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# SOME FINITE GROUPS WITH DIVISIBILITY GRAPH CONTAINING NO TRIANGLES

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Abstract. Let $G$ be a finite group. The graph $D(G)$ is a divisibility graph of $G$. Its vertex set is the non-central conjugacy class sizes of $G$ and there is an edge between vertices $a$ and $b$ if and only if $a|b$ or $b|a$. In this paper, we investigate the structure of the divisibility graph $D(G)$ for a non-solvable group with $\sigma^*(G) = 2$, a finite simple group $G$ that satisfies the one-prime power hypothesis, a group of type$(A)$,$(B)$ or $(C)$ and certain metacyclic $p-$groups and a minimal non-metacyclic $p-$group where $p$ is a prime number. We will show that the divisibility graph $D(G)$ for all of them has no triangles.

## 1. Introduction

Assigning a graph to a group is one of the interesting tools in investigating the structure of a group. There are some graphs related to finite groups and this graphs have been widely studied. See, for instance [3, 4, 5, 6, 7, 12, 18, 21]. In the following, we recall some important graphs associated with an arbitrary non-empty subset $X$ of positive integers.

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Let $X$ be a set of positive integers and $X^* = X \setminus \{1\}$. In [19], the author introduced two graphs related to $X$. The common divisor graph for $X$ is denoted by $\Gamma(X)$. The vertex set for $\Gamma(X)$ is $V(\Gamma(X)) = X^*$ and the edge set for $\Gamma(X)$ is $E(\Gamma(X)) = \{\{x, y\}; gcd(x, y) \neq 1 : x, y \in X^*\}$, where $gcd(x, y)$ is the greatest common divisor of $x$ and $y$. The prime vertex graph for $X$ is denoted by $\Delta(X)$ that its vertex set is $V(\Delta(X)) = \rho(X) = \cup_{x\in X}\pi(x)$, where $\pi(x) = \{p; p \ is\ a\ prime, p|x\}$ and the edge set for $\Delta(X)$ is $E(\Delta(X)) = \{\{p, q\}; pq|x, x \in X\}$. The connection between these two graphs has been clarified by [16] which defines a bipartite graph $B(X)$. Iranmanesh and Praeger in [16] introduced the bipartite divisor graph $B(X)$ for $X$, that has as vertex set the disjoint union $\rho(X) \cup X^*$ and its edge set is $E(B(X)) = \{\{p, x\}; p \in \rho(X), x \in X^* : p|x\}$.

In [10] A. R. Camina and R. D. Camina introduced a new graph. This graph is called divisibility graph $\overrightarrow{D}(X)$ for a set of positive integers $X$. Its vertex set is $V(\overrightarrow{D}(X)) = X^*$ and the edge set is $E(\overrightarrow{D}(X)) = \{(x, y); x, y \in X^*, x|y\}$. This graph is not strongly connected, so we consider simple graph $D(X)$ instead of $\overrightarrow{D}(X)$. In [1] the authors showed some relations between the structure of $D(X)$, and the structure of known graphs $\Gamma(X)$, $\Delta(X)$ and $B(X)$.

All groups considered here are finite groups. Let $G$ be a finite group and $x$ an element of $G$. $x^G$ denotes the $G$-conjugacy class containing $x$, $|x^G|$ denotes the size of $x^G$ and $cs(G) = \{|x^G|; x \in G\}$ denotes the set of $G$-conjugacy class sizes. Also we consider $D(G)$ instead of $D(cs(G))$. For $a$,$b$ in $G$ we denote by $[a, b] = a^{-1}b^{-1}ab$, the commutator of $a$ and $b$ and $a^b = b^{-1}ab$. We denote by $G^{'}$, the derived subgroup of $G$ and let $G$ be a finite $p-$group and for any positive integer $i$, $K_i(G) = \{g \in G : |G : C_G(g)| = p^i\}$. $K_n$ is the complete graph with $n$ vertices.

In this paper, we investigate the structure of the divisibility graph $D(G)$ for some finite groups such as:

*i)* a non-solvable group with $\sigma^*(G) = 2$, where $\sigma^*(G) = max\{|\pi(g^G)| : g \in G\}$.
*ii)* a finite simple group $G$ that satisfies the one-prime power hypothesis.
*iii)* a group of type$(A)$,$(B)$ or $(C)$.
*iv)* certain metacyclic $p-$groups where $p$ is a prime number.
*v)* some non-metacyclic $p-$groups where $p$ is a prime number.
*vi)* a minimal non-metacyclic $p-$group where $p$ is a prime number.

We will observe that the divisibility graph $D(G)$ for all of them has no triangles.

## 2. Preliminaries

In this section, we state some preliminary results which are necessary for the proofs. In the following theorems we need the definition of a metacyclic group. A group $G$ is called metacyclic, if it contains a normal cyclic subgroup whose quotient is also cyclic.

**Theorem 2.1.** [22] *Let $G$ be a metacyclic $2-$group. Then $G$ has one presentation of the following three kinds:*

*I) Groups*

$$G = \langle a, b : a^{2^{v+t'+u+1}} = 1, b^{2^{t+1}} = a^{2^{v+t'+1}}, a^b = a^{-1+2^{v+u+1}} \rangle,$$

*where $v, t, t', u$ are non-negative integers with $u \leq 1$, $t' \leq 1$, $tt' = tv = ut' = 0$, and if $t + u + v = 0$, then $t' = 0$.*

*II) Ordinary metacyclic 2-groups*

$$G = \langle a, b : a^{2^{r+s+u}} = 1, b^{2^{r+s+t}} = a^{2^{r+s}}, a^b = a^{1+2^r} \rangle,$$

*where $r, s, t, u$ are non-negative integers with $r \geq 2$ and $u \leq r$.*

*III) Exceptional metacyclic 2-groups*

$$G = \langle a, b : a^{2^{r+s+v+t'+u}} = 1, b^{2^{r+s+t}} = a^{2^{r+s+v+t'}}, a^b = a^{-1+2^{r+v}} \rangle,$$

*where $r, s, v, t, t', u$ are non-negative integers with $r \geq 2$, $t' \leq r$, $u \leq 1$, $tt' = sv = tv = 0$, and if $t' \geq r - 1$, then $u = 0$.*

*Groups of different types or of the same type but with different values of parameters are not isomorphic to each other. Furthermore, a Type I group $G \neq C_2 \times C_2$ is split if and only if $(t, u) \neq (0, 1)$; a Type II group is split if and only if either $s = 0$, or $t = 0$, or $u = 0$; a Type III group is split if and only if $u = 0$.*

**Theorem 2.2.** [14] *Let $G$ be a metacyclic $p-$group, where $p$ is an odd prime number, $|G'| = p^n \neq 1$. Then, for $i = 1, ..., n$, we have*

$$|K_i(G)| = |Z(G)|(p^{2i} - p^{2i-2})$$

*thus $G$ contains precisely $|Z(G)|(p^i - p^{i-2})$ conjugacy classes of length $p^i$.*

In [8] N. Blackburn has proved the following theorem:

**Theorem 2.3.** [8] *Let $G$ be a group of order $p^n$, where $p$ is an odd prime number, and suppose that $G$ has rank $d \leq 2$. Then one of the following hold:*

i) *$G$ is metacyclic.*

ii) *$G = \langle x, y, z : x^p = y^p = z^{p^{n-2}} = [x, z] = [y, z] = 1, x^{-1}yx = yz^{p^{n-3}} \rangle$ and $n \geq 3$.*

iii) *$G = \langle x, y, z : x^p = y^p = z^{p^{n-2}} = [y, z] = 1, x^{-1}yx = yz^{sp^{n-3}}, x^{-1}zx = yz \rangle$ and $n \geq 4$. Moreover either $s = 1$, or $s$ is a quadratic non-residue mod $p$.*

iv) *$G$ is a $3-$group of maximal class.*

**Lemma 2.4.** [11] *Let $G$ be a non-solvable group with $\sigma^*(G) = 2$, where*

$$\sigma^*(G) = max\{|\pi(g^G)|; g \in G\}.$$

*Then $G = A \times S$, where $A$ is abelian and $S$ is isomorphic to either $PSL_2(4)$ or $PSL_2(8)$.*

**Theorem 2.5.** [20] *Let $G$ be a finite group such that $B(G)$ is a cycle. Then $G = A \times S$, where $A$ is abelian, and $S \cong SL_2(q)$, $q = 4, 8$. Consequently $B(G)$ is a cycle if and only if $B(G)$ is the 6-cycle $2-12-3-15-5-20$ and $G \cong A \times SL_2(4)$, or is the 6-cycle $2-72-3-63-7-56-2$ and $G \cong A \times SL_2(8)$, where $A$ is abelian.*

**Theorem 2.6.** [1] *If $deg(v) \leq 2$ for every vertex $v$ of $B(X)$, then $D(X)$ is acyclic.*

**Lemma 2.7.** [20] *Let $G$ be a finite simple group. Then $G$ satisfies the one-prime power hypothesis if and only if $G \cong SL_2(4)$ or $SL_2(8)$.( A finite group $G$ satisfies the one-prime power hypothesis, if $m, n \in cs^*(G)$ ($cs^*(G) = cs(G) \setminus \{1\}$), then either $gcd(m, n) = 1$ or $gcd(m, n)$ is a power of a prime.)*

We recall the definition of three relevant families of groups $G$ that we need in the next theorem.

**Definition 2.8.** [9] Let $P$ be a $p$-group , $Q$ a $q$-group and $R$ an $r$-group, for distinct primes $p, q$ and $r$.

Type($A$): Let

$$G = P \rtimes (Q \times R)$$

with $P$ and $Q$ abelian, $r = 2$, $Z(G) = O_2(G)$, $\frac{G}{Z(G)}$ a Frobenius group and $\frac{R}{Z(G)} \simeq Q_8$.

Type($B$): Let

$$G = (P \times R) \rtimes Q$$

with $P$ and $Q$ abelian, $\frac{G}{Z(G)}$ a Frobenius group and $|Cs^*(R)| = 1$.

Type($C$): Let

$$G = R \rtimes PQ$$

with $R = C_G(R)$ minimal normal in $G$ and $PQ \leq \Gamma L(1, R)$ a Frobenius group.

**Theorem 2.9.** [9] *For a group $G$, $B(G) = P_5$( where $P_5$ is a path of length 5) if and only if, up to factoring out a subgroup $Z_0 \leq Z(G)$ with $Z_0 \cap G' = 1$, $G$ is a group of Type(A), (B) or (C).*

## 3. The investigation of divisibility graph for some finite groups

In this section, we consider some finite groups and investigate the structure of divisibility graph $D(G)$ for them. The divisibility graph $D(G)$ for all of them has no triangles.

**Theorem 3.1.** *Let $G$ be one of the following groups:*

*i) A non-solvable group with $\sigma^*(G) = 2$, where $\sigma^*(G) = max\{|\pi(g^G)| : g \in G\}$.*

*ii) A finite simple group and G satisfies the one-prime power hypothesis.*

*iii) A group of type(A),(B) or (C). Then the divisibility graph $D(G)$ has no triangles.*

*Proof.* $(i)$: Let $G$ be a non-solvable group with $\sigma^*(G) = 2$. Now by Lemma 2.4, $G \cong A \times S$, where $A$ is abelian and $S \cong PSL_2(q)$ where $q \in \{4, 8\}$. So by Theorem 2.5, $B(G)$ is a cycle. Thus $deg(v) = 2$ for every vertex $v$ of $B(G)$. Hence the divisibility graph $D(G)$ is acyclic by Theorem 2.6. Therefore the divisibility graph $D(G)$ has no triangles.

$(ii)$: Let $G$ be a finite simple group and $G$ satisfies the one-prime power hypothesis, therefore by Lemma 2.7, we have $G \cong SL_2(q)$ where $q \in \{4, 8\}$. Now according to [[2], Theorem 2.1] and $PSL_2(q) \cong SL_2(q)$ where $q$ is even, the divisibility graph $D(G)$ has no triangles.

$(iii)$: According to the hypothesis of theorem, we have that $G$ is one of the groups of type$(A)$, $(B)$ or $(C)$. By Theorem 2.9, $B(G) = P_5$. So $deg(v) \leq 2$ for every $v \in V(B(G))$. Therefore by Theorem 2.6, the divisibility graph $D(G)$ is acyclic. Thus the divisibility graph $D(G)$ contains no triangles. □

**Theorem 3.2.** *Let G be a metacyclic 2−group as in Theorem 2.1(I) or a metacyclic p−group, where p is an odd prime number and $|G'| = p^n$, where n is a positive integer and $n < 3$, then the divisibility graph $D(G)$ contains no triangles.*

*Proof.* We consider two cases for $p$. The first case $p = 2$ and the other case $p \neq 2$. For the first case, assume a non-abelian group $G$ has order $2^n$. According to Theorem 2.1(I) and by Satz I.14.9 of [15], $G$ is isomorphic to exactly one of the following:

*i)* $G = \langle a, b : a^{2^{n-1}} = 1, b^2 = a^{2^{n-2}}, b^{-1}ab = a^{-1}\rangle \cong Q_{2^n}$, where $n \geq 3$.

*ii)* $G = \langle a, b : a^{2^{n-1}} = b^2 = 1, b^{-1}ab = a^{-1}\rangle \cong D_{2^n}$, where $n \geq 3$.

*iii)* $G = \langle a, b : a^{2^{n-1}} = b^2 = 1, b^{-1}ab = a^{-1+2^{n-2}}\rangle \cong SD_{2^n}$, where $n \geq 4$.

*iv)* $G = \langle a, b : a^{2^{n-1}} = b^2 = 1, b^{-1}ab = a^{1+2^{n-2}}\rangle$, where $n \geq 4$.

Now we consider case $(i)$, in this case, $|G| = 2^n$. It is easy to see that $Z(Q_{2^n}) = \{1, b^2\}$, then $Q_{2^n}$ has two central conjugacy classes. Since $< a > \leq C_{Q_{2^n}} < a >$ and $a^j b a^{-j} = a^{2j} b$ and $a^j aba^{-j} = a^{2j+1} b$ for $0 \leq j \leq 2^{n-2} - 1$ , then the non central conjugacy classes of $Q_{2^n}$ are as the following:

*i)* $\{a^i, a^{-i}\}$, $1 \leq i \leq 2^{n-2} - 1$.

*ii)* $\{a^{2j} b : 0 \leq j \leq 2^{n-2} - 1\}$.

*iii)* $\{a^{2j+1} b : 0 \leq j \leq 2^{n-2} - 1\}$.

So $cs(Q_{2^n}) = \{1, 2, 2^{n-2}\}$, where $n$ is a positive integer such that $n \geq 3$. For case $(ii)$, $|G| = 2^n$. According to[[17], page108] , the non central conjugacy classes of $D_{2^n}$ are as the following:

*i)* $\{a^i, a^{-i}\}$, $1 \leq i \leq 2^{n-2} - 1$.

*ii)* $\{a^i b : 0 \leq i \leq 2^{n-1} - 1, 2 \nmid i\}$ and $(b)^G = \{a^i b : 0 \leq i \leq 2^{n-1} - 1, 2 \mid i\}$.

So $cs(D_{2^n}) = \{1, 2, 2^{n-2}\}$, where $n$ is a positive integer such that $n \geq 3$. For case $(iii)$, $|G| = 2^n$. Similar to the above cases $cs(G) = \{1, 2, 2^{n-2}\}$, where $n$ is a positive integer such that $n \geq 4$.

For cases $(i)$ and $(ii)$ if $n = 3$ , then $D(G) = K_1$. If $n \geq 4$, then $D(G) = K_2$. For case $(iii)$, we have $D(G) = K_2$.

For case $(iv)$, this group is of order $2^n$. Since $|G^{'}| = 2$, then $cs(G) = \{1, 2\}$ Now according to the definition of the divisibility graph $D(G)$, $V(D(G)) = \{2\}$, therefore $D(G) = K_1$.
Now we consider the second case, where $p \neq 2$. Let $G$ be a metacyclic $p-$group and $|G^{'}| = p^n \neq 1$. According to Theorem 2.2, for $i = 1, ..., n$, $G$ contains precisely $|Z(G)|(p^i - p^{i-2})$ conjugacy classes of length $p^i$. Therefore $cs(G) = \{1, p^i : i = 1, ..., n\}$. Now according to $n < 3$ and by the definition of the divisibility graph $D(G)$, we have $V(D(G)) = \{p, p^2\}$. So $D(G) = K_2$. □

Now we will investigate the structure of the divisibility graph $D(G)$ where $G$ is a non-metacyclic $p-$group as in Theorem 2.3. It is worth mentioning that we turn to case $(iv)$ in Theorem 2.3. Then $p = 3$, and G has maximal class. The structure of $G$ is described in Satz III.14.17 of [15]: $G$ is metabelian , and $G_1 := C_G(K_2(G)) \diagup K_4(G)$ is metacyclic of class $c \leq 2$. Moreover, $G$ is not an exceptional group of maximal class, in the sense of Definition III.14.5 of [15]. Now in case $(iii)$ in the next theorem, we consider the case where $G_1$ is abelian.

**Theorem 3.3.** *Suppose that $G$ is a non-metacyclic $p-$group of order $p^n$ and rank $d \leq 2$ where $p$ is an odd prime number. Moreover, suppose that $G_1 := C_G(K_2(G)) \diagup K_4(G)$ is abelian. Then $D(G) \cong K_1$ or $K_2$.*

*Proof.* Let $G$ be a non-metacyclic $p-$group of order $p^n$ and rank $d \leq 2$. Now according to Theorem 2.3, $G$ is one of the following groups:

$i)$ $G = \langle x, y, z : x^p = y^p = z^{p^{n-2}} = [x, z] = [y, z] = 1, x^{-1}yx = yz^{p^{n-3}} \rangle$ and $n \geq 3$.

$ii)$ $G = \langle x, y, z : x^p = y^p = z^{p^{n-2}} = [y, z] = 1, x^{-1}yx = yz^{sp^{n-3}}, x^{-1}zx = yz \rangle$ and $n \geq 4$. Moreover either $s = 1$, or $s$ is a quadratic non-residue mod $p$.

$iii)$ $G$ is a $3-$group of maximal class, with $n \geq 4$.

For the first case, by [[14], proposition 5.2$(ii)$], $G$ contains precisely $p^{n-2}$ conjugacy classes of length 1 and precisely $p^{n-1} - p^{n-3}$ conjugacy classes of length $p$. So $cs(G) = \{1, p\}$, then according to the definition of the divisibility graph $D(G)$, we have $V(D(G)) = \{p\}$. Therefore $D(G) = K_1$. Now for case $(ii)$, according to [[14], proposition 5.3$(ii)$], $G$ contains precisely $p^{n-3}$ conjugacy classes of length 1, precisely $p^{n-2} - p^{n-4}$ conjugacy classes of length $p$, and precisely $p^{n-2} - p^{n-3}$ conjugacy classes of length $p^2$. Therefore $cs(G) = \{1, p, p^2\}$, so by the definition of the divisibility graph $D(G)$, we have $V(D(G)) = \{p, p^2\}$. Then $D(G) = K_2$.

Finally for the last one, we have two different cases. Let $G$ as in [[14], proposition 5.4], then according to [[14], proposition 5.4($ii$)], $G$ contains precisely $p$ conjugacy classes of length 1, precisely $p^{n-2}-1$ conjugacy classes of length $p$, and precisely $p^2-p$ conjugacy classes of length $p^{n-2}$. So for $n \geq 4$, $cs(G) = \{1, p, p^{n-2}\}$. Then according to the definition of the divisibility graph $D(G)$, we have $V(D(G)) = \{p, p^{n-2}\}$. Therefore $D(G) = K_2$. □

For the next theorem, we need the definition of the minimal non-metacyclic $p-$group. A $p$-group $G$ is a minimal non-metacyclic $p-$group when $G$ is not metacyclic but all of its proper subgroups are metacyclic.

**Theorem 3.4.** *Suppose that $G$ is a minimal non-metacyclic $p-$group where $p$ is a prime number. Then the divisibility graph $D(G)$ has no triangles.*

*Proof.* Now suppose that $G$ is a minimal non-metacyclic $p-$group. By [[8], Theorem 3.2], $G$ is one of the following groups:

$i$) any group of order $p^3$ and exponent $p$.

$ii$) a group of order $3^4$ and class 3.

$iii$) the direct product $C_2 \times Q_8$.

$iv$) the central product $Q_8 * C_4$ of order $2^4$.

$v$) the group of order $2^5$ and class 2, $Z(G) = \Omega_1(G) = \Phi(G)$ is of order 4.

For the first case, if $G$ is abelian, then D(G) is a null graph. If $G$ is non-abelian, then $|Z(G)| = |G^{'}| = p$. Since $|x^G| \leq |G^{'}|$ for $x \in G$, then $cs(G) = \{1, p\}$. Now according to the definition of the divisibility graph $D(G)$, we have $V(D(G)) = \{p\}$. So $D(G) = K_1$.

For case ($ii$), since $G$ is a minimal non-metacyclic $3-$group, then $G$ is a non-metacyclic $3-$group. By [[14], proposition 5.4(ii)], we have $cs(G) = \{1, 3, 3^2\}$. Then $D(G) = K_2$.

For case ($iii$), since $Q_8$ has five conjugacy classes and $C_2$ is an abelian group, so $G \cong C_2 \times Q_8$ has the same conjugacy classes sizes as $Q_8$. Therefore $cs(G) = \{1, 2\}$. Thus $V(D(G)) = \{2\}$ and we have $D(G) = K_1$. For case ($iv$), $G$ is a non-metacyclic $2-$group of order $2^4$. According to [[13], Theorem 4.5], $D_{16}$, $Q_{16}$, and $SD_{16}$ are the only groups of order $2^4$ that $|G^{'}| = 2^2$. By Theorem 2.1 these groups are metacyclic $2-$groups. Therefore the other non-abelian groups of order $2^4$ have $|G^{'}| = 2$. So for this case, we have $|G^{'}| = 2$. Since $|x^G| \leq |G^{'}|$ for $x \in G$, therefore $cs(G) = \{1, 2\}$. Thus according to the definition of the divisibility graph $D(G)$, we have $D(G) = K_1$. For the last one, since $G^{'} = Z(G)$, then $|x^G| \leq 4$. It is easy to see that $cs(G) = \{1, 2, 2^2\}$. Now by the definition of the divisibility graph $D(G)$, we have $D(G) = K_2$.

□

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