $H_5 = \{y, w\}$, $H_6 = \{z, w\}$, $H_7 = \{x, y, z\}$, $H_8 = \{x, z, w\}$, $H_9 = \{x, y, w\}$, $H_{10} = \{y, z, w\}$, $H_{11} = \{x, y, z, w\}$.

Now density $(D_t(H), D_f(H))$ is $D(H_1) = (1.5, 2)$, $D(H_2) = (0, 0)$, $D(H_3) = (1.5, 2)$, $D(H_4) = (1.5, 2)$, $D(H_5) = (0, 0)$, $D(H_6) = (1.5, 2)$, $D(H_7) = (1.5, 2)$, $D(H_8) = (1.5, 2)$, $D(H_9) = (1.5, 2)$, $D(H_{10}) = (1.5, 2)$, $D(H_{11}) = (1.5, 2)$. So, $D(H) \leq D(G)$, for all subgraphs H of G. Hence, G is balanced irregular vague graph.

Theorem 3.22. *Let $G = (A, B)$ be an irregular vague graph and all the edges of G are strong then, G is balanced.*

Proof. Since all the edges are strong then $t_B(xy) = t_A(x) \wedge t_A(y)$ and $f_B(xy) = f_A(x) \vee f_A(y)$. Now by Theorem 3.19, $D(G) = (2, 2)$. For the subgraphs H, $(D_t(H), D_f(H)) = (2, 2)$ if the vertices of H having edges, otherwise $(D_t(H), D_f(H)) = (0, 0)$ i.e. $D_t(H) \leq D_t(G)$ and $D_f(H) \leq D_f(G)$ which implies G is balanced. ∎

Theorem 3.23. *Every irregular vague graph which is complete is balanced.*

Proof. Let G be an irregular vague graph and complete then, by Theorem 3.19, $D(G) = (2, 2)$. Let H be a non-empty subgraphs of G and G is complete. Then, all the edges are strong. Therefore, $D(H) = (2, 2)$, for every H in G. Also the subgraphs are totally irregular vague graphs. Hence, G is balanced. ∎

Remark 3.24. The converse need not be true. That is every balanced irregular vague graph need not be complete.

Example 3.25. Figure 4 is balanced irregular vague graph, but it is not complete.

4. Application Example of vague graph

Graph models find wide application in many areas of mathematics, computer science, and the natural and social sciences. Often these models need to incorporate more structure than simply the adjacencies between vertices. In studies of group behavior, it is observed that certain people can influence thinking of others. A directed graph, called an influence graph, can be used to model this behavior. Each person of a group is represented by a vertex. There is a directed edge from vertex x to vertex y, when the person represented by vertex x influence the person represented by vertex y. This graph does not contain loops and it does not contain multiple directed edges. We now explore vague influence graph model to find out the influential person within a social group. In influence graph, the vertex (node) represents a power (authority) of a person and the edge represents the influence of a person on another person in the social group. Consider a vague influence graph of a social group. In Figure 4, vague influence graph, the degree of power of a person is defined in terms of its trueness and falseness. The node of the vague influence graph shows the authority a person possesses in the group; for example, C has 50% authority in the group, but he does not have
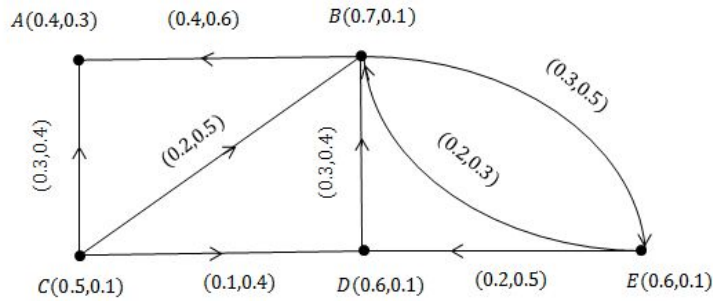
Figure 5. Vague influence graph

10% power, and 10% power is not decided, whereas the edges show the influence of a person on another in a group; for example C can influence B 20%, but he can not convince him 50%, and remaining 30% is hesitation part. The degree of a vertex and edge in a vague influence graph is also characterized by an interval $[t_A(x), 1 - f_A(x)]$. It is worth mentioning here that interval-valued fuzzy sets are not vague sets. In vague sets both are independently proposed by the decision maker. Thus a vague influence graph can be interpreted in the form of interval-valued membership. The node of the vague influence graph shows the likelihood of power a person possesses in the group; for example, C possesses $t_A = 50\%$ to $1 - f_A = 90\%$ power, whereas the edges show the interval of influence a person has on another person in a socia group. C has $t_A = 20\%$ to $1 - f_A = 50\%$ influence on B and B has $t_A = 30\%$ to $1 - f_A = 50\%$ influence on E.

5. Conclusion

Graph theory has several interesting applications in system analysis, operations research, computer applications, and economics. Since most of the time the aspects of graph problems are uncertain, it is nice to deal with these aspects via the methods of fuzzy systems. It is known that fuzzy graph theory has numerous applications in modern science and engineering, neural networks, expert systems, medical diagnosis, town planning and control theory. In this paper, we introduced the notions of product vague graph, balanced product vague graph, irregularity and total irregularity of any irregular vague graphs and some results are presented. Also, density, balanced irregular vague graphs are discussed and some of their properties are established. Finally we gave an application of vague digraphs.

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