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Research Paper

LEFT φ-BIPROJECTIVITY OF SOME CLASSES OF ABSTRACT SEGAL ALGEBRAS

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Abstract. In this paper, we investigate left φ-biprojectivity of Segal algebras and abstract Segal algebras. We show that for some abstract Segal algebras with some mild conditions left φ-biprojectivity is equivalent with left φ-contractibility. Also we characterize left φ-biprojectivity of a Segal algebra $S(G)$ in the terms of compactness of G, where G is a locally compact group. We introduce a class of abstract Segal algebras among Triangular Banach algebras. We show that some abstract Segal algebras related to triangular Banach algebras are not biprojective.

1. Introduction and preliminaries

Studying the structure of Banach algebras through the homology concepts was established by Helemskii. In fact he introduced the notions of biflatness and biprojectivity for Banach algebras. A Banach algebra A is called biprojective if there exists a bounded linear A-bimodule

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morphism $\rho : A \to A \otimes_p A$ which $\pi_A \circ \rho = id_A$, where $A \otimes_p A$ is denoted for the projective tensor product of A with A and $\pi_A : A \otimes_p A \to A$ is given for product morphism specified by $\pi_A(a \otimes b) = ab$ for all $a, b \in A$. It is shown that every biprojective Banach algebra with identity is contractible, that is, there exists $M \in A \otimes_p A$ such that $a \cdot M = M \cdot a$ and $\pi_A(M)a = a$, for all $a \in A$, see [9].

Recently Hu *et al.* defined the notion of contractibility related to a multiplicative linear functional for Banach algebras. In fact a Banach algebra A with respect to a multiplicative linear functional φ is called left φ-contractible, if there exists an element $m \in A$ such that $am = \varphi(a)m$ and $\varphi(m) = 1$ for all $a \in A$. Left φ contractibility of the Fourier algebras, measure algebra and abstract Segal algebras were studied, see [5] and [7].

Motivated by these considerations, author with M. Rostami and A. Pourabbas defined a notion of left φ-biprojectivity for Banach algebras. Indeed A Banach algebra A is left φ-biprojective if there exists a bounded linear map $\rho : A \to A \otimes_p A$ such that $a \cdot \rho(b) = \rho(ab) = \varphi(b)\rho(a)$ and $\varphi \circ \pi_A \circ \rho(a) = \varphi(a)$ for all $a \in A$. Group algebras and the Fourier algebras were studied under left φ-biprojectivity. Indeed the group algebra $L^1(G)$ is left φ-biprojective if and only if G is compact and the Fourier algebra $A(G)$ is left φ-biprojective if and only if G is discrete [10].

In this paper, we study the left φ-biprojectivity of abstract Segal algebras. We study the relation of left φ-contractibility and left φ-biprojectivity for abstract Segal algebras in the present of φ-inner amenability. We also give a new class of abstract Segal algebras in the class of matrix algebras. We show that this class of abstract Segal algebra is not left φ-biprojective, for some multiplicative linear functional φ.

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2. Left φ-biprojectivity and left φ-contractibility of abstract Segal algebras

In this paper, $\Delta(A)$ is denoted for the set of all non-zero multiplicative linear functionals.

In this section, we study the relation of left φ-biprojectivity and left φ-contractibility of abstract Segal algebras.

Lemma 2.1. [11, Lemma 2.1] *Let A be a Banach algebra and $\varphi \in \Delta(A)$. If A is left φ-contractible, then A is left φ-biprojective.*

Theorem 2.2. *Suppose that A is a Banach algebra which has a left approximate identity and $\varphi \in \Delta(A)$. Let B be an abstract Segal algebra with respect to A. Then B is left $\varphi|_B$-biprojective if and only if B is left $\varphi|_B$-contractible.*

Proof. Suppose that B is left φ-biprojective. So there is a bounded linear map $\rho : B \to B \otimes_p B$ which satisfies $\varphi \circ \pi_B \circ \rho(b) = \varphi(b)$ for all $b \in B$. Let i_0 and R_{i_0} be same as in the proof of Theorem 2.6. Here I is denoted for the inclusion map from B into A. Set

$$\lambda := (I \otimes I) \circ \rho \circ R_{i_0} : A \to A \otimes_p A.$$

Clearly λ is a bounded linear map with the following properties:

$$\begin{aligned} \varphi \circ \pi_A \circ \lambda(a) &= \varphi \circ \pi_A \circ (I \otimes I) \circ \rho \circ R_{i_0}(a) \\ &= \varphi \circ \pi_B \circ \rho \circ R_{i_0}(a) \\ &= \varphi(a), \quad (a \in A), \end{aligned} \tag{1}$$

$$\begin{aligned} b_1 \cdot \lambda(b_2) = b_1 \cdot (I \otimes I) \circ \rho \circ R_{i_0}(b_2) &= (I \otimes I) \circ \rho \circ R_{i_0}(b_1 b_2) \\ &= \varphi(b_2) \circ (I \otimes I) \circ \rho \circ R_{i_0}(b_1) \\ &= \varphi(b_2)\lambda(b_1), \quad (b_1, b_2 \in B). \end{aligned} \tag{2}$$

Let $K = \ker \varphi$ (in A). Here id_A is denoted for the identity map and $q : A \to \frac{A}{K}$ is denoted for the quotient map. Set $\zeta := (id_A \otimes q) \circ \lambda : A \to A \otimes_p \frac{A}{K}$. It is clear that ζ is a bounded linear map. Since A posses a left approximate identity, we have $\overline{AK}^{||\cdot||} = K$. Hence for each $k \in K$, we have

$$\begin{aligned} \zeta(k) = (id_A \otimes q) \circ \lambda(k) &= (id_A \otimes q) \circ \lambda(\lim_n a_n k_n) \\ &= \lim_n \varphi(k_n)(id_A \otimes q) \circ \lambda(a_n) = 0, \end{aligned} \tag{3}$$

where (a_n) is sequence in A and $(k_n)$is a sequence K. Therefore ζ gives a map on $\frac{A}{K}$ which we denote it again by ζ. Using $\frac{A}{K} \cong \mathbb{C}$, follows that $A \otimes_p \frac{A}{K} \cong A$. Hence we may assume that $m = \zeta(i_0 + K) \in A$. Consider

$$bm = b\zeta(i_0 + K) = \zeta(bi_0 + K) = \zeta(\varphi(b)i_0 + K) = \varphi(b)m, \quad (b \in B) \tag{4}$$

also

$$(\varphi \otimes \overline{\varphi}) \circ \lambda(b) = \varphi \circ \pi_B \circ \rho(b) = \varphi(b), \quad (b \in B)$$

and $\widetilde{\varphi} \circ (id_A \otimes \overline{\varphi}) = \varphi \otimes \overline{\varphi}$, where $\overline{\varphi}$ is a character on $\frac{A}{K}$ given by $\overline{\varphi}(a + K) = \varphi(a)$ for each $a \in A$. Using these facts follow that

$$\begin{aligned}\varphi(m) = \varphi\circ\zeta(i_0+K) &= \varphi\circ(id_A\otimes q)\circ\lambda(i_0)\\ &= (\varphi\otimes\overline{\varphi})\circ\lambda(i_0)\\ &= \varphi\circ\pi_B\circ\rho(i_0)\\ &= \varphi(i_0) = 1.\end{aligned} \tag{5}$$

Using the density of B in A and by (4) we have $am = \varphi(a)m$ for all $a\in A$. It gives that A is left φ-contractible. Replacing m with mi_0, we may assume that $m\in B$. Thus B is left $\varphi|_B$-contractible. By Lemma 2.3, we have B is left $\varphi|_B$-biprojective.

Converse is true by Lemma 2.1. □

As an easy consequence of above theorem we have the following result by considering $B = A$.

Lemma 2.3. *Suppose that A is a left φ-biprojective Banach algebra with a left approximate identity. Then A is left φ-contractible.*

A linear subspace $S(G)$ of $L^1(G)$ is said to be a Segal algebra on G, if it satisfies the following conditions:

- (i) $S(G)$ is dense in $L^1(G)$,
- (ii) $S(G)$ with a norm $||\cdot||_{S(G)}$ is a Banach space and $||f||_{L^1(G)} \leq ||f||_{S(G)}$ for every $f\in S(G)$.
- (iii) For $f\in S(G)$ and $y\in G$, we have $L_y(f)\in S(G)$ and the map $y\mapsto L_y(f)$ from G into $S(G)$ is continuous, where $L_y(f)(x) = f(y^{-1}x)$.
- (iv) $||L_y(f)||_{S(G)} = ||f||_{S(G)}$ for every $f\in S(G)$ and $y\in G$.

For various examples of Segal algebras, we refer the reader to [8]. It is well-known that $S(G)$ always has a left approximate identity. For a Segal algebra $S(G)$ it has been shown that

$$\Delta(S(G)) = \{\varphi_{|_{S(G)}} | \varphi\in\Delta(L^1(G))\},$$

see [1, Lemma 2.2]. Also it is known that each Segal algebra is an abstract Segal algebra with respect to $L^1(G)$. But the converse is not true. For instance, if G is an infinite compact group, then $L^\infty(G)$ is an abstract Segal algebra with respect to $L^1(G)$ but $L^\infty(G)$ is not a Segal algebra, see [12, Example 4.8].

Corollary 2.4. *For a locally compact group G, the following statements are equivalent:*

- (i) *$L^1(G)$ is left φ-biprojective.*
- (ii) *Each Segal algebra $S(G)$ is left φ-biprojective.*
- (iii) *There exists a left φ-biprojective Segal algebra.*

(iv) *G is compact.*

Proof. $(i) \Rightarrow (ii)$ Suppose that $L^1(G)$ is left φ-biprojective. Since $L^1(G)$ always have a bounded approximate identity, Lemma 2.3 follows that $L^1(G)$ is left φ-contractible. Now applying [1, Theorem 3.3] gives that $L^1(G)$ is left φ-contractible. By Lemma 2.1 $S(G)$ is left φ-contractible.

$(ii) \Rightarrow (iii)$ is clear.

$(iii) \Rightarrow (iv)$ Since $S(G)$ always has a left approximate identity, by Lemma 2.3 $S(G)$ is left φ-contractible. Using [1, Theorem 3.3] gives that G is compact.

$(iv) \Rightarrow (i)$ is clear by [1, Theorem 3.3]. $\square$

Let A be a Banach algebra with norm $||\cdot||_A$. We recall that a Banach algebra B with norm $||\cdot||_B$ is called an abstract Segal algebra with respect to A if

(i) B is a dense left ideal in A,
(ii) There exists $M > 0$ such that $||b||_A \leq M||b||_B$ for every $b \in B$,
(iii) There exists $C > 0$ such that $||ab||_B \leq C||a||_A||b||_B$ for every $a \in A$ and $b \in B$.

The character space $\Delta(B) = \{\varphi|_B : \varphi \in \Delta(A)\}$, see [1, Lemma 2.2]. For more information about the abstract Segal algebras, see [4].

As an easy consequence of previous corollary and Theorem 2.2 we have the following result:

Theorem 2.5. *Let G be a locally compact group and let $S(G)$ be a Segal algebra. Suppose that B is an arbitrary abstract Segal algebra with respect to $S(G)$ and $\varphi \in \Delta(S(G))$. Then B is φ-biprojective if and only if G is compact.*

Theorem 2.6. *Suppose that A is a Banach algebra with $\varphi \in \Delta(A)$. Let B be an abstract Segal algebra with respect to A which posses an approximate identity. Then the following statements are equivalent:*

(i) *A is left φ-biprojective,*
(ii) *B is left $\varphi|_B$-biprojective,*
(iii) *B is left $\varphi|_B$-contractible,*
(iv) *A is left φ-contractible.*

Proof. $(i) \Rightarrow (ii)$ Let A be left φ-biprojective. So there is a bounded liner map $\Gamma : A \to A \otimes_p A$ such that $\Gamma(ab) = a \cdot \Gamma(b) = \varphi(b)\Gamma(a)$ and $\varphi \circ \pi_A \circ \Gamma(a) = \varphi(a)$, for all $a, b \in A$. We know that B is dense in A. Thus we pick i_0 in B such that $\varphi(i_0) = 1$. Define $R_{i_0} : A \to B$ by $R_{i_0}(a) = ai_0$, for each $a \in A$. It is easy to se that R_{i_0} is a bounded linear map. Set

$$\rho := R_{i_0} \otimes R_{i_0} \circ \Gamma|_B : B \to B \otimes_p B.$$

Clearly ρ is a bounded linear map which satisfies

$$\rho(b_1 b_2) = b_1\rho(b_2) = \varphi(b_2)\rho(b_1), \quad (b_1, b_2 \in B),$$

$$\varphi|_B \circ \pi_B \circ \rho(b_1) = \varphi|_B \circ \pi_B \circ R_{i_0} \otimes R_{i_0} \circ \Gamma|_B(b_1) = \varphi \circ \pi_A \circ \Gamma(b_1) = \varphi(b_1).$$

It gives that B is left $\varphi|_B$-biprojective.

$(ii) \Rightarrow (iii)$ Since B has a left approximate identity, it is true by Lemma 2.3.

$(iii) \Rightarrow (iv)$ [1, Proposition 2.3] finishes this part.

$(iv) \Rightarrow (i)$ We have it by Lemma 2.1. □

A Banach algebra A with $\varphi \in \Delta(A)$ is called φ-inner contractible, if there is an element a_0 in A such that $aa_0 = a_0a$ and $\varphi(a_0) = 1$, for all $a \in A$. Note that each commutative Banach algebra A with $\varphi \in \Delta(A)$ is φ-inner contractible.

Proposition 2.7. *Let A be a Banach algebra and $\varphi \in \Delta(A)$. Suppose that A is φ-inner contractible. Let B be an abstract Segal algebra with respect to A. Then B is left $\varphi|_B$-biprojective if and only if B is left $\varphi|_B$-contractible.*

Proof. Let B be left φ-biprojective. Suppose that R_{i_0}, I, q, id_A, ρ, λ, and ζ are the same as in the proof of Theorem 2.6. We know that A is φ-inner contractible. So there is an element a_0 in A such that $aa_0 = a_0a$ and $\varphi(a_0) = 1$, for all $a \in A$. Define $m = \zeta(a_0) \in A \otimes \frac{A}{K} \cong A$. Consider

$$\begin{aligned}
&bm - \varphi(b)m \\
&= b(id_A \otimes q) \circ (I \otimes I) \circ \rho \circ R_{i_0}(a_0) - \varphi(b)(id_A \otimes q) \circ (I \otimes I) \circ \rho \circ R_{i_0}(a_0) \\
&= (id_A \otimes q) \circ (I \otimes I) \circ \rho \circ R_{i_0}(ba_0) - (id_A \otimes q) \circ (I \otimes I) \circ \rho \circ R_{i_0}(a_0 b) \\
&= (id_A \otimes q) \circ (I \otimes I) \circ \rho \circ R_{i_0}(ba_0 - a_0 b) = 0, \quad (b \in B).
\end{aligned}$$

Also

$$\varphi(m) = \varphi \circ (id_A \otimes q) \circ (I \otimes I) \circ \rho \circ R_{i_0}(a_0) = \varphi \circ \pi_B \circ \rho \circ R_{i_0}(a_0) = \varphi(a_0) = 1.$$

So m in A such that $bm = \varphi(b)m$ and $\varphi(m) = 1$, for all $b \in B$. Since B is dense in A, for all $a \in A$ we have $am = \varphi(a)m$ and $\varphi(m) = 1$. Therefore A is left φ-contractible. Applying [1, Proposition 2.5] gives that B is left $\varphi|_B$-contractible.

Converse is true by Lemma 2.1. □

3. Applications to a class of abstract Segal algebras related to Triangular Banach algebras

One of the most important Banach algebras among matrix algebras are Triangular Banach algebras. Suppose that A and B are Banach algebras and let X be a Banach (A,B)-module, that is, X is a Banach space, a left A-module and a right B-module with the compatible module action which satisfies $(a\cdot x)\cdot b = a\cdot(x\cdot b)$ and $||a\cdot x\cdot b||\leq ||a||||x||||b||$ for every $a\in A, x\in X, b\in B$. It is easy to see that with the usual matrix operations and $||\left(\begin{array}{cc} a & x\\ 0 & b\end{array}\right)||_{T} = ||a||_{A}+||x||_{X}+||b||_{B}$ as the norm, the matrix space $T=\left(\begin{array}{cc} A & X\\ 0 & B\end{array}\right)$ becomes a Banach algebra. These matrix algebras are called Triangular Banach algebra. Let $\varphi\in\Delta(B)$. Then the map $\psi_{\varphi}:T\rightarrow\mathbb{C}$ defined by $\psi(\left(\begin{array}{cc} a & x\\ 0 & b\end{array}\right))=\varphi(b)$ is a multiplicative linear functional on T for all $a\in A, x\in X, b\in B$. Some cohomological properties of Triangular Banach algebras were studied in [2] and [3].

Lemma 3.1. *Let A be a Banach algebra and B be an abstract Segal algebra with respect to A. Then $T_{B}=\left(\begin{array}{cc} B & B\\ 0 & B\end{array}\right)$ is an abstract Segal algebra with respect to $T_{A}=\left(\begin{array}{cc} A & A\\ 0 & A\end{array}\right)$.*

Proof. Suppose that a_1,a_2 and a_3 are arbitrary elements in A and also suppose that b_1,b_2 and b_3 are arbitrary elements in B. Since B is a left ideal of A, so

$$\left(\begin{array}{cc} a_1 & a_2\\ 0 & a_3\end{array}\right)\left(\begin{array}{cc} b_1 & b_2\\ 0 & b_3\end{array}\right)=\left(\begin{array}{cc} a_1b_1 & a_1b_2+a_2b_3\\ 0 & a_3b_3\end{array}\right)\in T_{B}.$$

Moreover, the density of B in A gives that T_B is dense in T_A. Also

$$\begin{aligned} ||\left(\begin{array}{cc} b_1 & b_2\\ 0 & b_3\end{array}\right)||_{T_A} = ||b_1||_A+||b_2||_A+||b_3||_A &\leq M(||b_1||_B+||b_2||_B+||b_3||_B)\\ &= M||\left(\begin{array}{cc} b_1 & b_2\\ 0 & b_3\end{array}\right)||_{T_B}, \end{aligned}$$

for some $M > 0$. Finally,

$$\begin{aligned}
||\begin{pmatrix} a_1 & a_2 \\ 0 & a_3 \end{pmatrix}\begin{pmatrix} b_1 & b_2 \\ 0 & b_3 \end{pmatrix}||_{T_B} &= ||\begin{pmatrix} a_1b_1 & a_1b_2 + a_2b_3 \\ 0 & a_3b_3 \end{pmatrix}||_{T_B} \\
&= ||a_1b_1||_B + ||a_1b_2 + a_2b_3||_B + ||a_3b_3||_B \\
&\leq ||a_1b_1||_B + ||a_1b_2||_B + ||a_2b_3||_B + ||a_3b_3||_B \\
&\leq C(||a_1||_A||b_1||_B + ||a_1||_A||b_2||_B + ||a_2||_A||b_3||_B + ||a_3||_A||b_3||_B) \\
&\leq C||\begin{pmatrix} a_1 & a_2 \\ 0 & a_3 \end{pmatrix}||_{T_A}||\begin{pmatrix} b_1 & b_2 \\ 0 & b_3 \end{pmatrix}||_{T_B},
\end{aligned}$$

for some $C > 0$. Thus T_B is an abstract Segal algebra of T_A. □

Corollary 3.2. *Let G be a locally compact group. Suppose that $S(G)$ is an arbitrary Segal algebra with respect to $L^1(G)$ with $\varphi \in \Delta(S(G))$. Let B be an abstract Segal algebra with respect to $S(G)$. Then T_B is not ψ_φ-biprojective.*

Proof. We assume in contradiction that T_B is ψ_φ-biprojective. Since $S(G)$ has a left approximate identity, $T_{S(G)}$ also has an approximate identity. Applying previous lemma and Theorem 2.2 follows that T_B is left ψ_φ-contractible. So there exists an element $m \in T_B$ such that $am = \psi_\varphi(a)m$ and $\psi_\varphi(a)m$ and $\psi_\varphi(m) = 1$, for all $a \in T_B$. Define $I = \begin{pmatrix} 0 & B \\ 0 & B \end{pmatrix}$. It is easy to see that I is a closed ideal of T_B which $\psi_\varphi|_I \neq 0$. Since T_B is left ψ_φ-contractible , by [7, Proposition 3.8] I is left ψ_φ-contractible. Then there is a $m = \begin{pmatrix} 0 & b_2 \\ 0 & b_3 \end{pmatrix}$ in I (where b_2 and $b_3 \in B$) such that

$$\begin{pmatrix} 0 & a_2 \\ 0 & a_3 \end{pmatrix}\begin{pmatrix} 0 & b_2 \\ 0 & b_3 \end{pmatrix} = \psi_\varphi(\begin{pmatrix} 0 & a_2 \\ 0 & a_3 \end{pmatrix})\begin{pmatrix} 0 & b_2 \\ 0 & b_3 \end{pmatrix} = \varphi(a_3)\begin{pmatrix} 0 & b_2 \\ 0 & b_3 \end{pmatrix}, \quad (a_1, a_2 \in B)$$

and

$$\psi_\varphi(m) = \psi_\varphi(\begin{pmatrix} 0 & b_2 \\ 0 & b_3 \end{pmatrix}) = \varphi(b_3) = 1.$$

Pick i_0 in B which $\varphi(i_0) = 1$. Let $a_2 = i_0$ and $a_3 = 0$ and put a_2 and a_3 at above equation. So we have $i_0b_3 = 0$. Apply φ on this fact we have $\varphi(b_3) = 0$ which is a contradiction. Therefore T_B is not left ψ_φ-biprojective. □

It is easy to see that every biprojective Banach algebra A with $\varphi \in \Delta(A)$ is left φ-biprojective. Thus we have the following result:

Theorem 3.3. *Let G be a locally compact group and let $S(G)$ be a Segal algebra. Suppose that B is an arbitrary abstract Segal algebra with respect to $S(G)$ and $\varphi \in \Delta(S(G))$. Then T_B is not biprojective.*

Remark 3.4. Note that using the same arguments as in the proof of Corollary 3.2, we can show that for every commutative Banach algebra A with $\varphi \in \Delta(A)$ (for example the Fourier algebra on a locally compact group) and for every abstract Segal algebra with respect to A, T_B is not ψ_φ-biprojective.

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