Journal of Algebraic Structures and Their Applications
ISSN: 2382-9761
www.as.yazd.ac.ir

Algebraic Structures and Their Applications Vol. 1 No. 1 (2014), pp 11-20.

ENLARGED FUNDAMENTALLY VERY THIN H_v-STRUCTURES

T. VOUGIOUKLIS

Communicated by B. Davvaz

Abstract. We study a new class of H_v-structures called Fundamentally Very Thin. This is an extension of the well known class of the Very Thin hyperstructures. We present applications of these hyperstructures.

1. Fundamentals and the Fundamental Relations

We deal with hyperstructures called *H_v-structures* introduced in 1990 [8], which satisfy the *weak axioms* where the non-empty intersection replaces the equality.

Some basic definitions are the following:

MSC(2010): Primary: 20N20. Secondary: 16Y99.

Keywords: Hyperstructures, H_v-structures, hopes, ∂-hopes.

Received: 31 May 2014, Accepted:25 August 2014.

∗Corresponding author

In a set H equipped with a hyperoperation (abbreviation *hyperoperation*=***hope***)

$$\cdot : H \times H \to P(H) - \{\emptyset\}$$

we abbreviate by *WASS* the *weak associativity*: $(xy)z \cap x(yz) \neq \varnothing,\ \forall x, y, z \in H$ and by *COW* the *weak commutativity*: $xy \cap yx \neq \varnothing,\ \forall x, y \in H$. The hyperstructure $(H, \cdot)$ is called H_v*-semigroup* if it is *WASS*, it is called H_v*-group* if it is reproductive H_v-semigroup, i.e., $xH = Hx = H, \forall x \in H$.

Motivation. We know that the quotient of a group with respect to an invariant subgroup is a group. F. Marty from 1934, states that, the quotient of a group with respect to any subgroup is a hypergroup. Finally, the quotient of a group with respect to any partition (or equivalently to any equivalence relation) is an H_v-group. This is the motivation to introduce H_v-structures [8],[10].

In an H_v-semigroup the powers of an element $h \in H$ are defined as follows: $h^1 = \{h\}, h^2 = h \cdot h, ..., h^n = h \circ h \circ ... \circ h$, where $(\circ)$ denotes the *n-ary circle hope*, i.e. take the union of hyperproducts, n times, with all possible patterns of parentheses put on them. An H_v-semigroup $(H, \cdot)$ is called *cyclic of period s*, if there exists an element g, called *generator*, and a natural number s, the minimum one, such that $H = h^1 \cup h^2 ... \cup h^s$. Analogously the cyclicity for the infinite period is defined. If there is an element h and a natural number s, the minimum one, such that $H = h^s$, then $(H, \cdot)$ is called *single-power cyclic of period s.*

In an a similar way more complicated hyperstructures can be defined:

$(R, +, \cdot)$ is called H_v*-ring* if $(+)$ and $(\cdot)$ are WASS, the reproduction axiom is valid of $(+)$ and $(\cdot)$ is weak distributive with respect to $(+)$:

$$x(y + z) \cap (xy + xz) \neq \varnothing,\ (x + y)z \cap (xz + yz) \neq \varnothing,\ \forall x, y, z \in R.$$

Let $(R, +, \cdot)$ be H_v-ring, $(\mathbf{M}, +)$ be a COW H_v-group and the is an external hope :

$$\cdot : \mathbf{R} \times \mathbf{M} \to \mathcal{P}(\mathbf{M} : (a, x)) \mapsto ax$$

such that $\forall a, b \in R$ and $\forall x, y \in \mathbf{M}$ we have

$$a(x + y) \cap (ax + ay) \neq \emptyset, (a + b)x \cap (ax + bx) \neq \emptyset, (ab)x \cap a(bx) \neq \emptyset$$

then $\mathbf{M}$ is called an H_v-module over F. In the case of an H_v-field F instead of H_v-ring R, then the H_v-vector space is defined.

For more definitions and applications on H_v-structures one can see the books [2],[4],[10] and survey papers as [1],[3],[14].

The main tool to study hyperstructures is the fundamental relation. In 1970 M. Koscas defined in hypergroups the relation β and its transitive closure β^*. β^* is turned to be very important since its connects the hyperstructures with the corresponding classical ones [2],[4],[10] and is also defined in H_v-groups as well. T. Vougiouklis [8],[10] introduced the relations γ^* and ε^*, defined, in H_v-rings and H_v-vector spaces, respectively. He also named all these relations β^*, γ^* and ε^*, *Fundamental Relations* because they play very important role to the study of hyperstructures especially in the representation theory of them.

Definition 1.1. The fundamental relations β^*, γ^* and ε^*, are defined, in H_v-groups, H_v-rings and H_v-vector space, respectively, as the smallest equivalences so that the quotient would be group, ring and vector space, respectively.

Specifying the above motivation we remark the following: Let $(G, \cdot)$ be a group and R be an equivalence relation (or a partition) in G, then $(G/R, \cdot)$ is an H_v-group, therefore we have the quotient $(G/R, \cdot)/\beta^*$ which is a group, the fundamental one. Remark that the classes of the fundamental group $(G/R, \cdot)/\beta^*$ are a union of some of the R-classes. Otherwise, the $(G/R, \cdot)/\beta^*$ has elements classes of G where they form a partition which classes are larger than the classes of the original partition R.

The way to find the fundamental classes is given by the following:

Theorem 1.2. *Let $(\boldsymbol{H}, \cdot)$ be an H_v-group and denote by $\boldsymbol{U}$ the set of all finite products of elements of H. We define the relation β in H by setting $x\beta y$ iff $\{x, y\} \subset \boldsymbol{u}$ where $\boldsymbol{u} \in \boldsymbol{U}$. Then β^* is the transitive closure of β.*

Analogous theorems for the fundamental relations γ^* in H_v-rings, ε^* in H_v-modules and H_v-vector spaces, are also proved. An element is called ***single*** if its fundamental class is singleton [10].

Fundamental relations are used for general definitions. Thus, an H_v-ring $(R, +, \cdot)$ is called H_v***-field*** if R/γ^* is a field. The general definition of an H_v-Lie algebra was given as follows [10],[13],[14]:

Let $(L, +)$ be an H_v-vector space over the H_v-field $(F, +, \cdot)$, $\varphi : F \to F/\gamma^*$ the canonical map and $\omega_F = \{x \in F : \varphi(x) = 0\}$, where 0 is the zero of the fundamental field F/γ^*. Similarly, let ω_L be the core of the canonical map $\varphi' : L \to L/\varepsilon^*$ and denote by the same symbol 0 the zero of L/ε^*. Consider the *bracket (commutator) hope*:

$$[,] : L \times L \to P(L) : (x, y) \to [x, y]$$

then **L** is an H_v*-Lie algebra* over F if the following axioms are satisfied:

(L1) The bracket hope is bilinear, i.e.
$[\lambda_1 x_1 + \lambda_2 x_2, y] \cap (\lambda_1[x_1, y] + \lambda_2[x_2, y]) \neq \varnothing$
$[x, \lambda_1 y_1 + \lambda_2 y_2] \cap (\lambda_1[x, y_1] + \lambda_2[x, y_2]) \neq \varnothing,$
$\forall x, x_1, x_2, y, y_1, y_2 \in L, \lambda_1, \lambda_2 \in F$

(L2) $[x, x] \cap \omega_L \neq \varnothing, \quad \forall x \in L$

(L3) $([x, [y, z]] + [y, [z, x]] + [z, [x, y]]) \cap \omega_L \neq \varnothing, \quad \forall x, y \in L$

Let $(H, \cdot), (H, *)$ be H_v-semigroups defined on the same set H. $(\cdot)$ is called *smaller* than $(*)$, and $(*)$ *greater* than $(\cdot)$, iff there exists an

$$f \in Aut(H, *) \text{ such that } xy \subset f(x * y),\ \forall x, y \in H.$$

Then we write $\cdot \leq *$ and we say that $(H, *)$ contains $(H, \cdot)$. If $(H, \cdot)$ is a structure then it is called *basic structure* and $(H, *)$ is called $H_b - structure$.

Theorem 1.3. ***(The Little Theorem)***. *Greater hopes than the ones which are WASS or COW, are also WASS or COW, respectively.*

This Theorem leads to a partial order on H_v-structures and mainly to a correspondence between hyperstructures and posets. The determination of all H_v-groups and H_v-rings is very interesting but hard. Fortunately, in this direction there are results as one can see, for example, in the paper [1] how many H_v-groups and H_v-rings there exist, up to isomorphism, for small sets, for several classes of hyperstructures, of two, three or four elements.

Using the partial ordering with the fundamental relations one can give several definitions to obtain constructions used in several applications [11]:

Let $(H, \cdot)$ be hypergroupoid. We *remove* $h \in H$, if we consider the restriction of $(\cdot)$ in the set $H - \{h\}$. $\underline{h} \in H$ *absorbs* $h \in H$ if we replace h by $\underline{h}$ and h does not appear in the structure. $\underline{h} \in H$ *merges* with $h \in H$, if we take as product of any $x \in H$ by $\underline{h}$, the union of the results of x with both h, $\underline{h}$, and consider h and $\underline{h}$ as one class with representative $\underline{h}$.

A large class of H_v-structures is the following [13]:

Let $(G, \cdot)$ be groupoid (resp. hypergroupoid) and $f : G \to G$ be a map. We define (∂), called ***theta-hope***, we write ∂***-hope***, on G as follows

$$x\partial y = \{f(x) \cdot y, x \cdot f(y)\},\ \forall x, y \in G. \text{ (resp. } x\partial y = (f(x) \cdot y) \cup (x \cdot f(y)),\ \forall x, y \in G$$

If $(\cdot)$ is commutative then (∂) is commutative. If $(\cdot)$ is COW then (∂) is COW.

Let $(G, \cdot)$ be groupoid (resp., hypergroupoid) and $f : G \to P(G) - \{\varnothing\}$ be any multivalued map. We define the (∂), on G as follows

$$x\partial y = (f(x) \cdot y) \cup (x \cdot f(y)), \forall x, y \in G$$

.

Let $(G, \cdot)$ be groupoid $f_i : G \to G, i \in I$, be a set of maps on G. The

$$f_\cup : G \to P(G) : f_\cup(x) = \{f_i(x) | i \in I\},$$

is the *union* of $f_i(x)$. We have the *union theta-hope* (∂), on G if we take $f_\cup(x)$. If we take $\underline{f} \equiv f \cup (id)$, then we have the *b-*$\partial$*-hope*.

Motivation for the definition of the ∂-hope is the map *derivative* where only the multiplication of functions can be used. The basic property is that if $(G, \cdot)$ is a semigroup then for every f, the (∂) is WASS.

Another well known and large class of hopes is given as follows [10],[12]:

Let $(G,\cdot)$ be a groupoid then for every $P \subset G$, $P \neq \varnothing$, we define the following hopes called P-*hopes*: for all $x, y \in G$

$$\underline{P} : x\underline{P}y = (xP)y \cup x(Py),$$

$$\underline{P}_r : x\underline{P}_r y = (xy)P \cup x(yP),$$

$$\underline{P}_l : x\underline{P}_l y = (Px)y \cup P(xy).$$

The $(G,\underline{P})$,$(G,\underline{P}_r)$ and $(G,\underline{P}_l)$ are called P-*hyperstructures.* The most usual case is if $(G,\cdot)$ is semigroup, then $x\underline{P}y = (xP)y \cup x(Py) = xPy$ and $(G,\underline{P})$ is a semihypergroup but we do not know about $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$. In some cases, depending on the choice of P, the $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$ can be associative or WASS. If more operations are defined in G, then for each operation several P-hopes can be defined.

In 1989 Corsini & Vougiouklis [10], introduced a method to obtain stricter algebraic structures, from given ones, through hyperstructures. This method was introduced before H_v-structures, but in fact the H_v-structures appeared in the procedure. The ***uniting elements*** method is the following: Let G be structure and d be property, which is not valid, and suppose that d is described by a set of equations. Consider the partition in G for which it is put together, in the same class, every pair of elements that causes the non-validity of d. The quotient G/d is an H_v-structure. Then quotient of G/d by the fundamental relation β^*, is a stricter structure $(G/d)\beta^*$ for which d is valid.

2. **Very Thin and Fundamentally Very Thin constructions**

An extreme class is the so called very thin [10]. This class was first introduced by Vougiouklis in [9] (1991) for hypergroups, and then was normally generalized and studied for every H_v-structure [6],[10].

Definition 2.1. An H_v-structure is called **Very Thin** (we rename and abbreviate them by **VT**-H_v-**structure**) iff all hopes are operations except one, which has all hyperproducts singletons except only one, which is a subset of cardinality more than one. Therefore, in a VT-H_v-structure in a set H there exists a hope $(\cdot)$ and a pair $(a,b) \in H^2$ for which $ab = A$, with $cardA > 1$, and all the other products, with respect to any other hopes (so they are operations), are singletons.

Theorem 2.2. *Let $(H,\cdot)$ be a finite VT-H_v-group, $cardA = n > 1$, and let a, b be the only elements of H such that $ab = A$, with $cardA > 1$. Then we have the following two cases:*

(i) *either $\forall u \in H - \{a\}, ua = a$; in which case:*

if $n = 2$, then there exists a group $(H,*)$ such that $a * b \in A$ and $x * y = xy, \forall(x,y) \in H^2 - \{(a,b)\}$,

if $n > 2$, then $a = b$, $H - \{a\}$ is a group and $A = H$ or $A = H - \{a\}$.

(ii) *or if there exists an element $u \in H$ such that $u \neq a$ and $ua \neq a$, then there exists an almost-associative group (i.e. the associativity is valid for all triples except the ones which contain the product $a * b$), such that*

$$a * b \in A \text{ and } x * y = xy, \forall (x, y) \in H^2 - \{(a, b)\}.$$

For a Proof see the book [10] *p.148.*

Since there is a partial order on H_v-groups, we obtain that there are minimals VT-H_v-groups and the rest are greater than the minimals. The types of minimals are described by the following:

Construction 2.3. The non-degenerate (not single valued) minimal VT-H_v-groups $(H, \cdot)$ are the following:

Type I. The non-degenerate minimal VT-H_v-group of Type I, is an H_b-group $(H, *)$ where the basic group is $(H, \cdot)$ and there are elements $a, b, v \in H$ with $ab \neq v$, for which the only one hyperproduct is $a * b = \{ab, v\}$.

Type II. The non-degenerate minimal VT-H_v-group of Type II, is a hypergroup where there is an element $v \in H$ such that $(H - \{v\}, \cdot)$ is a group and $vx = xv = v \forall x \in H - \{v\}$ and $vv = H - \{v\}$.

The Type II appeared first in research but one can see that the Type I is an extremely large class of hyperstructures. Moreover Type I can be applied in more complicated hyperstructures as we can easily see in the following.

The above Construction Type II is referred in H_v-groups. The Type I in fact is referred in weaker hyperstructures like the H_v-semigroups, therefore can be applied in hyperrings in the multiplicative part as well. In [10] one can see several cases of VT-H_v-rings appeared for several classes of hyperrings. The application on H_v-vector spaces and H_v-algebras in any operation internal or external gives also corresponding non-degenerate minimal VT-H_v-structures.

Now we can combine the enlarging theory [11], with the Very Thin one. We can achieve this as follows:

Definition 2.4. An H_v-structure is called **Fundamentally Very Thin** (abbreviate by **FVT-H_v-structure**) if all hopes are greater than the very thin hopes under the assumption that they have the same corresponding fundamental structure

It is easy to see that this class of hyperstructures is turn to be very large. The difficulty to find the fundamental classes can be faced, in some degree, by finding only some elements of several fundamental classes using several hyperproducts.

Special elements and special properties are still valid in the weak form. More properties weak or strong could appear and stricter hyperstructures can be obtained to be used in applied sciences. The most important situation is that the partition yielded connects the hyperstructure theory to the classical one.

The FVT-H_v-structures can be used in the representation theory of H_v-groups by H_v-matrices where the main results are completely related with the ones in the classical theory or the results from the classical theory of representations can be extended in the hyperstructure theory as well. Using FVT-H_v-rings or FVT-H_v-fields remains invariant the basic theorem of the representations. In the following we present the situation:

Representations (abbreviate ***rep***) of H_v-groups can be considered either by generalized permutations, by using the translations, or by H_v-matrices [10],[12],[13],[14]. $\mathbf{H_v}$***-matrix*** is called a matrix if has entries from an H_v-ring. The hyperproduct of H_v-matrices is defined in a usual manner. The problem of the H_v-matrix reps is the following:

Let $(H,\cdot)$ be an H_v-group. Find an H_v-ring $(R,+,\cdot)$, a set M_R=$\{(a_{ij})|a_{ij}{\in}R\}$, and a map

$$\mathbf{T}: H \to \mathbf{M}_R : h \to T(h) \text{ such that } \ T(h_1h_2) \cap T(h_1)T(h_2) \neq \varnothing, \forall h_1, h_2 \in H,$$

The map T is called $\mathbf{H_v}$-***matrix rep***.

If $T(h_1h_2) \subset T(h_1)T(h_2), \forall h_1, h_2 \in H$, then $\boldsymbol{T}$ is an *inclusion rep*. If $T(h_1h_2) = T(h_1)T(h_2) = \{T(h)|h \in h_1h_2\}, \forall h_1, h_2 \in H$, then $\boldsymbol{T}$ is called *good rep* and then an induced representation T^* for the hypergroup algebra is obtained. If T is one to one and good then it is a *faithful rep*. The problem of reps is complicated because the cardinality of the product of H_v-matrices is very big. But it can be simplified in special cases such as the following:

(a) In H_v-matrices over H_v-rings with 0 and 1 and if these are scalars.
(b) In H_v-matrices over VT-H_v-rings.
(c) In of 2×2 H_v-matrices, since the circle hope is the hyperaddition.
(d) In H_v-rings in which the strong associativity in hyperaddition is valid.
(e) In H_v-rings which contains singles, then these act as absorbings.

The main theorem of reps is the following [10],[12]:

Theorem 2.5. *A necessary condition in order to have an inclusion rep T of an H_v-group $(H,\cdot)$ by $n\times n$ H_v-matrices over the H_v-ring $(R,+,\cdot)$ is the following: $\forall \beta^*(x)$, $x \in H$ there must exist elements $a_{ij} \in H, i,j \in \{1,...,n\}$ such that*

$$T(\beta^*(a)) \subset \{A = (a'_{ij})|a'_{ij} \in \gamma^*(a_{ij}), i,j \in \{1,...,n\}\}$$

*So every inclusion rep $T: H \to M_R : a \mapsto T(a) = (a_{ij})$ induces an homomorphic rep T^* of the group H/β^* over the ring R/γ^* by setting $T^*(\beta^*(a)) = [\gamma^*(a_{ij})], \forall \beta^*(a) \in H/\beta^*$, where the $\gamma^*(a_{ij}) \in R/\gamma^*$ is the ij entry of the matrix $T^*(\beta^*(a))$. Then T^* is called **fundamental induced rep** of T.*

Remark. This theorem is completely valid for every FVT-H_v-ring for an obvious reason, this is why the FVT-H_v-rings are useful since they can represent more classes of H_v-groups.

Denote $tr_\varphi(T(x)) = \gamma^*(T(x_{ii}))$ the fundamental trace, then the mapping

$$X_T : H \to R/\gamma^* : x \mapsto X_T(x) = tr_\varphi(T(x)) = trT^*(x)$$

is called *fundamental character*. There are several types of traces.

For an attached H_v-field $(\underline{H_o}, +, \cdot)$, in $\Sigma a_{ik} \cdot b_{kj}$ the terms $a_{ik} \cdot b_{kj}$ could be $0, v, x$ or H (where $x \in H$). But any sum is only 0 or v or H. Thus, for finite H_v-fields $(\underline{H_o}, +, \cdot)$, if the set H appears in t entries then the cardinality of the hyperproducts is $(cardH)^t$.

3. Applications

During last decades hyperstructures seem to have a variety of applications not only in other branches of mathematics but also in many other sciences. These applications range from biomathematics - conchology, inheritance- and hadronic physics to mention but a few. The hyperstructures theory is closely related to fuzzy theory; consequently, hyperstructures can now be widely applicable in industry and production, too.

In several books and papers, such as [2],[4],[5],[7],[10],[14],[15], one can find numerous applications.

3.1. **The e-hyperstructures.** The Lie-Santilli theory on *isotopies* was born in 1970s to solve Hadronic Mechanics problems [5]. Santilli proposed a "lifting" of the n-dimensional trivial unit matrix of a normal theory into a nowhere singular, symmetric, real-valued, positive-defined, n-dimensional new matrix. The original theory is reconstructed such as to admit the new matrix as left and right unit. The *isofields* needed in this theory correspond into the hyperstructures were introduced by Santilli and Vougiouklis in 1996 and they are called *e-hyperfields*. The H_v-fields of the above types, can give e-hyperfields which can be used in the isotopy theory in applications as in physics or biology. We present in the following the main definitions and results restricted in the H_v-structures.

Definitions. A hyperstructure $(H, \cdot)$ which contain a unique scalar unit e, is called e-hyperstructure. In an e-hyperstructure, we assume that for every element x, there exists an inverse x^{-1}, i.e. $e \in x \cdot x^{-1} \cap x^{-1} \cdot x$.

A hyperstructure $(F, +, \cdot)$, where $(+)$ is an operation and $(\cdot)$ is a hope, is called ***e-hyperfield*** if the following axioms are valid: $(F, +)$ is an abelian group with the additive unit 0, $(\cdot)$ is WASS, $(\cdot)$ is weak distributive with respect to $(+)$, 0 is absorbing element: $0 \cdot x = x \cdot 0 = 0, \forall x \in F$, exists a multiplicative scalar unit 1, i.e. $1 \cdot x = x \cdot 1 = x, \forall x \in F$, for every $x \in F$ there exists a unique inverse x^{-1}, such that $1 \in x \cdot x^{-1} \cap x^{-1} \cdot x$. The elements of an e-hyperfield are called *e-hypernumbers*. In the case that the relation: $1 = x \cdot x^{-1} = x^{-1} \cdot x$, is valid, then we say that we have a *strong e-hyperfield*.

A general construction is *the Main e-Construction*: Given a group $(G, \cdot)$, where e is the unit, then we define in G, a large number of hopes $(\otimes)$ as follows:

$$x \otimes y = \{xy, g_1, g_2, ...\}, \forall x, y \in G - \{e\}, \text{ and } g_1, g_2, ... \in G - \{e\}$$

g_1, g_2,... are not necessarily the same for each pair (x,y). Then $(G, \otimes)$ becomes an H_v-group, in fact is H_b-group which contains the $(G, \cdot)$, which is an e-hypergroup. Moreover, if for each x,y such that $xy = e$, so we have $x \otimes y = xy$, then $(G, \otimes)$ becomes a strong e-hypergroup.

Example. Consider the finite quaternion group $Q = \{1, -1, i, -i, j, -j, k, -k\}$ whose multiplication $(\cdot)$ is the ordinary one. Using $(\cdot)$ we obtain several hopes which define very interesting e-groups. Denoting $\underline{i} = \{i, -i\}, \underline{j} = \{j, -j\}, \underline{k} = \{k, -k\}$ we may define, for example the $(*)$ hope, which is a FVT by the table:

$*$	1	-1	i	-i	j	-j	k	-k
1	1	-1	i	-i	j	-j	k	-k
-1	-1	1	-i	i	-j	j	$\underline{k}$	k
i	i	-i	-1	1	k	-k	-j	j
-i	-i	$\underline{i}$	1	-1	-k	k	j	-j
j	j	-j	-k	k	-1	1	i	-i
-j	-j	j	k	-k	1	-1	-i	i
k	k	$\underline{k}$	$\underline{j}$	-j	-i	i	-1	1
-k	-k	k	-j	j	i	-i	1	-1

The rep theory and the Lie algebras as well as in hypermatrix theory large classes of e-hyperstructures appear and can offer to Lie-Santilli algebraic theory models to represent their theory.

3.2. **The Type I hyperstructures. Minimal cases.** As the uniting elements procedure does, the minimal very thin hopes can lead to stricter structures by a quotient. Normally one can apply these procedures with intention to obtain structures with more properties. However, the procedure can be applied with no intension. The only one suggestion is to apply "small" hopes because the quotients could be degenerate. In order to avoid degenerate cases the very thin case, as well as the FVT, is the suggested one. This is a construction analogous to the one given in the book [10] p.75.

Construction 3.1. Consider the additive group of integers $(\mathbf{Z}, +)$ and we take the minimal very thin hope defined by enlarging the only one result of the elements 1 and $n-1$ setting

$$1 \oplus (n-1) = \{0, n\} \text{ and } x \oplus y = x + y \text{ in all the other cases.}$$

One can prove that we have $(\mathbf{Z}, \oplus)/\beta^* \cong (\mathbf{Z}_n, +)$.

Therefore the fundamental group is of finite order, which is completely different from the initial H_v-group, which is of infinitive order. Remark that the reason to enlarge only this result instead of the one in the book, where the enlarged result is the one of the elements 0 and n, is because this construction gives an e-construction.

3.3. **Hyperstructures in social sciences: questionnaires.** In Social Sciences, where questionnaires are used, seem to have hyperstructure theory as an organized devise. An important new application, which combines hyperstructure theory and fuzzy theory, is to replace in questionnaires the scale of Likert by the bar of Vougiouklis & Vougiouklis. The suggestion is the following [15]:

Definition 3.2. "In every question substitute the Likert scale with 'the bar' whose poles are defined with '0' on the left end, and '1' on the right end:

0 ________________ 1

The subjects/participants are asked instead of deciding and checking a specific grade on the scale, to cut the bar at any point s/he feels expresses her/his answer to the specific question".

The use of the bar of Vougiouklis & Vougiouklis instead of a scale of Likert has several advantages during both the filling-in and the research processing. The final suggested length of the bar, according to the Golden Ratio, is 6.2cm. The hyperstructure theory, especially minimal construction, gives innovating new suggestions to connect several groups of objects. These suggestions are obtained from properties and special elements inside the hyperstructure.

References

[1] R. Bayon, N. Lygeros,*Advanced results in enumeration of hyperstructures,* J. Algebra, 320 (2008) 821-835.

[2] P. Corsini, V. Leoreanu, *Application of Hyperstructure Theory,* Kluwer Academic Pub., 2003.

[3] B. Davvaz, *A brief survey of the theory of H_v-structures,* 8th AHA, Greece, Spanidis (2003), 39-70.

[4] B. Davvaz, V. Leoreanu, *Hyperring Theory and Applications,* International Academic Press, 2007.

[5] B. Davvaz, R.M Santilli and T. Vougiouklis, *Studies of multivalued hyper-structures for the characterization of matter-antimatter systems and their extension*, Algebras, Groups and Geometries 28 (3) (2011) 05–116.

[6] C. Gutan, *Proprietes des semi-groupes, Applicationes a l'etude des hyperstructures tres fines*, These Docteur dUniversite, Universite Blaise Pascal, 1994.

[7] T. Vougiouklis, *On representations of algebraic multivalued structures,* Rivista Mat. Pura Appl., N.7, 1990, 87-92.

[8] T. Vougiouklis *The fundamental relation in hyperrings. The general hyperfield,* 4thAHA, Xanthi 1990, World Scientific (1991), 203-211.

[9] T. Vougiouklis, *The very thin hypergroups and the S-construction*, Combinatorics 88, Incidence Geometries Comb. Str., 2, 1991, 471-477.

[10] T. Vougiouklis,*Hyperstructures and their Representations,* Monographs in Mathematics, Hadronic, 1994.

[11] T. Vougiouklis, *Enlarging H_v-structures*, Algebras and Combinatorics, ICAC97, Hong Kong, Springer Verlag, 1999, 455-463.

[12] T. Vougiouklis, *On H_v-rings and H_v-representations,* Discrete Mathematics, Elsevier, 208/209 (1999), 615-620.

[13] T. Vougiouklis, *∂-operations and H_v-fields,* Acta Mathematica Sinica, English S., vol. 23, 6 (2008), 965-972.

[14] T. Vougiouklis, T*he Lie-hyperalgebras and their fundamental relations*, Southeast Asian Bulletin of Mathematics, V.37(4), 2013, 601-614.

[15] T. Vougiouklis, P. K.ambaki-Vougioukli, *On the use of the bar*, China- USA Business Review, Vol.10, No. 6, 2011, 484-489.

T. Vougiouklis

Democritus University of Thrace, School of Science of Education, 68100 Alexandroupolis, Greece

Email: tvougiou@eled.duth.gr