Journal of Algebraic Structures and Their Applications
ISSN: 2382-9761
www.as.yazd.ac.ir

Algebraic Structures and Their Applications Vol. 3 No. 2 (2016) pp 71-79.

THE REMOTENESS OF THE PERMUTATION CODE OF THE GROUP U_{6n}

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Communicated by Saeid Alikhani

Abstract. Recently, a new parameter of a code, referred to as the remoteness, has been introduced. This parameter can be viewed as a dual to the covering radius. It is exactly determined for the cyclic and dihedral groups. In this paper, we consider the group U_{6n} as a subgroup of S_{2n+3} and obtain its remoteness. We show that the remoteness of the permutation code U_{6n} is $2n+2$. Moreover, it is proved that the covering radius of U_{6n} is also $2n+2$.

1. Introduction

Let S_n be the symmetric group acting on the set $\{1, \ldots, n\}$. We can consider any element $\sigma \in S_n$ in the list form $\sigma(1)\ \sigma(2)\ \cdots\ \sigma(n)$. The distance between any two elements σ and τ of S_n, called the Hamming distance, is given by

$$d(\sigma, \tau) = |\{1 \leq i \leq n \mid \sigma(i) \neq \tau(i)\}|.$$

It follows that d is a metric on S_n such that $0 \leq d(\sigma, \tau) \leq n$ and $d(\sigma, \tau) \neq 1$. This metric, as we know, was first considered by Farahat [7]. The function d is invariant under left and right translation, i.e., $d(\sigma, \tau) = d(\upsilon\sigma, \upsilon\tau) = d(\sigma\upsilon, \tau\upsilon)$ for any $\sigma, \tau, \upsilon \in S_n$. Denote by $\mathrm{Fix}(\sigma)$ the set of the fixed points of

MSC(2010): 94B60; 05A05; 05B40; 20B05.

Keywords: permutation code, permutation array, remoteness, group U_{6n}.

Received: 29 Jun 2017, Accepted: 03 Dec 2017.

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σ. Thus, $d(\sigma, \tau) = n - |\mathrm{Fix}(\sigma\tau^{-1})|$. The weight $\mathrm{wt}(\sigma)$ of a permutation σ is defined to be the number of non-fixed points of σ, i.e., $\mathrm{wt}(\sigma) = d(1, \sigma)$, where 1 is the identity permutation. Any nonempty subset $\mathcal{C} \subseteq S_n$ is said to be a permutation code of length n over $\{1, 2, \ldots, n\}$. The elements of $\mathcal{C}$ are called codewords. The minimum distance $d(\mathcal{C})$ of $\mathcal{C}$ is the minimum distance between any two distinct codewords in $\mathcal{C}$, i.e., $d(\mathcal{C}) = \min_{\sigma \neq \tau \in \mathcal{C}} d(\sigma, \tau)$. When $\mathcal{C}$ is a subgroup of S_n, it is easy to see that

$$d(\mathcal{C}) = n - \max_{1 \neq \sigma \in \mathcal{C}} |\mathrm{Fix}(\sigma)|,$$

which the right-hand side of the equality is known as the minimum degree of the group $\mathcal{C}$. If the permutation code $\mathcal{C}$ is of length n that contains M codewords and has the minimum distance d, we will refer to it as an (n, M, d)-permutation code. The numbers n, M and d are called the parameters of the code. Notice that M is the size of $\mathcal{C}$, that is, $|\mathcal{C}| = M$. We can associate an $(M \times n)$-array A to the any (n, M, d)-permutation code $\mathcal{C}$ such that the rows of A are the codewords. Any symbol of $1, 2, \ldots, n$ occurs exactly in one entry of every row and any two distinct rows disagree in at least d columns. The array A is also called an (n, M, d)-permutation array (PA). For example, S_n is itself an $(n, n!, 2)$-permutation code, the alternating group A_n is an $(n, n!/2, 3)$-permutation code and the cyclic subgroup $\langle (12 \cdots n) \rangle$ is an (n, n, n)-permutation code. For further results, see [1, 2, 5].

For any $\sigma \in S_n$ and any integer $r \geq 0$, the ball of radius r centered at σ, denoted by $B_r(\sigma)$, is the set $\{\tau \in S_n \mid d(\sigma, \tau) \leq r\}$. The diameter of $\mathcal{C}$ is the maximum distance between any two codewords in $\mathcal{C}$:

$$\delta(\mathcal{C}) = \max_{\sigma, \tau \in \mathcal{C}} d(\sigma, \tau).$$

The radius of $\mathcal{C}$, denoted by $\rho(\mathcal{C})$, is the minimum radius of a ball centered at a codeword needed to cover $\mathcal{C}$:

$$\rho(\mathcal{C}) = \min_{\sigma \in \mathcal{C}} \max_{\tau \in \mathcal{C}} d(\sigma, \tau).$$

Again, if $\mathcal{C} \subseteq S_n$ is a subgroup then $\delta(\mathcal{C}) = \rho(\mathcal{C}) = n - \min_{\sigma \in \mathcal{C}} |\mathrm{Fix}(\sigma)|$. The covering radius of the code $\mathcal{C}$ is defined to be the minimum radius such that the balls centered around the codewords cover the whole S_n:

$$\mathrm{cr}(\mathcal{C}) = \max_{x \in S_n} \min_{\sigma \in \mathcal{C}} d(x, \sigma).$$

For the covering radius of permutation codes, see [4]. Recently, Cameron and Gadouleau [3] interoduced a new parameter of a code, called the remoteness, which can be interpreted as a dual to the covering radius. The remoteness of $\mathcal{C}$ is the minimum radius of a single ball that covers the whole $\mathcal{C}$:

$$r(\mathcal{C}) = \min_{x \in S_n} \max_{\sigma \in \mathcal{C}} d(x, \sigma).$$

It is noticeable that we have the following equivalent definitions for the mentioned parameters:

$$\delta(\mathcal{C}) = \min\{r \mid \mathcal{C} \subseteq B_r(\sigma) \text{ for any } \sigma \in \mathcal{C}\},$$

$$\rho(\mathcal{C}) = \min\{r \mid \mathcal{C} \subseteq B_r(\sigma) \text{ for some } \sigma \in \mathcal{C}\},$$

$$\mathrm{cr}(\mathcal{C}) = \min\{r \mid S_n \subseteq \cup_{\sigma \in \mathcal{C}} B_r(\sigma)\},$$

$$r(\mathcal{C}) = \min\{r \mid \mathcal{C} \subseteq B_r(\sigma) \text{ for some } \sigma \in S_n\}.$$

Moreover, there are the following inequalities [3, 8]:

$$\rho(\mathcal{C}) \leq \delta(\mathcal{C}) \leq 2\rho(\mathcal{C}),$$

$$\delta(\mathcal{C})/2 \leq r(\mathcal{C}) \leq \rho(\mathcal{C}),$$

$$\rho(S_n) \leq r(\mathcal{C}) + \mathrm{cr}(\mathcal{C}) \leq \rho(S_n) + \delta(S_n),$$

and if $\mathcal{C}$ is neither a singleton nor S_n then $r(\mathcal{C}) + \mathrm{cr}(\mathcal{C}) \geq n+1$. For example, if $\mathcal{C} = \{\sigma\} \subseteq S_n$ is a singleton then $\delta(\mathcal{C}) = \rho(\mathcal{C}) = r(\mathcal{C}) = 0$. Moreover, $\mathrm{cr}(\mathcal{C})$ is 0 if $n = 1$ and n otherwise. Also, $\delta(S_n) = \rho(S_n) = r(S_n) = n$ and $\mathrm{cr}(S_n) = 0$.

An automorphism of a code $\mathcal{C}$ is any permutation of the coordinate positions that maps codewords to codewords. The set of all the automorphisms of $\mathcal{C}$ is a group and is denoted by $Aut(\mathcal{C})$. Clearly, $Aut(\mathcal{C}) \leq S_n$ if the length of $\mathcal{C}$ is n. In fact, the automorphism group of any permutation code of a group G is G.

In [3], the authors considered the cyclic and dihedral groups as permutation codes and obtained their remoteness. In this paper, motivated by [3], we consider the permutation group U_{6n} and focus on its remoteness. We show that $r(U_{6n}) = 2n+2$ if U_{6n} is considered as a subgroup of S_{2n+3}. Moreover, it is proved that minimum distance, diameter, radius and covering radius of U_{6n} are 3, $2n+3$, $2n+3$ and $2n+2$, respectively.

2. The remoteness of some permutation codes

The following results in this section deal with the remoteness of some permutation groups. Firstly, it is clear that the remoteness of a permutation code of length n is at most n. If

$$\sigma(i_1) = j_1, \sigma(i_2) = j_2, \ldots, \sigma(i_k) = j_k$$

for all codewords $\sigma \in \mathcal{C}$ then by translation we can assume that $i_1 = j_1, i_2 = j_2, \ldots, i_k = j_k$. So, $r(\mathcal{C})$ is unchanged by restriction to $\{1, \ldots, n\} \setminus \{i_1, \ldots, i_k\}$. Moreover, for computing the remoteness of any pair of permutations, it is sufficient to consider pairs of the form $\{1, \sigma\}$ by translation and it is shown that the remoteness depends on the cycle structure of σ. Notice that if $\mathcal{C} \subseteq \mathcal{D}$ then $r(\mathcal{C}) \leq r(\mathcal{D})$.

Theorem 2.1. [3, Proposition 4] *Let σ be a permutation, $d = d(1, \sigma)$ is even and $\mathcal{C} = \{1, \sigma\}$. Suppose that the cycle lengths $l_1, l_2, \ldots, l_k$ of σ, where $l_i \geq 2$, can be ordered in a way that there exists s such that $\sum_{i=1}^{s} l_i = \sum_{i=s+1}^{k} l_i = d/2$. Then $r(\mathcal{C}) = d/2$. Otherwise, $r(\mathcal{C}) = \lfloor d/2 \rfloor + 1$.*

Theorem 2.2. [3, Theorem 1] *Let $G = \langle a \rangle$. Suppose that $a \in S_n$ is a product of k distinct cycles with no fixed points. Then*

$$r(G) = \begin{cases} n-k & \text{if } a \text{ is an even permutation,} \\ n-k+1 & \text{if } a \text{ is an odd permutation.} \end{cases}$$

It is noticeable that the parameters of a code are not intrinsic but they depend on the permutation representation. For example, the cyclic group of order six can be generated by (1 2 3 4 5 6) or (1 2)(3 4 5) in S_6. By Theorem 2.2, in the first case the remoteness is 6 and in the second is 5.

Theorem 2.3. [3, Proposition 13] *Consider the dihedral group of order $2n$, i.e. D_{2n}, as a subgroup of S_n. Then*

$$r(D_{2n}) = \begin{cases} n-1 & \text{if } n \equiv 1 \text{ or } 5 \pmod 6, \\ n & \text{otherwise.} \end{cases}$$

Theorem 2.4. [3, Proposition 4] *A transitive group G has remoteness $n-1$ if and only if it has covering radius $n-1$. Otherwise, the remoteness of G is n.*

A latin square of order n is an $n \times n$ array with entries from the set $\{1, 2, \ldots, n\}$ so that each entry appears exactly once in any row and any column. A latin square is called cyclic if the first row contains $1, 2, \ldots, n$ in increasing order and each row is derived from the previous one by a cyclic shift to the left. A $k \times n$ Latin rectangle ($k \leq n$) is a $k \times n$ array with entries from $\{1, 2, \ldots, n\}$ such that each entry be exactly once in any row and at most once in any column. Notice that the rows of a latin square of order n define a code of length n and the minimum distance n.

Theorem 2.5. [3, Proposition 7] *If $\mathcal{C}$ is a $k \times n$ latin rectangle then $r(\mathcal{C}) \geq n - \lfloor n/k \rfloor$. Moreover, if $k \equiv 0 \pmod 2$, $n \equiv k \pmod 0$ and $\mathcal{C}$ consists of the first k rows of the cyclic latin square of order n then $r(\mathcal{C}) \geq n - \frac{n}{k} + 1$.*

3. Remoteness of the group U_{6n}

In this section, we consider the group U_{6n} as a permutation code and study its remoteness and other parameters.

Let $n \geq 1$ be an integer. The group U_{6n} is a group of order $6n$ generated by two elements a and b in which the relations $a^{2n} = b^3 = 1$ and $a^{-1}ba = b^{-1}$ between generators are hold, i.e.

$$U_{6n} = \langle a, b \mid a^{2n} = b^3 = 1, a^{-1}ba = b^{-1} \rangle.$$

Clearly, the subgroup $\langle b \rangle$ is normal in U_{6n} and $U_{6n} = \langle b \rangle \rtimes \langle a \rangle \cong \mathbb{Z}_3 \rtimes \mathbb{Z}_{2n}$. It means that U_{6n} is the semidirect product of a cyclic group of order 3 by a cyclic group of order $2n$ and we can write

$$U_{6n} = \{a^r, a^r b, a^r b^2 \mid 0 \leq r \leq 2n-1\}. \tag{1}$$

Notice that $U_6 \cong S_3$.

Let

$$\sigma = (1\ 2\ \cdots\ 2n)(2n+1\ 2n+2)$$

and

$$\tau = (2n+1\ 2n+2\ 2n+3)$$

be two permutations in the symmetric group S_{2n+3}. It is easily seen that $\sigma^{2n} = 1$, $\tau^3 = 1$ and $\sigma^{-1}\tau\sigma\tau = 1$. Then, as noted in [6], the mappings $a \mapsto (1\ 2\ \cdots\ 2n)(2n+1\ 2n+2)$ and $b \mapsto (2n+1\ 2n+2\ 2n+3)$ show that we can embed U_{6n} in S_{2n+3}. So, $U_{6n} = \langle \sigma, \tau \rangle$ and it is possible to consider U_{6n} as a subgroup of S_{2n+3}. Now, U_{6n} is a permutation code of length $2n+3$ and size $6n$.

For further works, let M be the $\big((6n) \times (2n+3)\big)$-array associated to the permutation code U_{6n}. Using (1), it is easy to see that

$$M = \left(\begin{array}{cccc|ccc}
1 & 2 & \cdots & 2n & 2n+1 & 2n+2 & 2n+3 \\
2 & 3 & \cdots & 1 & 2n+2 & 2n+1 & 2n+3 \\
\cline{5-7}
\vdots & \vdots & & \vdots & & \vdots & \\
\cline{5-7}
2n-1 & 2n & \cdots & 2n-2 & 2n+1 & 2n+2 & 2n+3 \\
2n & 1 & \cdots & 2n-1 & 2n+2 & 2n+1 & 2n+3 \\
\hline
1 & 2 & \cdots & 2n & 2n+2 & 2n+3 & 2n+1 \\
2 & 3 & \cdots & 1 & 2n+1 & 2n+3 & 2n+2 \\
\cline{5-7}
\vdots & \vdots & & \vdots & & \vdots & \\
\cline{5-7}
2n-1 & 2n & \cdots & 2n-2 & 2n+2 & 2n+3 & 2n+1 \\
2n & 1 & \cdots & 2n-1 & 2n+1 & 2n+3 & 2n+2 \\
\hline
1 & 2 & \cdots & 2n & 2n+3 & 2n+1 & 2n+2 \\
2 & 3 & \cdots & 1 & 2n+3 & 2n+2 & 2n+1 \\
\cline{5-7}
\vdots & \vdots & & \vdots & & \vdots & \\
\cline{5-7}
2n-1 & 2n & \cdots & 2n-2 & 2n+3 & 2n+1 & 2n+2 \\
2n & 1 & \cdots & 2n-1 & 2n+3 & 2n+2 & 2n+1
\end{array}\right).$$

It is clear that

$$(2) \qquad m_{ij} = \begin{cases} (i+j-2 \bmod 2n)+1 & \text{if } 1 \leq i \leq 2n, \\ (i+j-2n-2 \bmod 2n)+1 & \text{if } 2n+1 \leq i \leq 4n, \\ (i+j-4n-2 \bmod 2n)+1 & \text{if } 4n+1 \leq i \leq 6n, \end{cases}$$

for any $1 \leq i \leq 6n$ and $1 \leq j \leq 2n$. Here, the first $2n$ columns is consist of all of the cyclic shifts of the permutation $(1\ 2\ \cdots\ 2n)$ in which the rows $2n+1$ through $6n$ are constructed by repetition of the first $2n$ rows. For the last three columns, as it is seen, each of the blocks

$$\begin{pmatrix} 2n+1 & 2n+2 \\ 2n+2 & 2n+1 \\ 2n+3 & 2n+3 \end{pmatrix}^T, \quad \begin{pmatrix} 2n+2 & 2n+1 \\ 2n+3 & 2n+3 \\ 2n+1 & 2n+2 \end{pmatrix}^T, \quad \begin{pmatrix} 2n+3 & 2n+3 \\ 2n+1 & 2n+2 \\ 2n+2 & 2n+1 \end{pmatrix}^T,$$

are repeated n times. Set A, B and C be $(2n) \times (2n+3)$ submatrices that consist of the first, the second and the third $2n$ rows of M. Then,

$$M = \begin{pmatrix} A \\ \hline B \\ \hline C \end{pmatrix}.$$

In fact, $(i+1)$-th rows of A, B and C $(0 \le i \le 2n-1)$ are correspond to the elements a^i, a^ib and a^ib^2 from the group U_{6n}, respectively. It is easily seen that

$$d(a^ib^j, a^kb^l) = \begin{cases} 3 & \text{if } i = k, \\ 2n+2 & \text{if } i \not\equiv k \pmod 2, \\ 2n & \text{if } i \neq k, i \equiv k \pmod 2,\ j = l, \\ 2n+3 & \text{if } i \neq k, i \equiv k \pmod 2,\ j \neq l, \end{cases}$$

for any two distinct elements $a^ib^j, a^kb^l \in U_{6n}$, where $0 \le i, k \le 2n-1$ and $0 \le j, l \le 2$. So, $\delta(U_{6n}) = \rho(U_{6n}) = 2n+3$ and the minimum distance of U_{6n} is 3.

Theorem 3.1. *The remoteness of the permutation code U_{6n} is $2n+2$.*

Proof. Firstly, as noted in above, we have

$$\langle \underbrace{(1\ 2 \cdots\ 2n)(2n+1\ 2n+2)}_{\sigma} \rangle \leqslant U_{6n} \leqslant S_{2n+3}.$$

On the other hand, Theorem 2.2 shows that $r(\sigma) = (2n+2) - 2 = 2n$. So,

$$2n \le r(U_{6n}) \le 2n+3.$$

Now, we complete the proof by the following three steps.

Step 1: Let $\pi = \pi_1\ \pi_2\ \cdots\ \pi_{2n+3}$ be an arbitrary permutation in S_{2n+3} and recall that the rows of M are the codewords of U_{6n}. Denote by m_{ij} the entry in row i and column j of the matrix M and let $m_1, m_2, \ldots, m_{6n}$ be the rows of M. In fact, m_i is the permutation $m_{i1}\ m_{i2}\ \cdots\ m_{i(2n+3)}$. Consider the sum of distances between π and the rows of M, i.e. $\sum_{i=1}^{6n} d(\pi, m_i)$, which is equal to $\sum_{i=1}^{6n} \sum_{j=1}^{2n+3} d(\pi_j, m_{ij})$. Notice that $d(\pi_j, m_{ij})$ is 0 if $\pi_j = m_{ij}$ and 1 otherwise. We can find a lower bound for this sum by counting the number of different entries in any column. It is easy to see that if $1 \le j \le 2n$ then each of the numbers $1, 2, \ldots, 2n$ exactly occurs 3 times at column j, and if

$2n+1 \leq j \leq 2n+3$ then column j contains each of the numbers $2n+1, 2n+2, 2n+3$ exactly $2n$ times. Thus,

$$\sum_{i=1}^{6n} d(\pi_j, m_{ij}) \geq 3(2n-1) = 6n-3$$

for all $1 \leq j \leq 2n$ and

$$\sum_{i=1}^{6n} d(\pi_j, m_{ij}) \geq 2(2n) = 4n$$

for $j = 2n+1, 2n+2, 2n+3$. Now, we have

$$\begin{aligned}
\sum_{i=1}^{6n} d(\pi, m_i) &= \sum_{i=1}^{6n} \sum_{j=1}^{2n+3} d(\pi_j, m_{ij}) \\
&= \sum_{j=1}^{2n} \sum_{i=1}^{6n} d(\pi_j, m_{ij}) + \sum_{j=2n+1}^{2n+3} \sum_{i=1}^{6n} d(\pi_j, m_{ij}) \\
&\geq 2n(6n-3) + 3(4n) \\
&= 12n^2 + 6n. \qquad (3)
\end{aligned}$$

So, there exists $1 \leq i \leq 6n$ such that $d(\pi, m_i) \geq \lceil (12n^2 + 6n)/(6n) \rceil = 2n+1$. This implies that $r(U_{6n}) \geq 2n+1$.

Step 2: We want to show that $r(U_{6n}) \geq 2n+2$. Suppose to the contrary that there exsists $\pi \in S_{2n+3}$ such that for all i, $d(\pi, m_i) \leq 2n+1$. Now, (3) implies that $d(\pi, m_i) = 2n+1$ for all i. Hence, π agrees with each m_i in exactly two positions and this means that π dose not map $\{2n+1, 2n+2, 2n+3\}$ to itself. So, there exists $1 \leq s \leq 3$ such that $\pi_{2n+s} = k$, $\pi_l = 2n+s$, where $1 \leq k, l \leq 2n$. Then, we have

$$\begin{aligned}
12n^2 + 6n &= \sum_{i=1}^{6n} d(\pi, m_i) \\
&= \sum_{i=1}^{6n} \sum_{j=1}^{2n+3} d(\pi_j, m_{ij}) \\
&= \sum_{\substack{j=1 \\ j \neq l}}^{2n} \sum_{i=1}^{6n} d(\pi_j, m_{ij}) + \sum_{i=1}^{6n} d(\pi_l, m_{il}) \\
&\quad + \sum_{\substack{j=2n+1 \\ j \neq 2n+s}}^{2n+3} \sum_{i=1}^{6n} d(\pi_j, m_{ij}) + \sum_{i=1}^{6n} d(\pi_{2n+s}, m_{i(2n+s)}) \\
&\geq \sum_{\substack{j=1 \\ j \neq l}}^{2n} (6n-3) + 6n + \sum_{\substack{j=2n+1 \\ j \neq 2n+s}}^{2n+3} (4n) + 6n \\
&= (2n-1)(6n-3) + 6n + 2(4n) + 6n \\
&= 12n^2 + 8n + 3,
\end{aligned}$$

which is desired contradiction. Thus, $r(U_{6n}) \geq 2n+2$.

Step 3: Finally, we construct a permutation $\pi \in S_{2n+3}$ such that $d(\pi, m_i) \leq 2n+2$ for all m_i. Define $\pi_j \equiv 3j-2 \pmod{2n}$ for any $1 \leq j \leq 2n$. Notice that π is identity on the set $\{2n+1, 2n+2, 2n+3\}$. It is sufficient to show that this permutation agrees with each row of the matrix M in at least one position. By looking to the last three columns of M, it is seen that π agrees with the rows of A, the even rows of B and the even rows of C in at least one position. On the other hand, the first $2n$ columns of B and C are the same and their $(2i-1, j)$-th entry is $j+2i-2$, where $1 \leq i \leq n$ and $1 \leq j \leq 2n$. Now, the equation $3j-2 \equiv j+2i-2 \pmod{2n}$ gives us $j = i$ or $i+n$. In other words, π agrees with the odd rows of B and C in exactly two positions. □

Remark 3.1. Notice that in step 3 of the proof of Theorem 3.1, we can replace σ by the permutations $\pi\tau$ and $\pi\tau^2$, because they are at distance at most $2n+2$ from all codewords. Moreover, if we define $\pi_j \equiv 3j-1 \pmod{2n}$ for any $1 \leq j \leq 2n$ then similar arguments show that the permutations $\pi(2n+1\ 2n+2)$, $\pi(2n+1\ 2n+3)$ and $\pi(2n+2\ 2n+3)$ have the desired property too.

Theorem 3.2. *The covering radius of* U_{6n} *is* $2n+2$.

Proof. It is easily seen that for any π in S_{2n+3} there exsist $1 \leq i \leq 2n$ and $1 \leq j \leq 2n+3$ such that $\pi_j = m_{ij}$, where $M = \left(m_{ij}\right)$ is the array associated to U_{6n}. Hence, $\min_i d(\pi, m_i) \leq 2n+2$.

Now, we construct a permutation $\mu \in S_{2n+3}$ such that $\min_i d(\mu, m_i) = 2n+2$. We define

$$\mu(j) = \begin{cases} 2j & \text{if } 1 \leq j \leq n, \\ 2(j-n)+1 & \text{if } n+1 \leq j \leq 2n-3, \\ 2n+1 & \text{if } j = 2n-2, \\ 2n+2 & \text{if } j = 2n-1, \\ 2n+3 & \text{if } j = 2n, \\ 1 & \text{if } j = 2n+1, \\ 2n-3 & \text{if } j = 2n+2, \\ 2n-1 & \text{if } j = 2n+3. \end{cases}$$

The construction of μ and (2) show that μ is agreement with any m_i in at most one position. So, μ is at a distance of $2n+2$ and $2n+3$ from all codewords of U_{6n}. This completes the proof. □

It is noticeable that we can construct more permutations with the property discussed in the proof of Theorem 3.2 and permutation μ is not unique. Theorems 3.1 and 3.2 show that U_{6n} is a non-transitive permutation group with the same remoteness and covering radius.

Acknowledgements

The authors would like to thank the anonymous referees for carefully reading the paper and specially for providing useful comments which have much improved the quality of this paper. This work is partially supported by the University of Kashan under grant number 572764/1.

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