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THE REMOTENESS OF THE PERMUTATION CODE OF THE GROUP U_{6n}

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Abstract. Recently, a new parameter of a code, referred to as the remoteness, has been introduced. This parameter can be viewed as a dual to the covering radius. It is exactly determined for the cyclic and dihedral groups. In this paper, we consider the group U_{6n} as a subgroup of S_{2n+3} and obtain its remoteness. We show that the remoteness of the permutation code U_{6n} is $2n+2$. Moreover, it is proved that the covering radius of U_{6n} is also $2n+2$.

1. Introduction

Let S_n be the symmetric group acting on the set $\{1,\ldots,n\}$. We can consider any element $\sigma \in S_n$ in the list form $\sigma(1)\ \sigma(2)\ \cdots\ \sigma(n)$. The distance between any two elements σ and τ of S_n, called the Hamming distance, is given by

$$d(\sigma,\tau) = |\{1 \leq i \leq n \mid \sigma(i) \neq \tau(i)\}|.$$

It follows that d is a metric on S_n such that $0 \leq d(\sigma,\tau) \leq n$ and $d(\sigma,\tau) \neq 1$. This metric, as we know, was first considered by Farahat [7]. The function d is invariant under left and right translation, i.e., $d(\sigma,\tau) = d(\upsilon\sigma,\upsilon\tau) = d(\sigma\upsilon,\tau\upsilon)$ for any $\sigma,\tau,\upsilon \in S_n$. Denote by $\mathrm{Fix}(\sigma)$ the set of the fixed points of

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σ. Thus, $d(\sigma,\tau) = n - |\mathrm{Fix}(\sigma\tau^{-1})|$. The weight $\mathrm{wt}(\sigma)$ of a permutation σ is defined to be the number of non-fixed points of σ, i.e., $\mathrm{wt}(\sigma) = d(1,\sigma)$, where 1 is the identity permutation. Any nonempty subset $\mathcal{C} \subseteq S_n$ is said to be a permutation code of length n over $\{1, 2, \ldots, n\}$. The elements of $\mathcal{C}$ are called codewords. The minimum distance $d(\mathcal{C})$ of $\mathcal{C}$ is the minimum distance between any two distinct codewords in $\mathcal{C}$, i.e., $d(\mathcal{C}) = \min_{\sigma\neq\tau\in\mathcal{C}} d(\sigma,\tau)$. When $\mathcal{C}$ is a subgroup of S_n, it is easy to see that

$$d(\mathcal{C}) = n - \max_{1\neq\sigma\in\mathcal{C}} |\mathrm{Fix}(\sigma)|,$$

which the right-hand side of the equality is known as the minimum degree of the group $\mathcal{C}$. If the permutation code $\mathcal{C}$ is of length n that contains M codewords and has the minimum distance d, we will refer to it as an (n, M, d)-permutation code. The numbers n, M and d are called the parameters of the code. Notice that M is the size of $\mathcal{C}$, that is, $|\mathcal{C}| = M$. We can associate an $(M \times n)$-array A to the any (n, M, d)-permutation code $\mathcal{C}$ such that the rows of A are the codewords. Any symbol of $1, 2, \ldots, n$ occurs exactly in one entry of every row and any two distinct rows disagree in at least d columns. The array A is also called an (n, M, d)-permutation array (PA). For example, S_n is itself an $(n, n!, 2)$-permutation code, the alternating group A_n is an $(n, n!/2, 3)$-permutation code and the cyclic subgroup $\langle(12\cdots n)\rangle$ is an (n, n, n)-permutation code. For further results, see [1, 2, 5].

For any $\sigma \in S_n$ and any integer $r \geq 0$, the ball of radius r centered at σ, denoted by $B_r(\sigma)$, is the set $\{\tau \in S_n \mid d(\sigma,\tau) \leq r\}$. The diameter of $\mathcal{C}$ is the maximum distance between any two codewords in $\mathcal{C}$:

$$\delta(\mathcal{C}) = \max_{\sigma,\tau\in\mathcal{C}} d(\sigma,\tau).$$

The radius of $\mathcal{C}$, denoted by $\rho(\mathcal{C})$, is the minimum radius of a ball centered at a codeword needed to cover $\mathcal{C}$:

$$\rho(\mathcal{C}) = \min_{\sigma\in\mathcal{C}} \max_{\tau\in\mathcal{C}} d(\sigma,\tau).$$

Again, if $\mathcal{C} \subseteq S_n$ is a subgroup then $\delta(\mathcal{C}) = \rho(\mathcal{C}) = n - \min_{\sigma\in\mathcal{C}} |\mathrm{Fix}(\sigma)|$. The covering radius of the code $\mathcal{C}$ is defined to be the minimum radius such that the balls centered around the codewords cover the whole S_n:

$$\mathrm{cr}(\mathcal{C}) = \max_{x\in S_n} \min_{\sigma\in\mathcal{C}} d(x,\sigma).$$

For the covering radius of permutation codes, see [4]. Recently, Cameron and Gadouleau [3] interoduced a new parameter of a code, called the remoteness, which can be interpreted as a dual to the covering radius. The remoteness of $\mathcal{C}$ is the minimum radius of a single ball that covers the whole $\mathcal{C}$:

$$r(\mathcal{C}) = \min_{x\in S_n} \max_{\sigma\in\mathcal{C}} d(x,\sigma).$$

It is noticeable that we have the following equivalent definitions for the mentioned parameters:

$$\delta(\mathcal{C}) = \min\{r \mid \mathcal{C} \subseteq B_r(\sigma) \text{ for any } \sigma \in \mathcal{C}\},$$

$$\rho(\mathcal{C}) = \min\{r \mid \mathcal{C} \subseteq B_r(\sigma) \text{ for some } \sigma \in \mathcal{C}\},$$

$$\mathrm{cr}(\mathcal{C}) = \min\{r \mid S_n \subseteq \cup_{\sigma\in\mathcal{C}} B_r(\sigma)\},$$

$$r(\mathcal{C}) = \min\{r \mid \mathcal{C} \subseteq B_r(\sigma) \text{ for some } \sigma \in S_n\}.$$

Moreover, there are the following inequalities [3, 8]:

$$\rho(\mathcal{C}) \leq \delta(\mathcal{C}) \leq 2\rho(\mathcal{C}),$$

$$\delta(\mathcal{C})/2 \leq r(\mathcal{C}) \leq \rho(\mathcal{C}),$$

$$\rho(S_n) \leq r(\mathcal{C}) + \mathrm{cr}(\mathcal{C}) \leq \rho(S_n) + \delta(S_n),$$

and if $\mathcal{C}$ is neither a singleton nor S_n then $r(\mathcal{C}) + \mathrm{cr}(\mathcal{C}) \geq n + 1$. For example, if $\mathcal{C} = \{\sigma\} \subseteq S_n$ is a singleton then $\delta(\mathcal{C}) = \rho(\mathcal{C}) = r(\mathcal{C}) = 0$. Moreover, $\mathrm{cr}(\mathcal{C})$ is 0 if $n = 1$ and n otherwise. Also, $\delta(S_n) = \rho(S_n) = r(S_n) = n$ and $\mathrm{cr}(S_n) = 0$.

An automorphism of a code $\mathcal{C}$ is any permutation of the coordinate positions that maps codewords to codewords. The set of all the automorphisms of $\mathcal{C}$ is a group and is denoted by $Aut(\mathcal{C})$. Clearly, $Aut(\mathcal{C}) \leq S_n$ if the length of $\mathcal{C}$ is n. In fact, the automorphism group of any permutation code of a group G is G.

In [3], the authors considered the cyclic and dihedral groups as permutation codes and obtained their remoteness. In this paper, motivated by [3], we consider the permutation group U_{6n} and focus on its remoteness. We show that $r(U_{6n}) = 2n + 2$ if U_{6n} is considered as a subgroup of S_{2n+3}. Moreover, it is proved that minimum distance, diameter, radius and covering radius of U_{6n} are 3, $2n + 3$, $2n + 3$ and $2n + 2$, respectively.

2. The remoteness of some permutation codes

The following results in this section deal with the remoteness of some permutation groups. Firstly, it is clear that the remoteness of a permutation code of length n is at most n. If

$$\sigma(i_1) = j_1, \sigma(i_2) = j_2, \ldots, \sigma(i_k) = j_k$$

for all codewords $\sigma \in \mathcal{C}$ then by translation we can assume that $i_1 = j_1, i_2 = j_2, \ldots, i_k = j_k$. So, $r(\mathcal{C})$ is unchanged by restriction to $\{1, \ldots, n\} \setminus \{i_1, \ldots, i_k\}$. Moreover, for computing the remoteness of any pair of permutations, it is sufficient to consider pairs of the form $\{1, \sigma\}$ by translation and it is shown that the remoteness depends on the cycle structure of σ. Notice that if $\mathcal{C} \subseteq \mathcal{D}$ then $r(\mathcal{C}) \leq r(\mathcal{D})$.

Theorem 2.1. [3, Proposition 4] *Let σ be a permutation, $d = d(1, \sigma)$ is even and $\mathcal{C} = \{1, \sigma\}$. Suppose that the cycle lengths $l_1, l_2, \ldots, l_k$ of σ, where $l_i \geq 2$, can be ordered in a way that there exists s such that $\sum_{i=1}^{s} l_i = \sum_{i=s+1}^{k} l_i = d/2$. Then $r(\mathcal{C}) = d/2$. Otherwise, $r(\mathcal{C}) = \lfloor d/2 \rfloor + 1$.*

Theorem 2.2. [3, Theorem 1] *Let $G = \langle a \rangle$. Suppose that $a \in S_n$ is a product of k distinct cycles with no fixed points. Then*

$$r(G) = \begin{cases} n-k & \text{if } a \text{ is an even permutation,} \\ n-k+1 & \text{if } a \text{ is an odd permutation.} \end{cases}$$

It is noticeable that the parameters of a code are not intrinsic but they depend on the permutation representation. For example, the cyclic group of order six can be generated by (1 2 3 4 5 6) or (1 2)(3 4 5) in S_6. By Theorem 2.2, in the first case the remoteness is 6 and in the second is 5.

Theorem 2.3. [3, Proposition 13] *Consider the dihedral group of order $2n$, i.e. D_{2n}, as a subgroup of S_n. Then*

$$r(D_{2n}) = \begin{cases} n-1 & \text{if } n \equiv 1 \text{ or } 5 \pmod 6, \\ n & \text{otherwise.} \end{cases}$$

Theorem 2.4. [3, Proposition 4] *A transitive group G has remoteness $n-1$ if and only if it has covering radius $n-1$. Otherwise, the remoteness of G is n.*

A latin square of order n is an $n \times n$ array with entries from the set $\{1, 2, \ldots, n\}$ so that each entry appears exactly once in any row and any column. A latin square is called cyclic if the first row contains $1, 2, \ldots, n$ in increasing order and each row is derived from the previous one by a cyclic shift to the left. A $k \times n$ Latin rectangle ($k \leq n$) is a $k \times n$ array with entries from $\{1, 2, \ldots, n\}$ such that each entry be exactly once in any row and at most once in any column. Notice that the rows of a latin square of order n define a code of length n and the minimum distance n.

Theorem 2.5. [3, Proposition 7] *If $\mathcal{C}$ is a $k \times n$ latin rectangle then $r(\mathcal{C}) \geq n - \lfloor n/k \rfloor$. Moreover, if $k \equiv 0 \pmod 2$, $n \equiv k \pmod 0$ and $\mathcal{C}$ consists of the first k rows of the cyclic latin square of order n then $r(\mathcal{C}) \geq n - \frac{n}{k} + 1$.*

3. Remoteness of the group U_{6n}

In this section, we consider the group U_{6n} as a permutation code and study its remoteness and other parameters.

Let $n \geq 1$ be an integer. The group U_{6n} is a group of order $6n$ generated by two elements a and b in which the relations $a^{2n} = b^3 = 1$ and $a^{-1}ba = b^{-1}$ between generators are hold, i.e.

$$U_{6n} = \langle a, b \mid a^{2n} = b^3 = 1, a^{-1}ba = b^{-1} \rangle.$$

Clearly, the subgroup $\langle b \rangle$ is normal in U_{6n} and $U_{6n} = \langle b \rangle \rtimes \langle a \rangle \cong \mathbb{Z}_3 \rtimes \mathbb{Z}_{2n}$. It means that U_{6n} is the semidirect product of a cyclic group of order 3 by a cyclic group of order $2n$ and we can write

$$U_{6n} = \{a^r, a^r b, a^r b^2 \mid 0 \leq r \leq 2n-1\}. \tag{1}$$

Notice that $U_6 \cong S_3$.

Let

$$\sigma = (1\ 2\ \cdots\ 2n)(2n+1\ 2n+2)$$

and

$$\tau = (2n+1\ 2n+2\ 2n+3)$$

be two permutations in the symmetric group S_{2n+3}. It is easily seen that $\sigma^{2n} = 1$, $\tau^3 = 1$ and $\sigma^{-1}\tau\sigma\tau = 1$. Then, as noted in [6], the mappings $a \mapsto (1\ 2\ \cdots\ 2n)(2n+1\ 2n+2)$ and $b \mapsto (2n+1\ 2n+2\ 2n+3)$ show that we can embed U_{6n} in S_{2n+3}. So, $U_{6n} = \langle \sigma, \tau \rangle$ and it is possible to consider U_{6n} as a subgroup of S_{2n+3}. Now, U_{6n} is a permutation code of length $2n+3$ and size $6n$.

For further works, let M be the $\big((6n) \times (2n+3)\big)$-array associated to the permutation code U_{6n}. Using (1), it is easy to see that

$$M = \left(\begin{array}{cccc|ccc}
1 & 2 & \cdots & 2n & 2n+1 & 2n+2 & 2n+3 \\
2 & 3 & \cdots & 1 & 2n+2 & 2n+1 & 2n+3 \\ \cline{5-7}
\vdots & \vdots & & \vdots & \multicolumn{3}{c}{\vdots} \\ \cline{5-7}
2n-1 & 2n & \cdots & 2n-2 & 2n+1 & 2n+2 & 2n+3 \\
2n & 1 & \cdots & 2n-1 & 2n+2 & 2n+1 & 2n+3 \\ \hline
1 & 2 & \cdots & 2n & 2n+2 & 2n+3 & 2n+1 \\
2 & 3 & \cdots & 1 & 2n+1 & 2n+3 & 2n+2 \\ \cline{5-7}
\vdots & \vdots & & \vdots & \multicolumn{3}{c}{\vdots} \\ \cline{5-7}
2n-1 & 2n & \cdots & 2n-2 & 2n+2 & 2n+3 & 2n+1 \\
2n & 1 & \cdots & 2n-1 & 2n+1 & 2n+3 & 2n+2 \\ \hline
1 & 2 & \cdots & 2n & 2n+3 & 2n+1 & 2n+2 \\
2 & 3 & \cdots & 1 & 2n+3 & 2n+2 & 2n+1 \\ \cline{5-7}
\vdots & \vdots & & \vdots & \multicolumn{3}{c}{\vdots} \\ \cline{5-7}
2n-1 & 2n & \cdots & 2n-2 & 2n+3 & 2n+1 & 2n+2 \\
2n & 1 & \cdots & 2n-1 & 2n+3 & 2n+2 & 2n+1
\end{array}\right).$$

It is clear that

$$m_{ij} = \begin{cases} (i+j-2 \bmod 2n)+1 & \text{if } 1 \le i \le 2n, \\ (i+j-2n-2 \bmod 2n)+1 & \text{if } 2n+1 \le i \le 4n, \\ (i+j-4n-2 \bmod 2n)+1 & \text{if } 4n+1 \le i \le 6n, \end{cases} \tag{2}$$

for any $1 \le i \le 6n$ and $1 \le j \le 2n$. Here, the first $2n$ columns is consist of all of the cyclic shifts of the permutation $(1\ 2\ \cdots\ 2n)$ in which the rows $2n+1$ through $6n$ are constructed by repetition of the first $2n$ rows. For the last three columns, as it is seen, each of the blocks

$$\begin{pmatrix} 2n+1 & 2n+2 \\ 2n+2 & 2n+1 \\ 2n+3 & 2n+3 \end{pmatrix}^T, \quad \begin{pmatrix} 2n+2 & 2n+1 \\ 2n+3 & 2n+3 \\ 2n+1 & 2n+2 \end{pmatrix}^T, \quad \begin{pmatrix} 2n+3 & 2n+3 \\ 2n+1 & 2n+2 \\ 2n+2 & 2n+1 \end{pmatrix}^T,$$

are repeated n times. Set A, B and C be $(2n) \times (2n+3)$ submatrices that consist of the first, the second and the third $2n$ rows of M. Then,

$$M = \begin{pmatrix} A \\ \hline B \\ \hline C \end{pmatrix}.$$

In fact, $(i+1)$-th rows of A, B and C $(0 \leq i \leq 2n-1)$ are correspond to the elements a^i, a^ib and a^ib^2 from the group U_{6n}, respectively. It is easily seen that

$$d(a^ib^j, a^kb^l) = \begin{cases} 3 & \text{if } i = k, \\ 2n+2 & \text{if } i \not\equiv k \pmod 2, \\ 2n & \text{if } i \neq k, i \equiv k \pmod 2,\ j = l, \\ 2n+3 & \text{if } i \neq k, i \equiv k \pmod 2,\ j \neq l, \end{cases}$$

for any two distinct elements $a^ib^j, a^kb^l \in U_{6n}$, where $0 \leq i, k \leq 2n-1$ and $0 \leq j, l \leq 2$. So, $\delta(U_{6n}) = \rho(U_{6n}) = 2n+3$ and the minimum distance of U_{6n} is 3.

Theorem 3.1. *The remoteness of the permutation code U_{6n} is $2n+2$.*

Proof. Firstly, as noted in above, we have

$$\langle \underbrace{(1\ 2 \cdots\ 2n)(2n+1\ 2n+2)}_{\sigma} \rangle \leqslant U_{6n} \leqslant S_{2n+3}.$$

On the other hand, Theorem 2.2 shows that $r(\sigma) = (2n+2) - 2 = 2n$. So,

$$2n \leq r(U_{6n}) \leq 2n+3.$$

Now, we complete the proof by the following three steps.

Step 1: Let $\pi = \pi_1\ \pi_2\ \cdots\ \pi_{2n+3}$ be an arbitrary permutation in S_{2n+3} and recall that the rows of M are the codewords of U_{6n}. Denote by m_{ij} the entry in row i and column j of the matrix M and let $m_1, m_2, \ldots, m_{6n}$ be the rows of M. In fact, m_i is the permutation $m_{i1}\ m_{i2}\ \cdots\ m_{i(2n+3)}$. Consider the sum of distances between π and the rows of M, i.e. $\sum_{i=1}^{6n} d(\pi, m_i)$, which is equal to $\sum_{i=1}^{6n} \sum_{j=1}^{2n+3} d(\pi_j, m_{ij})$. Notice that $d(\pi_j, m_{ij})$ is 0 if $\pi_j = m_{ij}$ and 1 otherwise. We can find a lower bound for this sum by counting the number of different entries in any column. It is easy to see that if $1 \leq j \leq 2n$ then each of the numbers $1, 2, \ldots, 2n$ exactly occurs 3 times at column j, and if

$2n+1 \leq j \leq 2n+3$ then column j contains each of the numbers $2n+1, 2n+2, 2n+3$ exactly $2n$ times. Thus,

$$\sum_{i=1}^{6n} d(\pi_j, m_{ij}) \geq 3(2n-1) = 6n-3$$

for all $1 \leq j \leq 2n$ and

$$\sum_{i=1}^{6n} d(\pi_j, m_{ij}) \geq 2(2n) = 4n$$

for $j = 2n+1, 2n+2, 2n+3$. Now, we have

$$\begin{aligned}
\sum_{i=1}^{6n} d(\pi, m_i) &= \sum_{i=1}^{6n} \sum_{j=1}^{2n+3} d(\pi_j, m_{ij}) \\
&= \sum_{j=1}^{2n} \sum_{i=1}^{6n} d(\pi_j, m_{ij}) + \sum_{j=2n+1}^{2n+3} \sum_{i=1}^{6n} d(\pi_j, m_{ij}) \\
&\geq 2n(6n-3) + 3(4n) \\
&= 12n^2 + 6n. \qquad (3)
\end{aligned}$$

So, there exists $1 \leq i \leq 6n$ such that $d(\pi, m_i) \geq \lceil (12n^2 + 6n)/(6n) \rceil = 2n+1$. This implies that $r(U_{6n}) \geq 2n+1$.

Step 2: We want to show that $r(U_{6n}) \geq 2n+2$. Suppose to the contrary that there exsists $\pi \in S_{2n+3}$ such that for all i, $d(\pi, m_i) \leq 2n+1$. Now, (3) implies that $d(\pi, m_i) = 2n+1$ for all i. Hence, π agrees with each m_i in exactly two positions and this means that π dose not map $\{2n+1, 2n+2, 2n+3\}$ to itself. So, there exists $1 \leq s \leq 3$ such that $\pi_{2n+s} = k$, $\pi_l = 2n+s$, where $1 \leq k, l \leq 2n$. Then, we have

$$\begin{aligned}
12n^2 + 6n &= \sum_{i=1}^{6n} d(\pi, m_i) \\
&= \sum_{i=1}^{6n} \sum_{j=1}^{2n+3} d(\pi_j, m_{ij}) \\
&= \sum_{\substack{j=1 \\ j \neq l}}^{2n} \sum_{i=1}^{6n} d(\pi_j, m_{ij}) + \sum_{i=1}^{6n} d(\pi_l, m_{il}) \\
&\quad + \sum_{\substack{j=2n+1 \\ j \neq 2n+s}}^{2n+3} \sum_{i=1}^{6n} d(\pi_j, m_{ij}) + \sum_{i=1}^{6n} d(\pi_{2n+s}, m_{i(2n+s)}) \\
&\geq \sum_{\substack{j=1 \\ j \neq l}}^{2n} (6n-3) + 6n + \sum_{\substack{j=2n+1 \\ j \neq 2n+s}}^{2n+3} (4n) + 6n \\
&= (2n-1)(6n-3) + 6n + 2(4n) + 6n \\
&= 12n^2 + 8n + 3,
\end{aligned}$$

which is desired contradiction. Thus, $r(U_{6n}) \geq 2n+2$.

Step 3: Finally, we construct a permutation $\pi \in S_{2n+3}$ such that $d(\pi, m_i) \leq 2n+2$ for all m_i. Define $\pi_j \equiv 3j-2 \pmod{2n}$ for any $1 \leq j \leq 2n$. Notice that π is identity on the set $\{2n+1, 2n+2, 2n+3\}$. It is sufficient to show that this permutation agrees with each row of the matrix M in at least one position. By looking to the last three columns of M, it is seen that π agrees with the rows of A, the even rows of B and the even rows of C in at least one position. On the other hand, the first $2n$ columns of B and C are the same and their $(2i-1, j)$-th entry is $j + 2i - 2$, where $1 \leq i \leq n$ and $1 \leq j \leq 2n$. Now, the equation $3j - 2 \equiv j + 2i - 2 \pmod{2n}$ gives us $j = i$ or $i + n$. In other words, π agrees with the odd rows of B and C in exactly two positions. □

Remark 3.1. Notice that in step 3 of the proof of Theorem 3.1, we can replace σ by the permutations $\pi\tau$ and $\pi\tau^2$, because they are at distance at most $2n+2$ from all codewords. Moreover, if we define $\pi_j \equiv 3j-1 \pmod{2n}$ for any $1 \leq j \leq 2n$ then similar arguments show that the permutations $\pi(2n{+}1\ 2n{+}2)$, $\pi(2n+1\ 2n+3)$ and $\pi(2n+2\ 2n+3)$ have the desired property too.

Theorem 3.2. *The covering radius of U_{6n} is $2n+2$.*

Proof. It is easily seen that for any π in S_{2n+3} there exsist $1 \leq i \leq 2n$ and $1 \leq j \leq 2n+3$ such that $\pi_j = m_{ij}$, where $M = \left(m_{ij}\right)$ is the array associated to U_{6n}. Hence, $\min_i d(\pi, m_i) \leq 2n+2$.

Now, we construct a permutation $\mu \in S_{2n+3}$ such that $\min_i d(\mu, m_i) = 2n+2$. We define

$$\mu(j) = \begin{cases} 2j & \text{if } 1 \leq j \leq n, \\ 2(j-n)+1 & \text{if } n+1 \leq j \leq 2n-3, \\ 2n+1 & \text{if } j = 2n-2, \\ 2n+2 & \text{if } j = 2n-1, \\ 2n+3 & \text{if } j = 2n, \\ 1 & \text{if } j = 2n+1, \\ 2n-3 & \text{if } j = 2n+2, \\ 2n-1 & \text{if } j = 2n+3. \end{cases}$$

The construction of μ and (2) show that μ is agreement with any m_i in at most one position. So, μ is at a distance of $2n+2$ and $2n+3$ from all codewords of U_{6n}. This completes the proof. □

It is noticeable that we can construct more permutations with the property discussed in the proof of Theorem 3.2 and permutation μ is not unique. Theorems 3.1 and 3.2 show that U_{6n} is a non-transitive permutation group with the same remoteness and covering radius.

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